Dark Matter

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Question 0 (Weighing the Universe)

We can estimate the total amount of luminous matter in the Universe, using information about the number density of galaxies and the mass-to light ratio, Υ, for each type of galaxy. The mass-to-light ratio is defined, in Solar units, as $\Upsilon = (M/M_\odot)/(L/L_\odot)$. The mass-to-light ratio is well known for stars in the main branch and stellar evolution inside galaxies can be used to determine this quantity for a whole galaxy. The exact value of Υ depends on the type of galaxy (e.g., for elliptical galaxies $\Upsilon \approx 6.5$, for spiral galaxies $\Upsilon \approx 1-5$, and irregular galaxies have $\Upsilon \approx 1$).

The number of galaxies per volume and per unit luminosity (a quantity referred to as "galaxy luminosity density function", $\Phi(L)$) can be measured on small scales and extrapolated to the whole Universe. An analytical fitting function (Schechter function) can be found that reads

$$\Phi(L) = \frac{\Phi_*}{L_*}\left(\frac{L}{L_*}\right)^{\alpha} e^{-L/L_*} ,$$

with parameters $L_* = 2.53 \times 10^{10}\ L_\odot$, $\Phi_* = 4.1 \times 10^{-3}$ Mpc^{-3}, and $\alpha = -1.25$.

How much mass is concentrated in luminous galaxies (compared to the critical density)? Hint: estimate the mean luminosity density as $\mathcal{L} = \int L\Phi(L)$ and then the matter density multiplying by the mass-to-light ratio $\rho = \mathcal{L}\Upsilon$.

Use $M_\odot = 1.116 \times 10^{57}$ GeV $\rho_c = 8.0992 \times h^2 \times 10^{-47}$ GeV^{-4}, and 1 Mpc$=1.5637 \times 10^{38}$ GeV^{-1}.

Question 1 (Boltzman Equation 1)

In the derivation of Boltzmann's equation for the evolution of the number density of massive species inthe Early Universe we expanded the Liouville operator, $\hat{L}$, that acts on the density distribution function. Prove that for a FRW universe the following relation holds

$$\begin{aligned} \hat{L} &= p^\mu \frac{\partial}{\partial x^\mu} - \Gamma^\mu_{\sigma\rho} p^\sigma p^\rho \frac{\partial}{\partial p^\mu} \\ &= E\frac{\partial}{\partial t} - H|\vec{p}|^2 \frac{\partial}{\partial E} \end{aligned} \tag{1}$$

Question 2 (Boltzman Equation 2)

Later, we performed an integral in momentum space that yielded

$$\frac{g}{(2\pi)^3}\int \frac{d^3\vec{p}}{E}\left[E\frac{\partial f}{\partial t} - H|\vec{p}|^2 \frac{\partial f}{\partial E}\right] = \frac{dn}{dt} + 3Hn \tag{2}$$

Check this result.

Question 2+$\frac{1}{2}$ (Yield at freeze-out)

It is easy to estimate the value of the Yield that we need in order to reproduce the correct DM relic abundance, $\Omega h^2 \approx 0.1$, since

$$\Omega h^2 = \frac{\rho_\chi}{\rho_c} h^2 = \frac{m_\chi n\chi h^2}{\rho_c} = \frac{m_\chi Y_\infty s_0 h^2}{\rho_c} \ , \tag{3}$$

where Y_∞ corresponds to the DM Yield today and s_0 is todays entropy density. We can assume that the Yield did not change since DM freeze-out and therefore

$$\Omega h^2 = \frac{m_\chi Y_f s_0 h^2}{\rho_c} \ . \tag{4}$$

Using the measured value $s_0 = 2970\ \mathrm{cm}^{-3}$, and the value of the critical density $\rho_c = 1.054 \times 10^{-5}\, h^2$ GeV cm^{-3}, as well as Plancks result on the DM relic abundance, estimate the correct range of values of the yield at freeze-out and the approximate values of x_f.

Question 3 (Freeze out of DM particles)

Using Botlzmann equation, expressed in terms of the yield $Y = n/s$, which reads

$$\frac{dY}{dx} = -\frac{\lambda \langle \sigma v \rangle}{x^2} \left(Y^2 - Y_{eq}^2 \right) \ , \tag{5}$$

define the quantity $\Delta_Y \equiv Y - Y_{eq}$ and show that, for non-relativistic particles, the solution can be approximated as

$$\Delta_Y = -\frac{\frac{dY_{eq}}{dx}}{Y_{eq}} \frac{x^2}{2\lambda\langle \sigma v \rangle}, \qquad 1 < x \ll x_f \tag{6}$$

$$\Delta_{Y_\infty} = Y_\infty = \frac{x_f}{\lambda \left(a + \frac{b}{2\, x_f^2} \right)}, \qquad x \gg x_f \tag{7}$$

Question 4 (Dark Matter relic density 1)

Consider a simple model in which the Dark Matter is a Dirac fermion, χ, which only couples to the Standard Model sector through the exchange of the a pseudoscalar particle A. The pseudoscalar A has couplings g_χ to the dark matter and g_b to b quarks as described by the Lagrangian

$$\mathcal{L} = i \left(g_\chi \bar{\chi} \gamma^5 \chi + g_b \bar{b} \gamma^5 b \right) A$$

