Quantum phase transitions in condensed matter:

from
the Higgs boson
to
holography

Higgs Center for Theoretical Physics, Edinburgh,
January 10, 2013

Subir Sachdev

Talk online at sachdev.physics.harvard.edu

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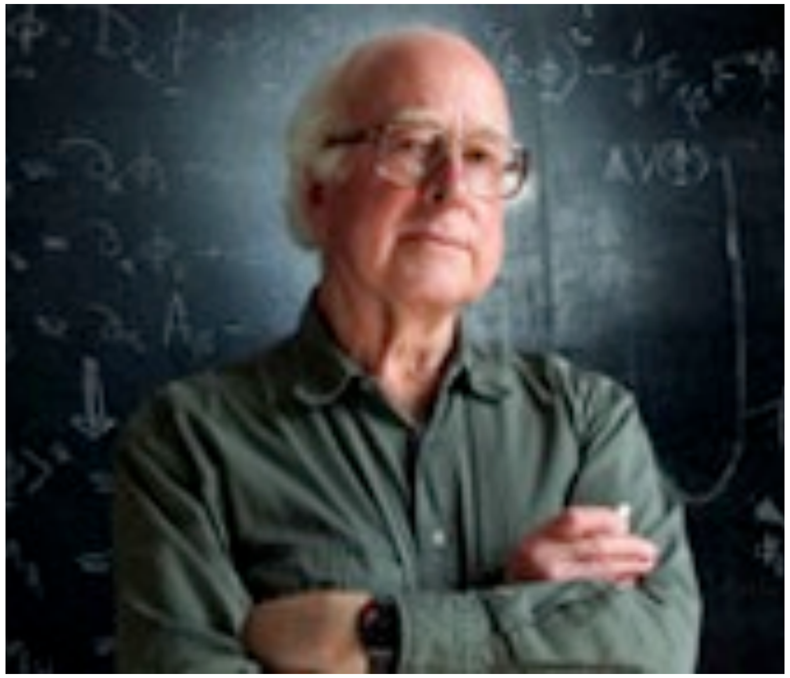
From



Higgs

to

Hawking

Outline

1. Higgs boson in a 3-dimensional antiferromagnet

2. Higgs "pole" near the superfluid-insulator transition in 2 dimensions

3. Quantum criticality and conformal field theories

4. Holography and the quasi-normal modes of black-hole horizons

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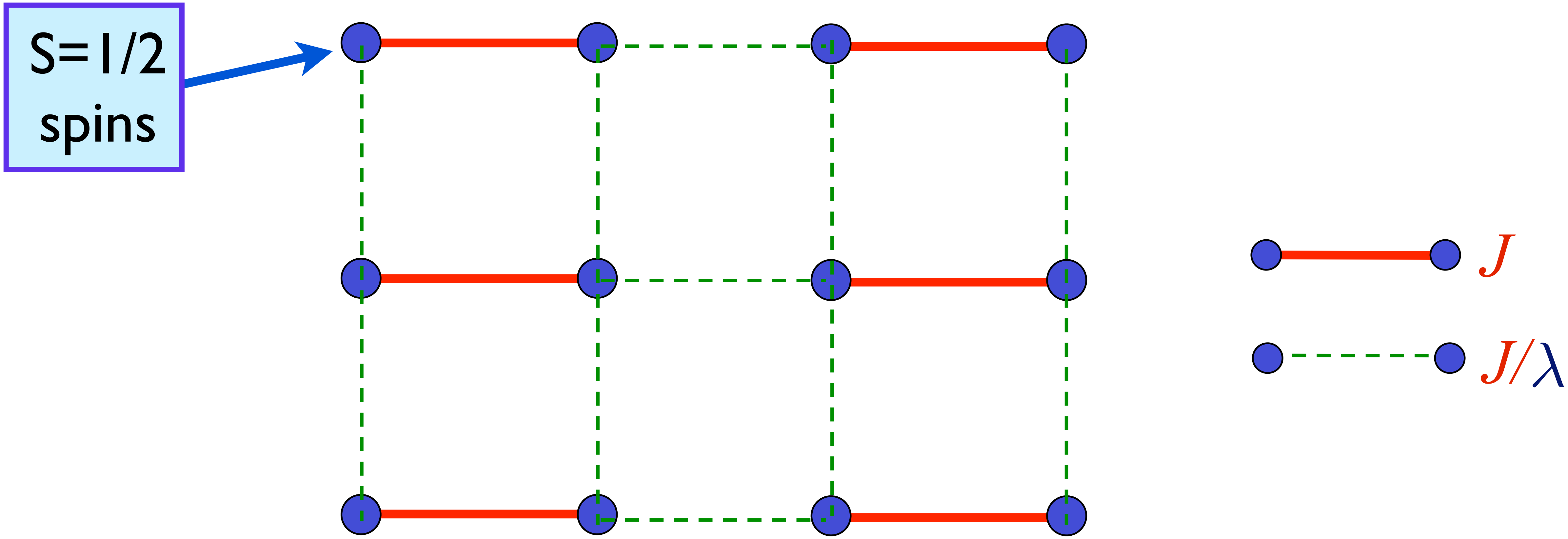
2. Higgs "pole" near the superfluid-insulator transition in 2 dimensions

3. Quantum criticality and conformal field theories

4. Holography and the quasi-normal modes of black-hole horizons

Spinning electrons localized on a cubic lattice

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Examine ground state as a function of λ

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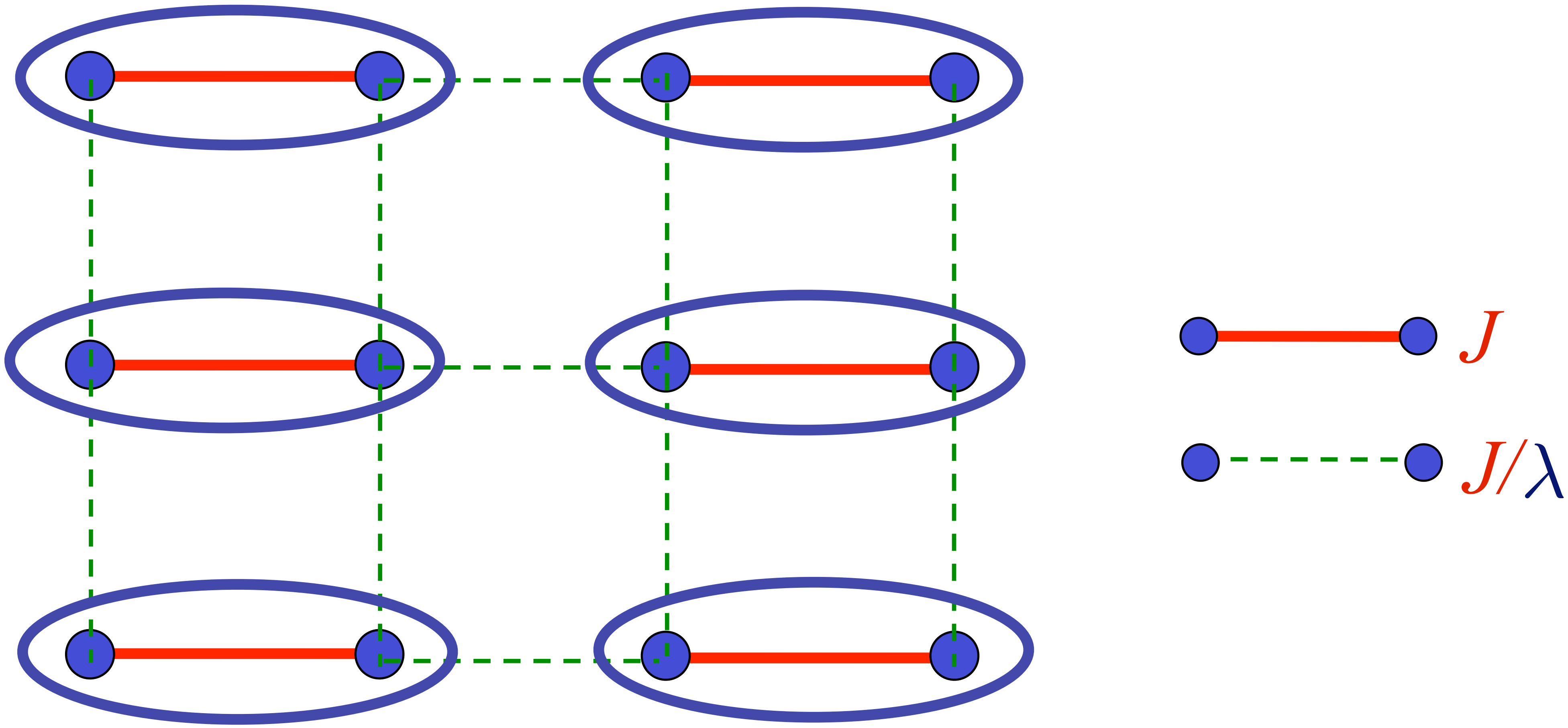

