SCET and Collider Physics — Tutorial 2

Method of regions — 24. 5. 2016

1. Using the method of regions, verify that the leading term in the expansion of the integral I discussed in the lecture is given by

$$I = \int_0^\infty dk \, \frac{k}{(k^2+m^2)\,(k^2+M^2)} = -\frac{1}{M^2}\left[\ln\frac{m}{M} + \mathcal{O}\left(\frac{m^2}{M^2}\right)\right].$$

The following integral is useful

$$\int_0^\infty dx \, \frac{x^a}{(1+x)^b} = \frac{\Gamma(a+1)\Gamma(b-a-1)}{\Gamma(b)}.$$

2. Compute the soft triangle loop integral

$$I_s = i\pi^{-d/2}\mu^{4-d}\int d^dk \, \frac{1}{k^2(n\cdot k\,\bar{n}\cdot p + p^2)(\bar{n}\cdot k\, n\cdot l + l^2)}.$$

For progagators linear in k such as $b = n\cdot k\,\bar{n}\cdot p + p^2$, it is useful to work with modified Feynman parameterizations such as

$$\frac{1}{ab} = \int_0^\infty d\eta \, \frac{1}{(a+\eta b)^2}, \qquad \frac{1}{abc} = \int_0^\infty d\eta_1 \int_0^\infty d\eta_2 \, \frac{2}{(a+\eta_1 b+\eta_2 c)^3}.$$

To perform the momentum integration, one can use the formula

$$\int d^dk \frac{1}{(k^2+2k\cdot Q-M^2)^\alpha} = (-1)^\alpha \frac{i\pi^{\frac{d}{2}}}{(M^2+Q^2)^{\alpha-\frac{d}{2}}}\frac{\Gamma\left(\alpha-\frac{d}{2}\right)}{\Gamma\left(\alpha\right)}.$$

3. Compute the leading term in the expansion of the integral

$$I(a) = \int_0^\infty dt \, \frac{\sin(t)}{(t+a)^2}$$

for small a. To do so, introduce a factor t^ϵ into the integrand and use the method of regions.

To obtain the result, the integral given in Problem 1 as well as the identity

$$\int_0^\infty dt \, t^\alpha \sin(t) = \Gamma(\alpha+1)\cos\frac{\pi\alpha}{2},$$

are useful. This second integral converges for $-2 < \alpha < 0$.