- Draw the Feynman diagram that corresponds to the pair-annihilation of two dark matter particles into $b\bar{b}$.
- Considering only Dark Matter annihilation into $b\bar{b}$, the annihilation cross section in the Early Universe can be expanded in plane waves as $\langle \sigma v \rangle \approx a_{b\bar{b}} + b_{b\bar{b}}\, x$, with (see e.g, Ref.[2])

$$a_{b\bar{b}} = \frac{1}{m_\chi^2} \left(\frac{N_c}{32\pi} \left(1 - \frac{4m_b^2}{s} \right)^{1/2} \frac{1}{2} \int_{-1}^{1} d\cos\theta_{CM} |\mathcal{M}_{\chi\chi \to bb}|^2 \right)_{s = 4m_\chi^2}$$

Show that to leading order in velocity (i.e., $x = 0$)

$$\langle \sigma v \rangle \approx \frac{3}{2\pi} \frac{(g_\chi g_b)^2 m_\chi^2 \sqrt{1 - m_b^2/m_\chi^2}}{(4m_\chi^2 - m_A^2)^2 + m_A^2 \Gamma_A^2}$$

Remember to average over initial spins and sum over final ones. You will also need the following trace, $\mathrm{Tr}\left[(\not{p}_1 - m_\chi)\gamma^5(\not{p}_2 + m_\chi)\gamma^5\right] = 4(-p_1 \cdot p_2 - m_\chi^2)$.

- Show that if the mediator is a scalar particle instead of a pseudoscalar then $a_{b\bar{b}} = 0$.
- Given a dark matter mass $m_\chi = 100$ GeV and a pseudoscalar mass $m_A = 1000$ GeV, estimate the value of the coupling $g_\chi g_b$ for which the correct relic density is obtained. Neglect the pseudoscalar decay width, Γ_A and use that

$$\Omega_\chi h^2 \approx \frac{3 \times 10^{-10}\ \mathrm{GeV}^{-2}}{\langle \sigma v \rangle}$$

Question 5 (Dark Matter detection 1)

Consider now a scalar Dark Matter model, ϕ, which only couples to the Standard Model sector through the exchange of the Higgs boson. The coupling, $C_{\phi\phi H^0_{SM}}$ (which can be understood as coming from a quartic term $\phi\phi H^0_{SM} H^0_{SM}$) is fixed by imposing that the relic density is correct, $\Omega_\phi h^2 \approx 0.1$, obtaining $C_{\phi\phi H^0_{SM}} \approx 20 GeV$. Compute the prediction for the spin-independent scattering cross-section off protons, $\sigma^{SI}_{\phi-p}$, and compare it with current experimental constraints from LUX and SuperCDMS. Is this cadidate viable or is it excluded if it has a mass $m_\phi < 20$ GeV?

To do this,

- Write down the effective Lagrangian that describes the elastic scattering of ϕ with quarks and express the interaction strength, α_q, in terms of the fundamental coupling $C_{\phi\phi H^0_{SM}}$.
- Assume that the scattering off protons can be computed assuming that the contribution of s quarks is dominant.
- The expression for the scattering cross-section of scalar dark matter can be found, e.g., in Section 3.4 of Ref. [6].

$$\sigma^{SI}_{\phi-p} = \frac{f_p m_p^2}{4\pi(m_\phi + m_p)^2}\ , \tag{8}$$

where

$$\frac{f_p}{m_p} = \sum_{q_i=u,d,s} f^p_{T_{q_i}} \frac{\alpha_{q_i}}{m_{q_i}} + \frac{2}{27} f^p_{TG} \sum_{q_i=c,b,t} \frac{\alpha_{q_i}}{m_{q_i}}\ . \tag{9}$$

We can consider for simplicity that the s quark contribution dominates, and use $f_{T_{q_s}} = 0.229$.

Question 6 (Dark Matter detection 2)

In the previous question we noticed that the predictions for $\sigma^{SI}_{\chi-p}$ exceed the current experimental limits from the direct detection experiments LUX and SuperCDMS. Is there any way in which we can "fix" this model?

- Think about why the annihilation cross section and the scattering cross section are related in the example above. How can we break this relation?
- Consider enlarging the "exotic" sector by including more particles.

Question 7 (Neutrino decoupling 1)

In the Early Universe, neutrinos remain in equilibrium through the process $e^+ + e^- \longleftrightarrow \nu_e + \bar{\nu}_e$. Using that both the electron-positron and neutrino populations are relativistic and therefore their number density scales as $n \sim T^3$, the decoupling temperature of neutrinos can be roughly estimated by equating the annihilation rate $\Gamma = n\langle\sigma v\rangle$ and the Hubble expansion rate $H = \sqrt{8\pi G\rho/3}$. The energy density of the Universe scales as $\rho \sim T^4$. Show that neutrinos decouple at approximately $T \sim 1$ MeV.

Question 8 (Neutrino decoupling 2)

From the question above, we know that when neutrinos decouple, they are still relativistic. The other relativistic species in the thermal bath are electrons, positrons, photons and the three neutrinos and antineutrinos. With this information the relic density of neutrinos in the Universe today can be estimated as a function of the neutrino mass.

To do that, remember that for relativistic species the Yield at equilibrium can be written as $y_{eq} = \frac{45}{2\pi^4}\zeta(3)\frac{g_{eff}}{g_{*s}} \approx 0.278\frac{g_{eff}}{g_{*s}}$.

References

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