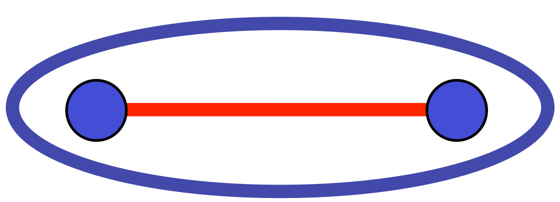

$$= \frac{1}{\sqrt{2}} \Big(\left| \uparrow \downarrow \right\rangle - \left| \downarrow \uparrow \right\rangle \Big)$$

At large λ ground state is a "quantum paramagnet" with spins locked in valence bond singlets

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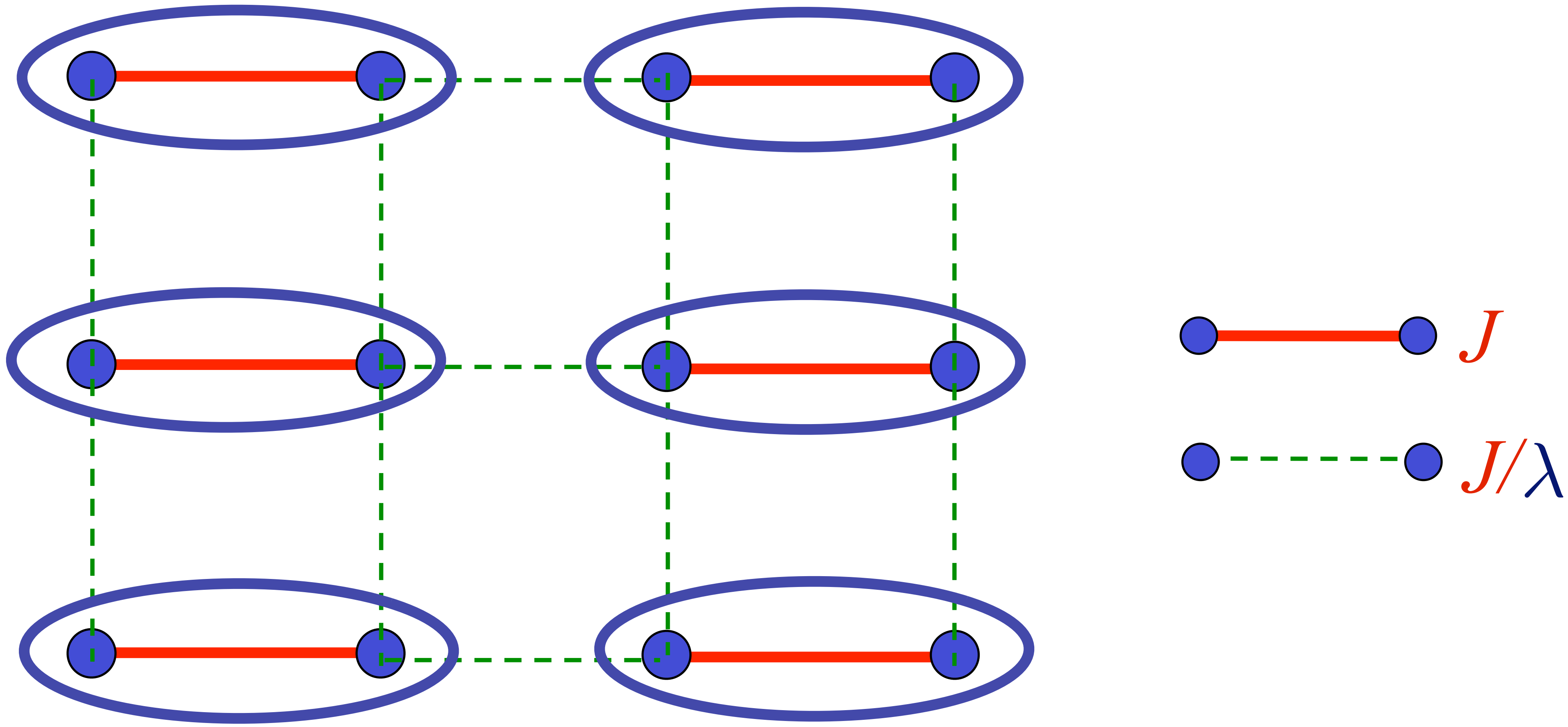

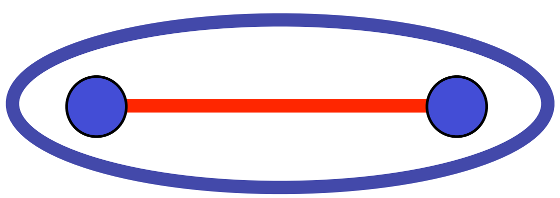

$$= \frac{1}{\sqrt{2}} \left(\left| \uparrow \downarrow \right\rangle - \left| \downarrow \uparrow \right\rangle \right)$$

Nearest-neighor spins are "entangled" with each other.

