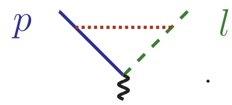
SCET and Collider Physics — Tutorial 3

Scalar SCET — 25. 5. 2016

1. Verify that the Feynman diagram with a soft exchange



indeed gives rise to the soft region loop integral you computed in Tutorial 2.

To get this result, derive first the Feynman rule for the soft-collinear interaction term

$$-i\frac{\lambda}{2!}\int d^4x\, \phi_c^2(x)\phi_s(x_-) \tag{1}$$

in the Lagrangian. Because the soft field gets evaluated at x_-, not all its momentum components are conserved. Due of this subtlety, instead of suppressing momentum conservation δ-functions in the Feynman rules, we must keep track of them. One therefore introduces a momentum integration for *each* internal propagator line in the diagram and supplies each vertex with the appropriate momentum conservation δ-function.

2. In Scalar SCET, we encountered the scalar current operator

$$J_2(x) = \int ds\, dt\, C_2(s,t)\, \phi_c(x+s\bar{n})\phi_{\bar{c}}(x+tn)$$

Compute its matrix element

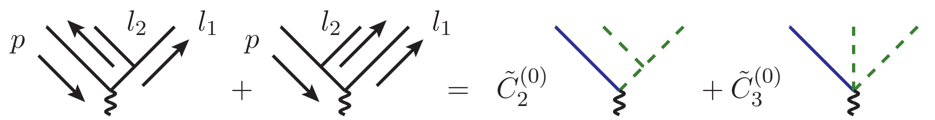
$$\langle p|J_2(0)|l\rangle$$

with a single particle in the initial state with anti-collinear momentum l and a single one in the final state with collinear momentum p. Show that the matrix element yields the Fourier transform of the Wilson coefficient $C_2(s,t)$ and that the Fourier transform only depends on $Q^2 = \bar{n}\cdot p\, n\cdot l$.

3. Consider the SCET current operator with two anti-collinear and one collinear field

$$J_3(x) = \int ds \int dt_1 \int dt_2\, C_3(s,t_1,t_2,\mu)\phi_c(x+s\bar{n})\phi_{\bar{c}}(x+t_1n)\phi_{\bar{c}}(x+t_2n)\,. \tag{2}$$

Determine the Fourier transform of the Wilson coefficient $C_3(s,t_1,t_2,\mu)$ at tree level by performing a matching computation, i.e. by computing (on-shell) matrix element of the current for an external state with one particle with momentum p with collinear scaling and two momenta l_1 ande l_2 with anti-collinear scaling. The relevant diagrammatic equation was shown in the lecture



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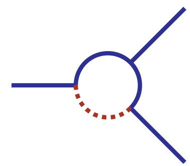
4. When we showed that there are no matching corrections to the terms in the Lagrangian, it was important that the following diagram,



which describes a soft loop correction to the ϕ_c^3 interaction, vanishes. Show that the corresponding diagram is indeed scaleless and vanishes in dimensional regularization.