Spinning electrons localized on a cubic lattice

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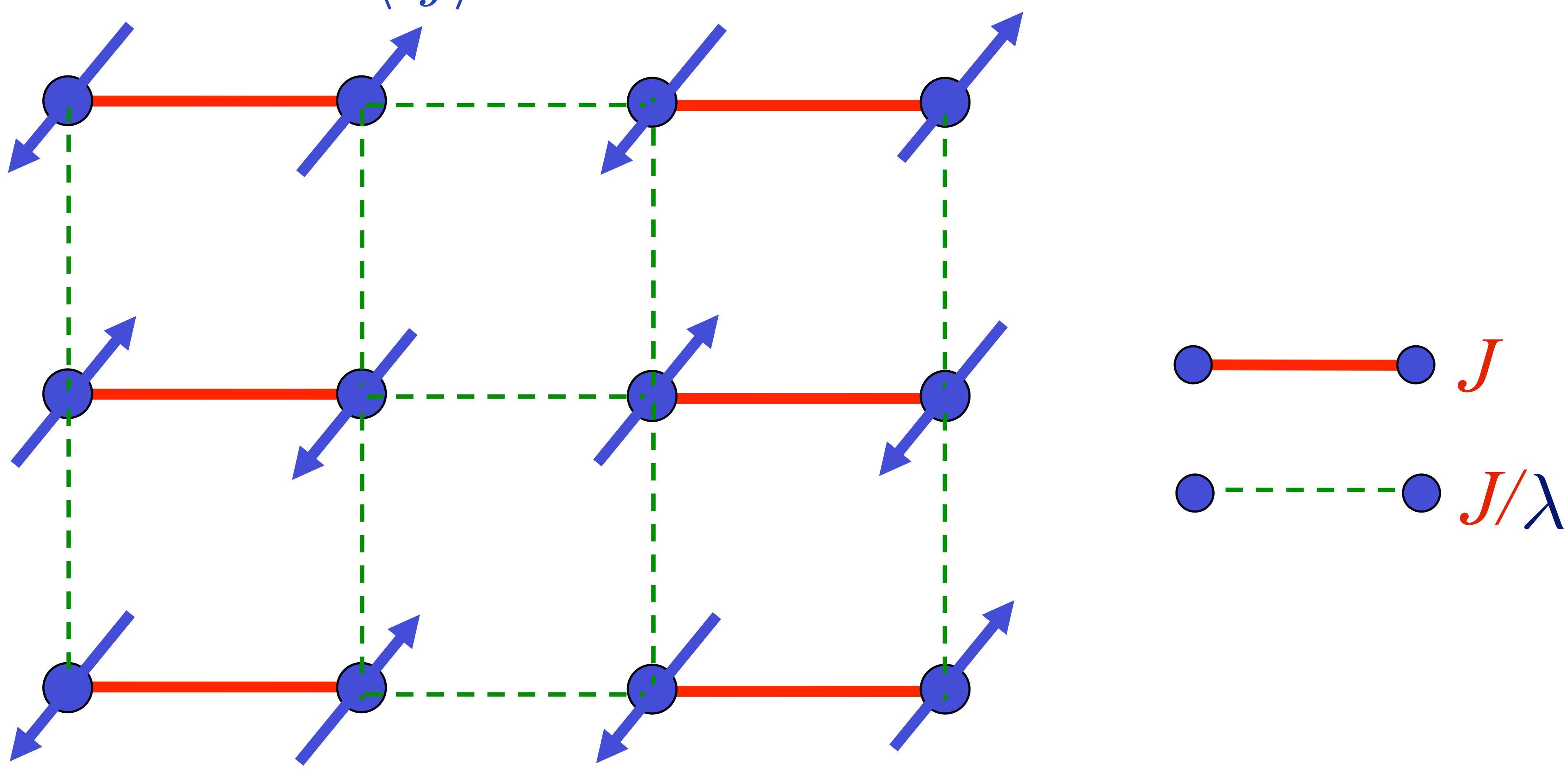
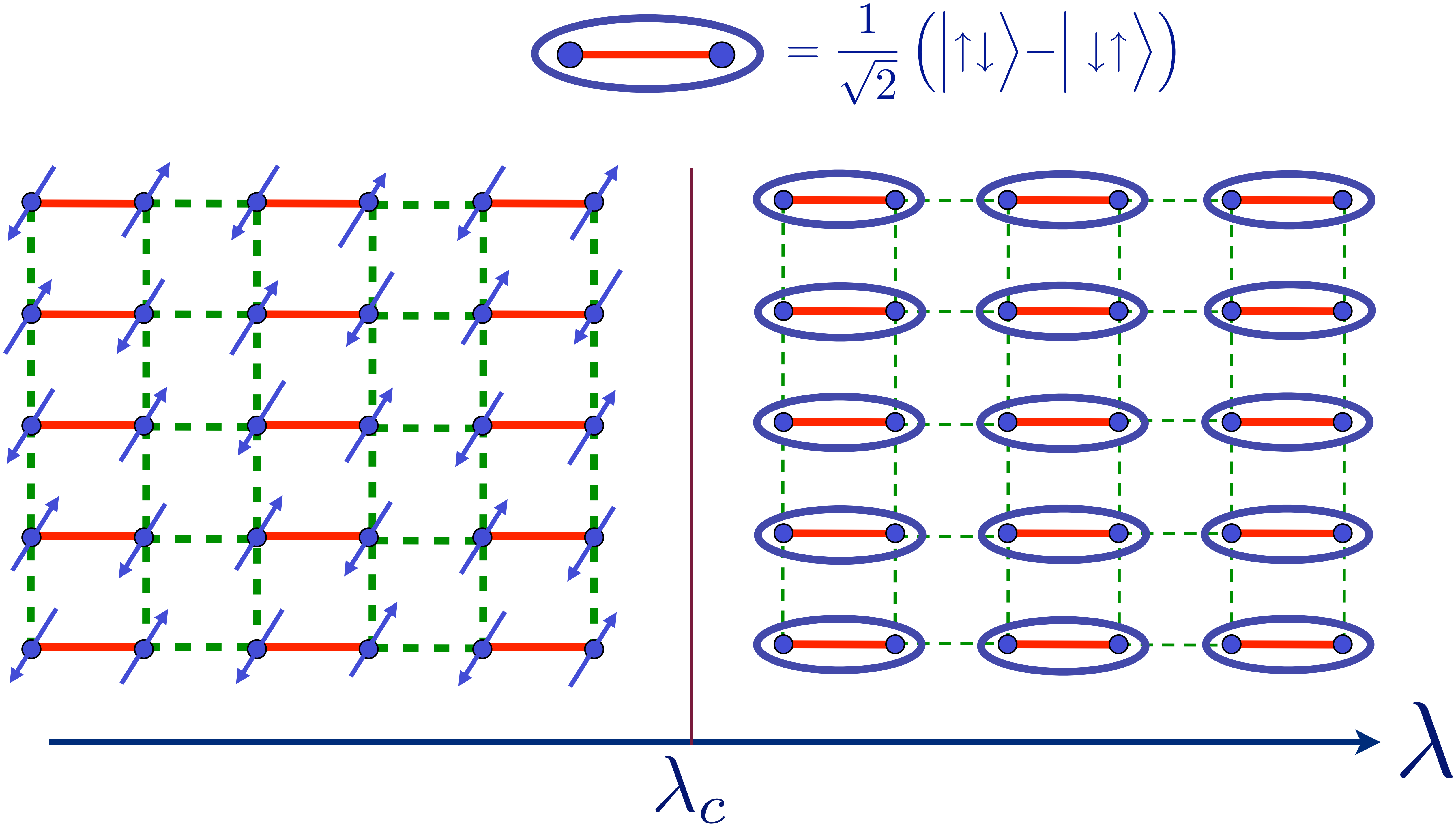
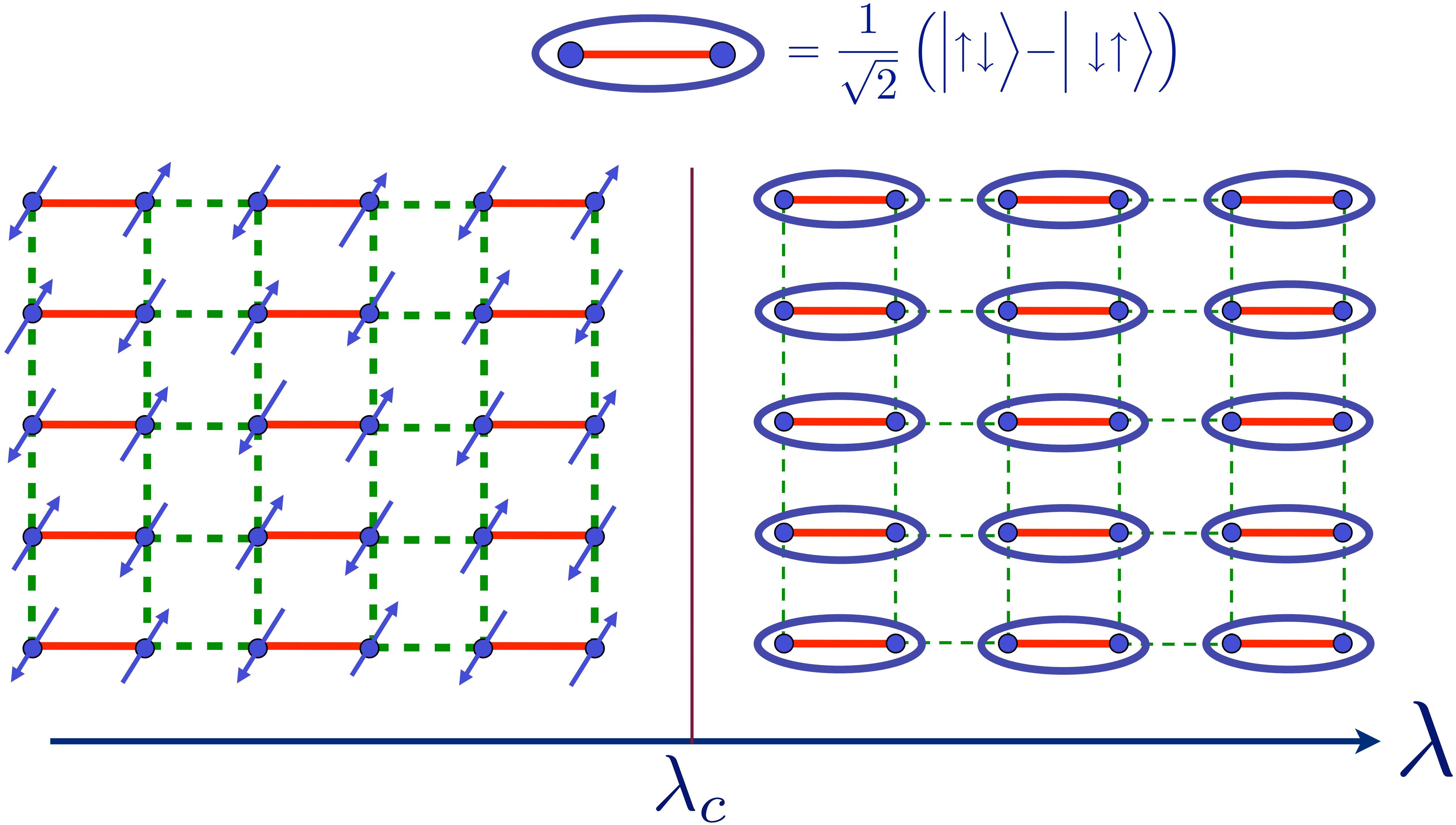


For $\lambda \approx 1$, the ground state has antiferromagnetic ("Néel") order, and the spins align in a checkerboard pattern.

There is a broken O(3) symmetry characterized by an order parameter $\vec{\varphi} \sim (-1)^{i_x + i_y} \vec{S}_i$



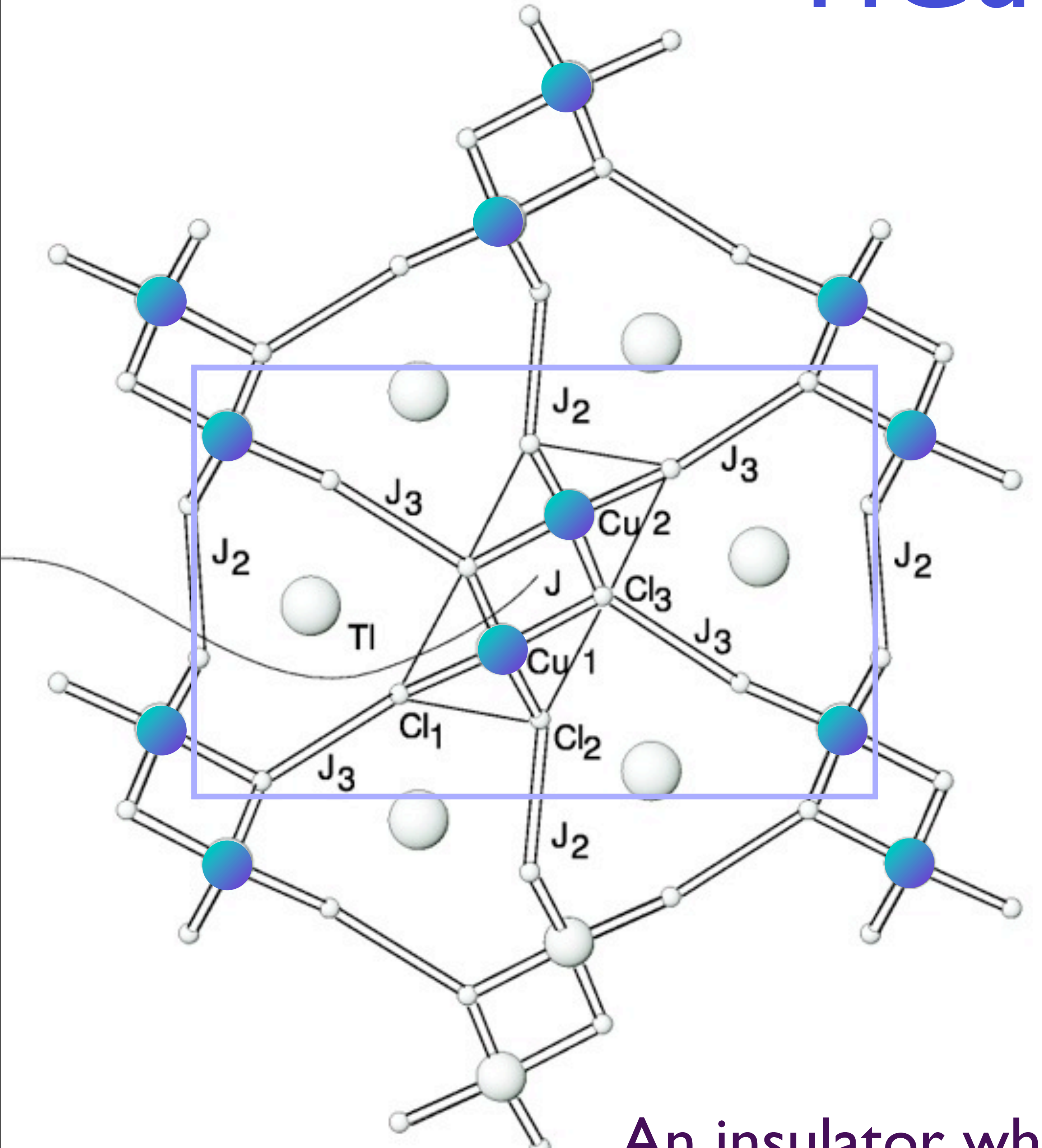
$= \frac{1}{\sqrt{2}}\Big(\big|\uparrow\downarrow\big\rangle - \big|\downarrow\uparrow\big\rangle\Big)$
λ_c
λ

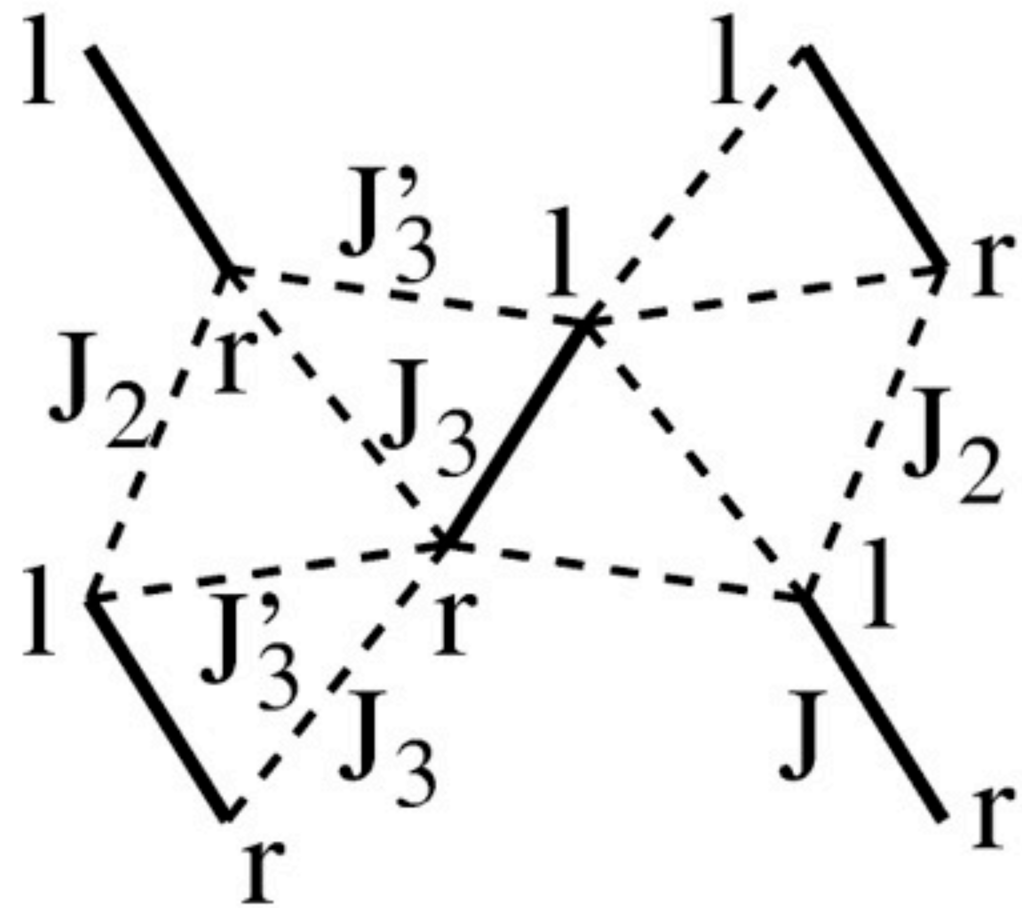


Pressure in $TlCuCl_3$

A. Oosawa, K. Kakurai, T. Osakabe, M. Nakamura, M. Takeda, and H. Tanaka, *Journal of the Physical Society of Japan*, **73**, 1446 (2004).

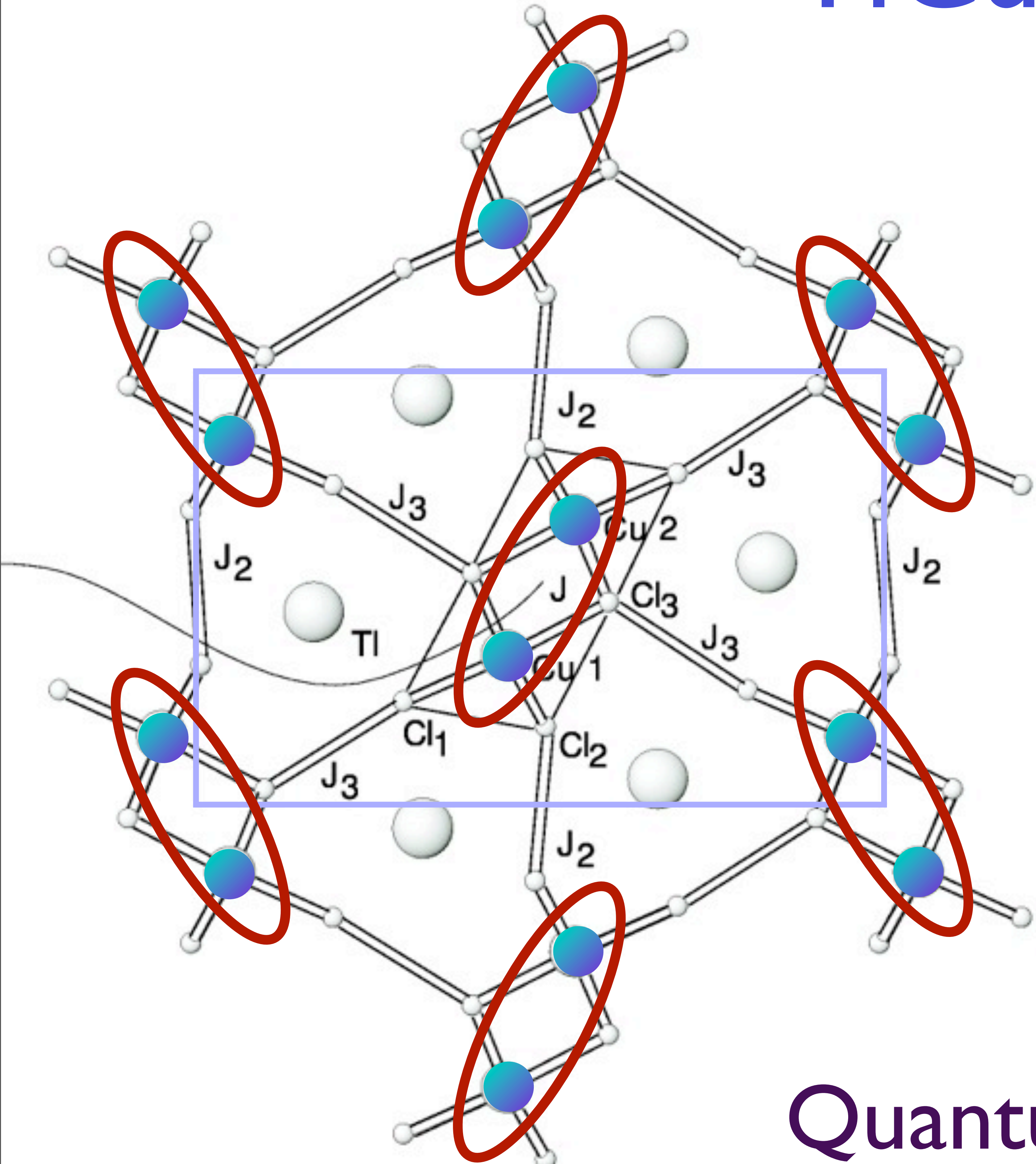
TlCuCl$_3$

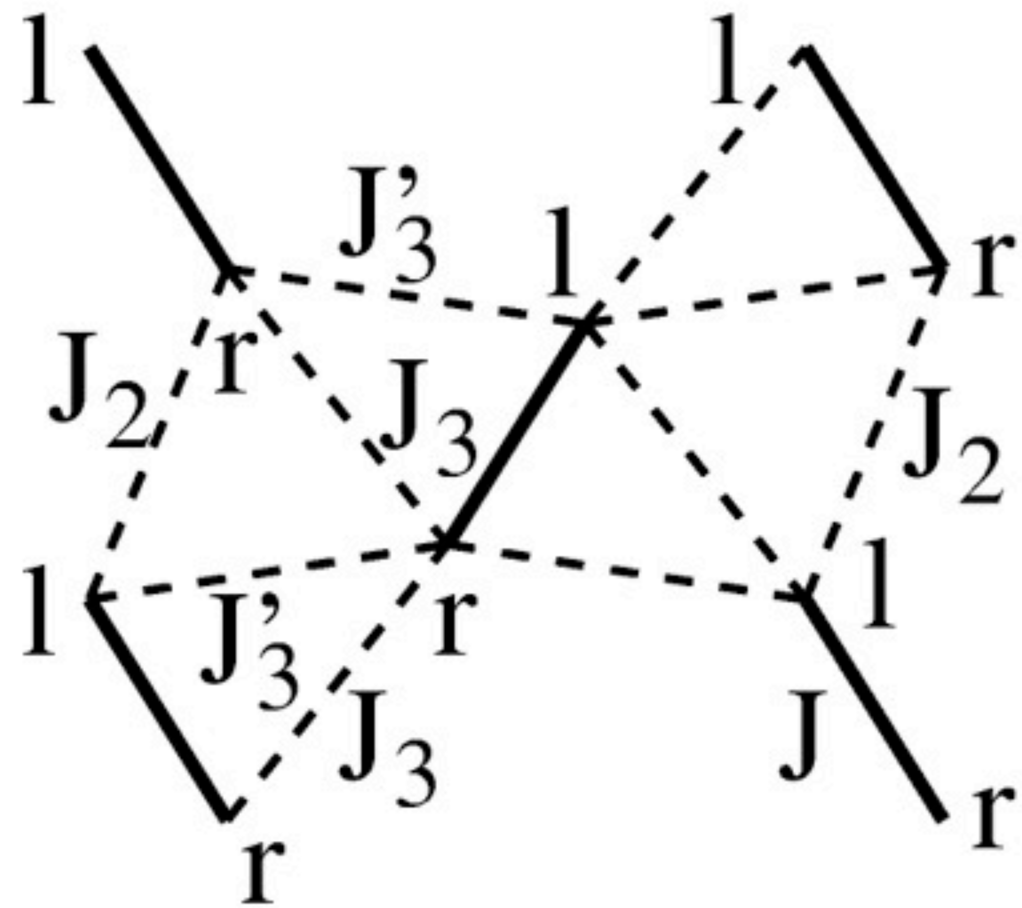




An insulator whose spin susceptibility vanishes exponentially as the temperature T tends to zero.

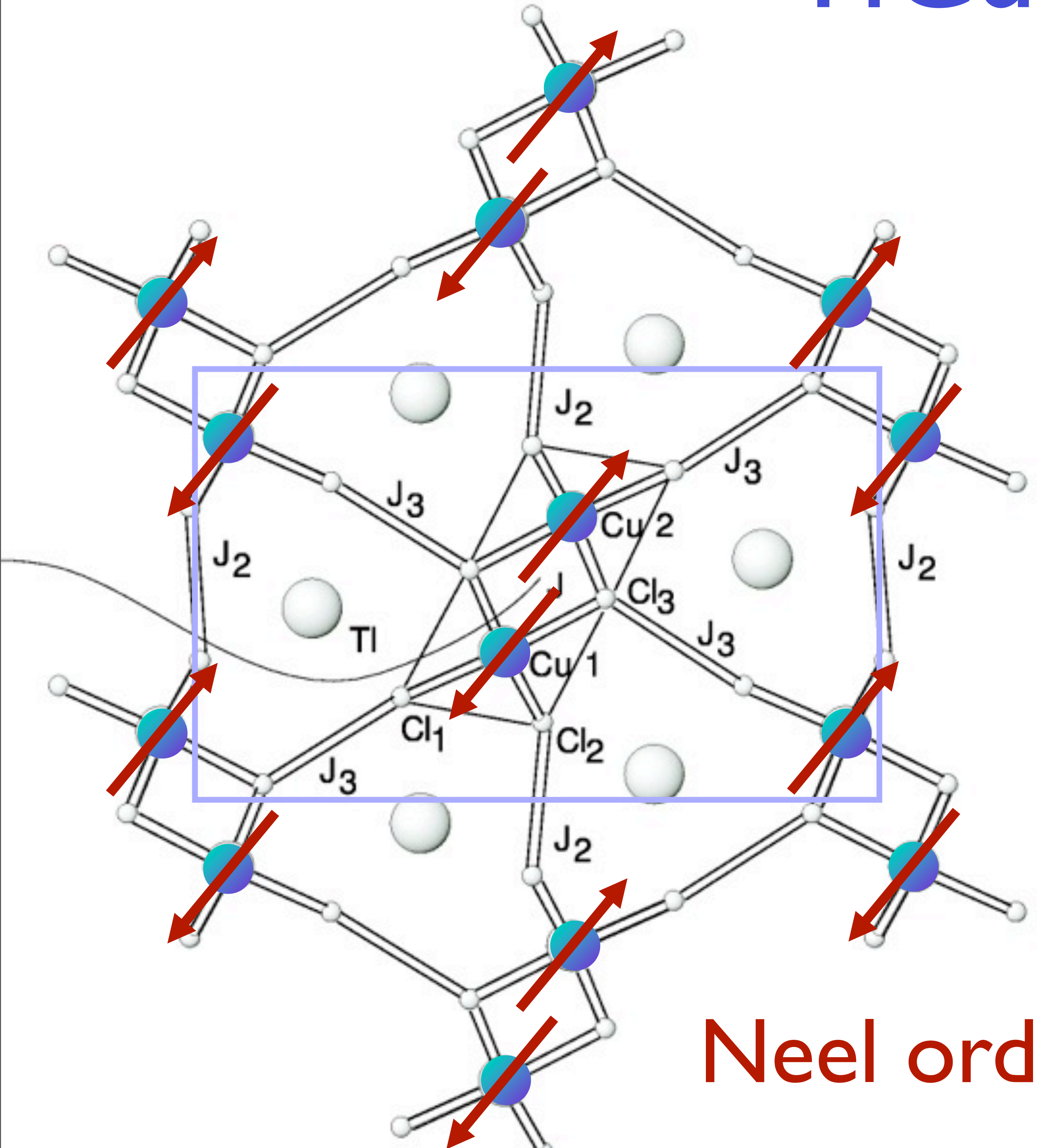
$TlCuCl_3$

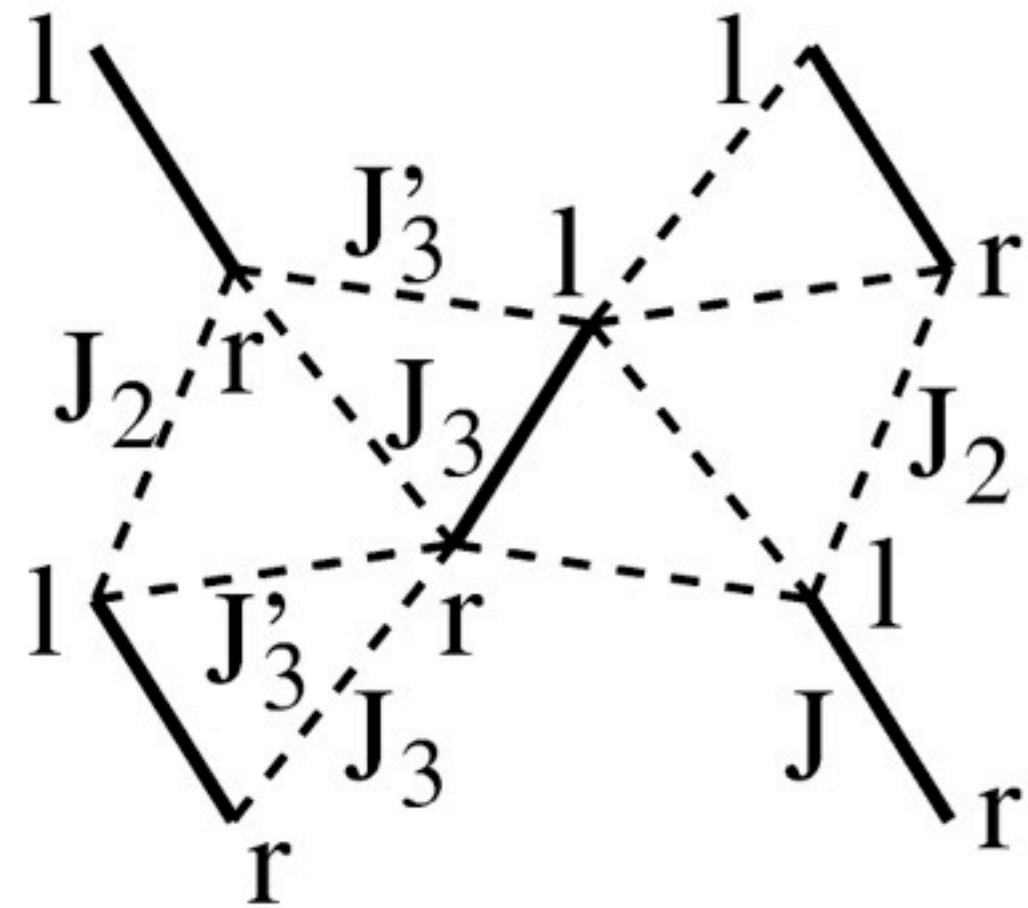




Quantum paramagnet at ambient pressure

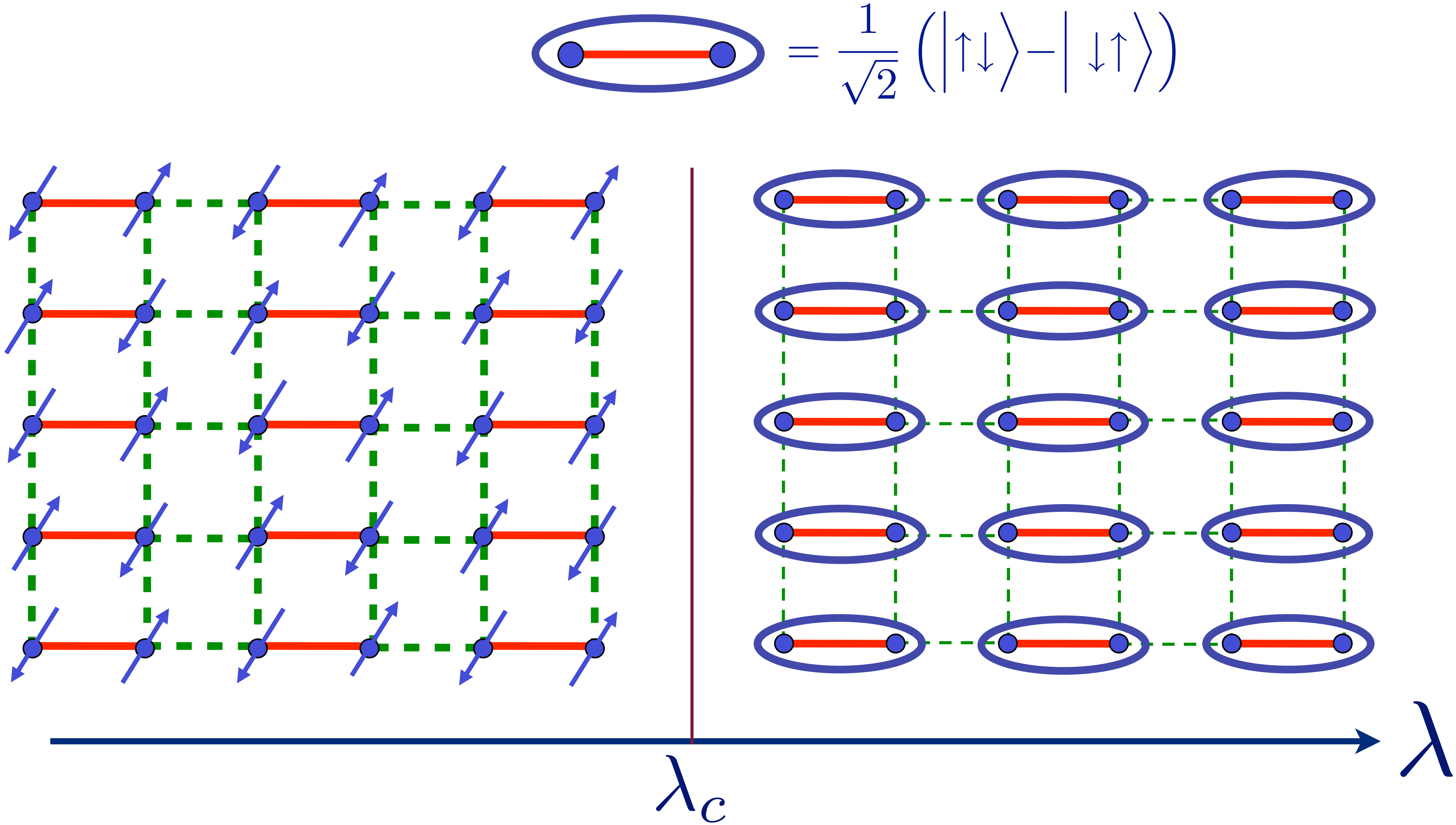
$TlCuCl_3$





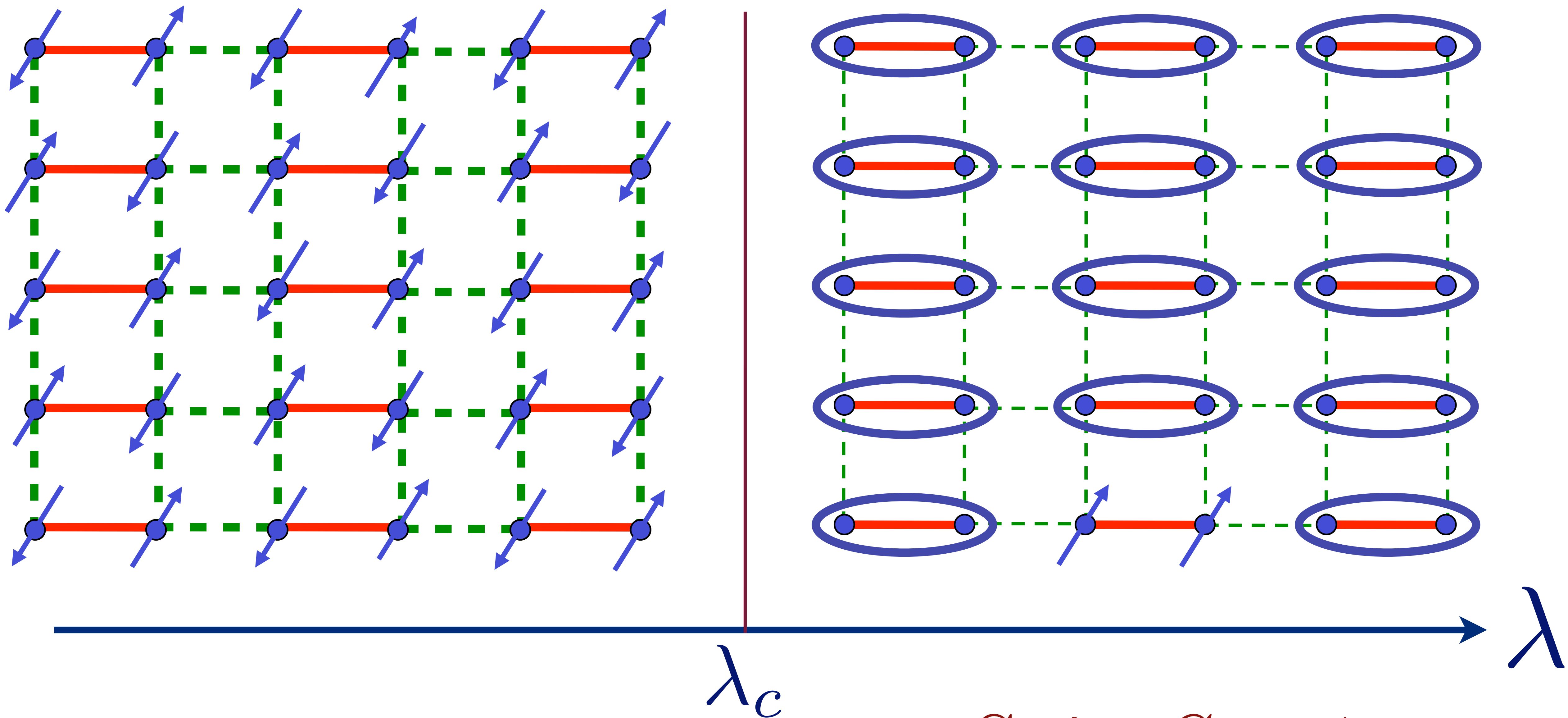
Neel order under pressure

A. Oosawa, K. Kakurai, T. Osakabe, M. Nakamura, M. Takeda, and H. Tanaka, *Journal of the Physical Society of Japan*, **73**, 1446 (2004).



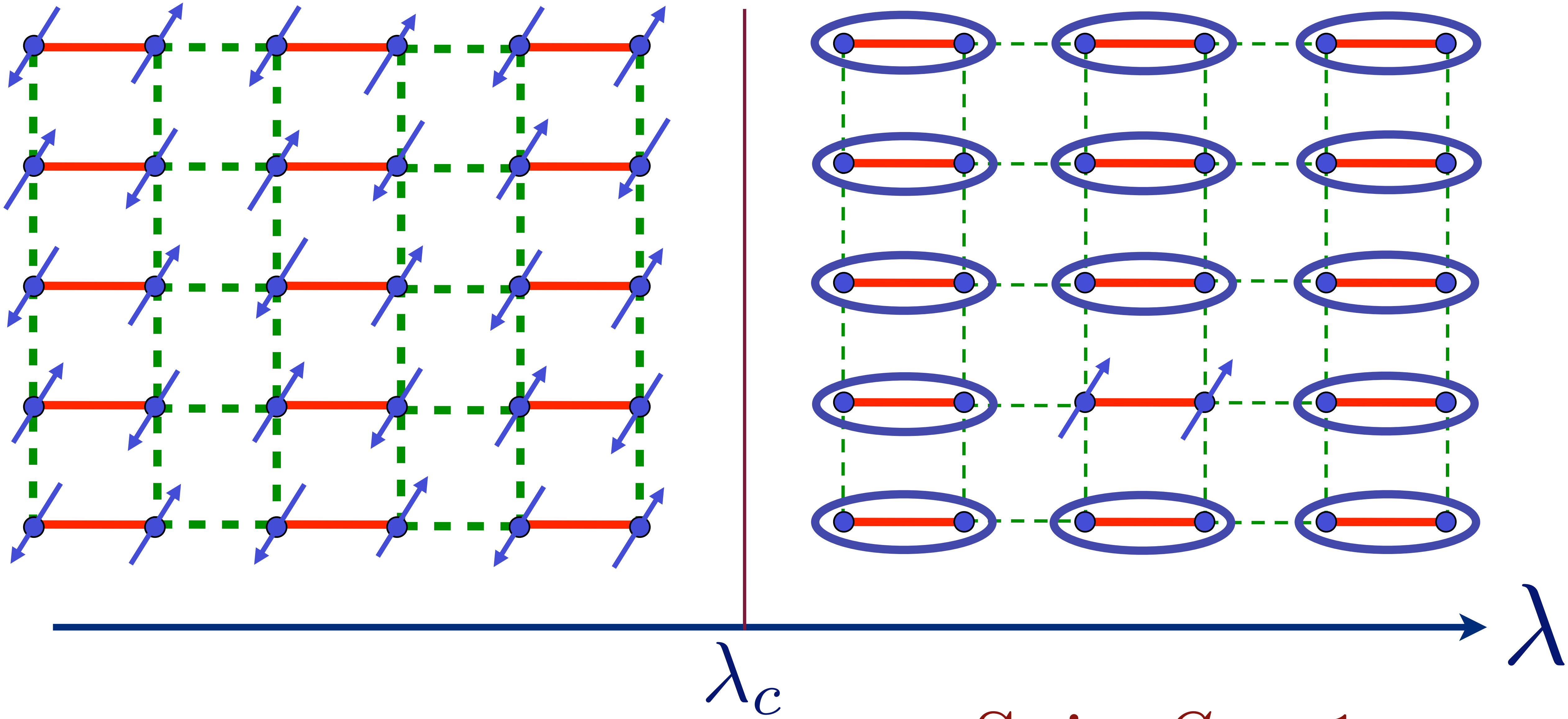
$= \frac{1}{\sqrt{2}}\Big(\big|\uparrow\downarrow\big\rangle - \big|\downarrow\uparrow\big\rangle\Big)$
λ_c
λ

Excitation spectrum in the paramagnetic phase



Spin $S = 1$
"triplon"

Excitation spectrum in the paramagnetic phase



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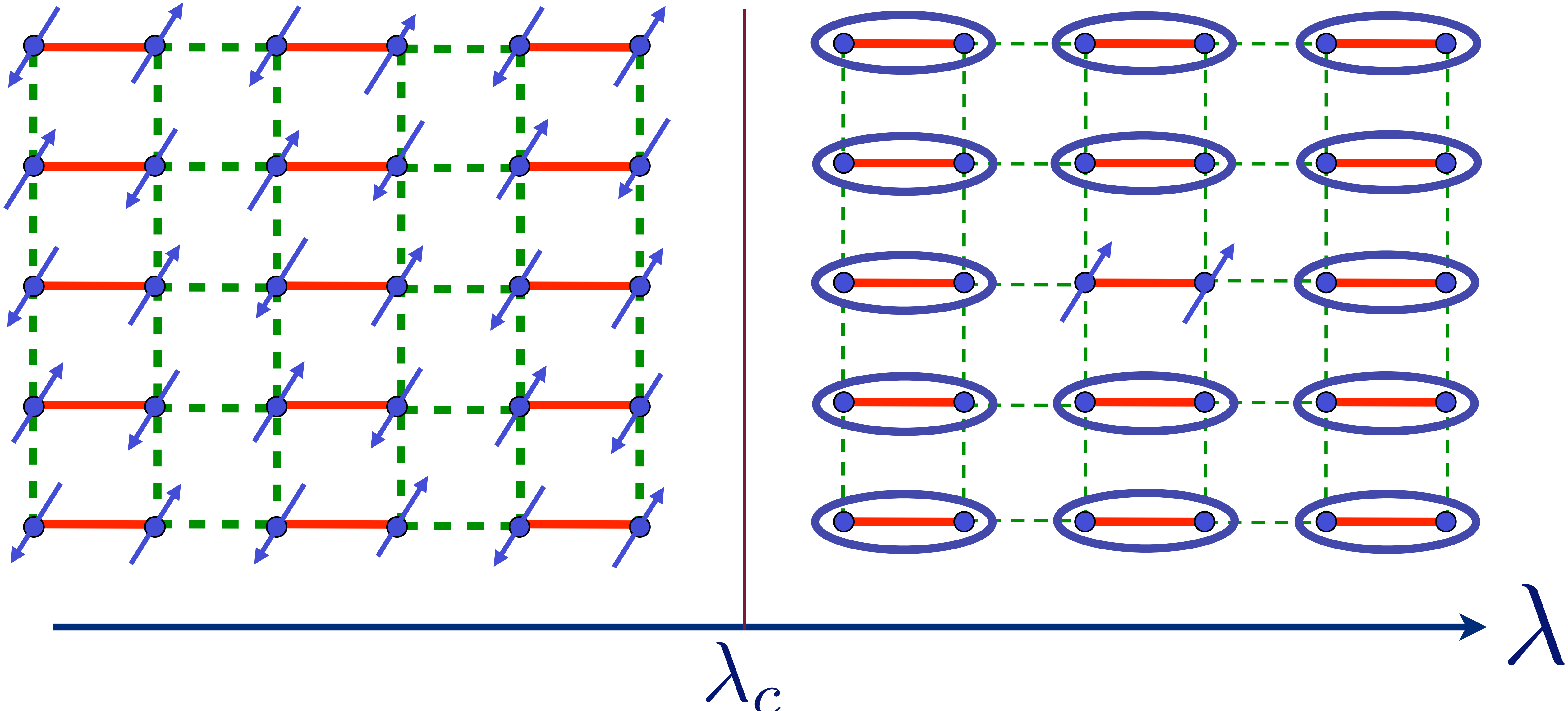

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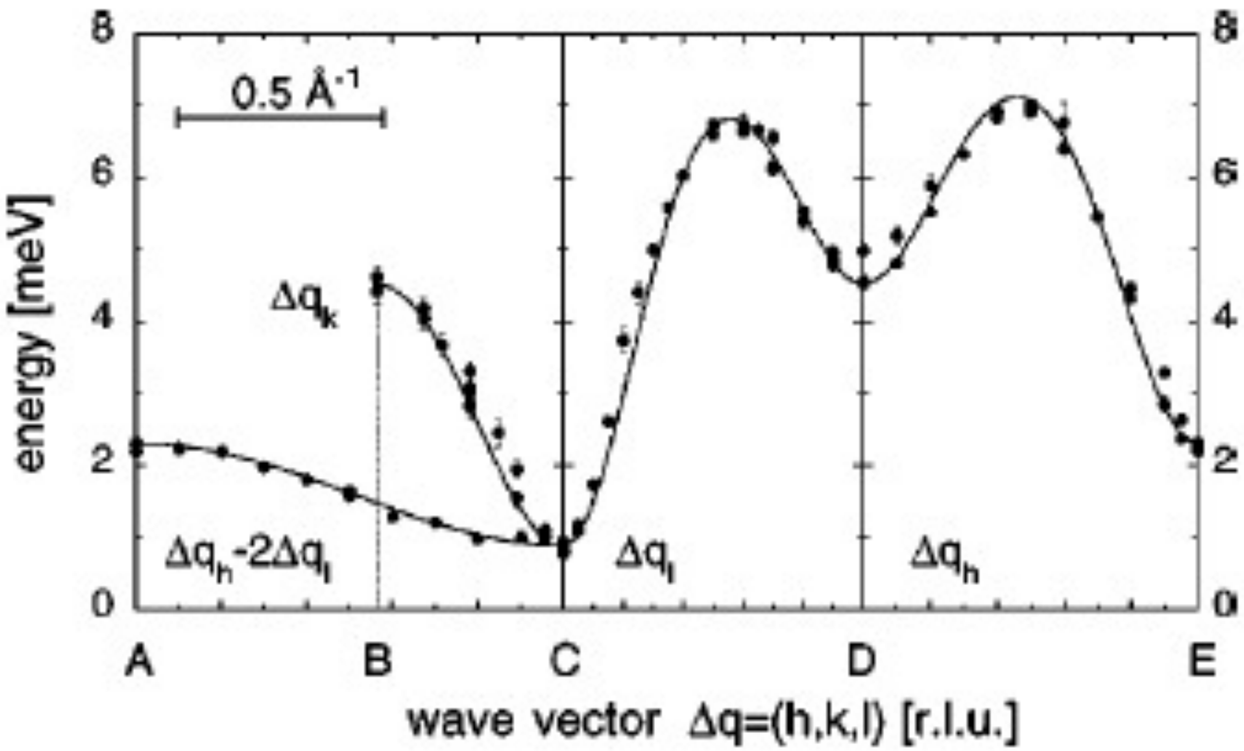
$TlCuCl_3$ at ambient pressure



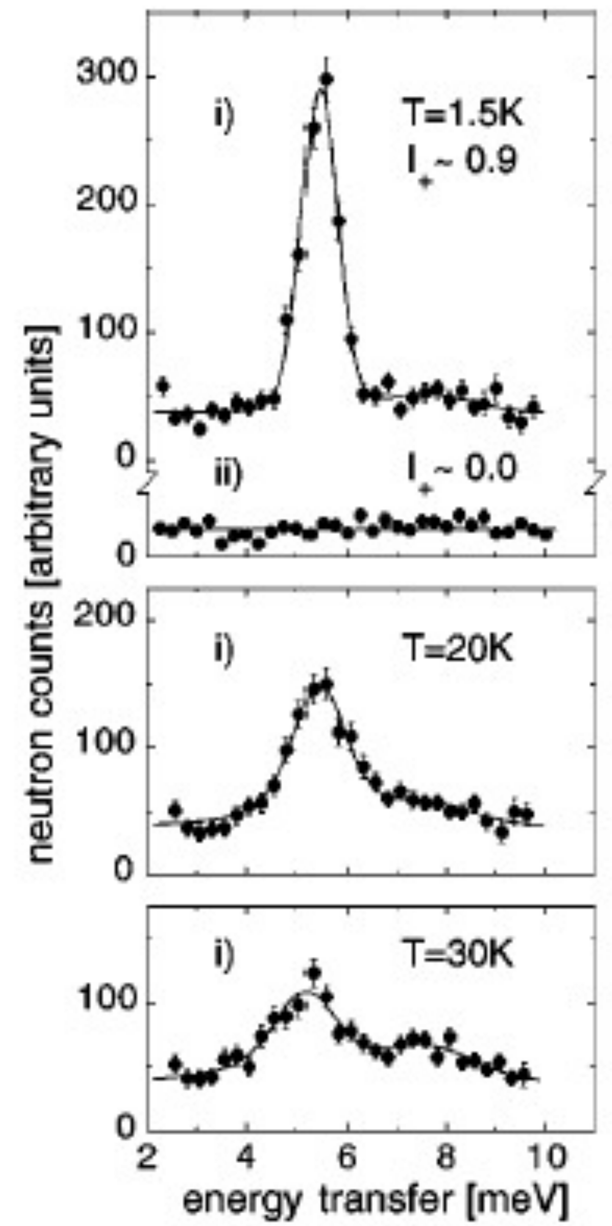
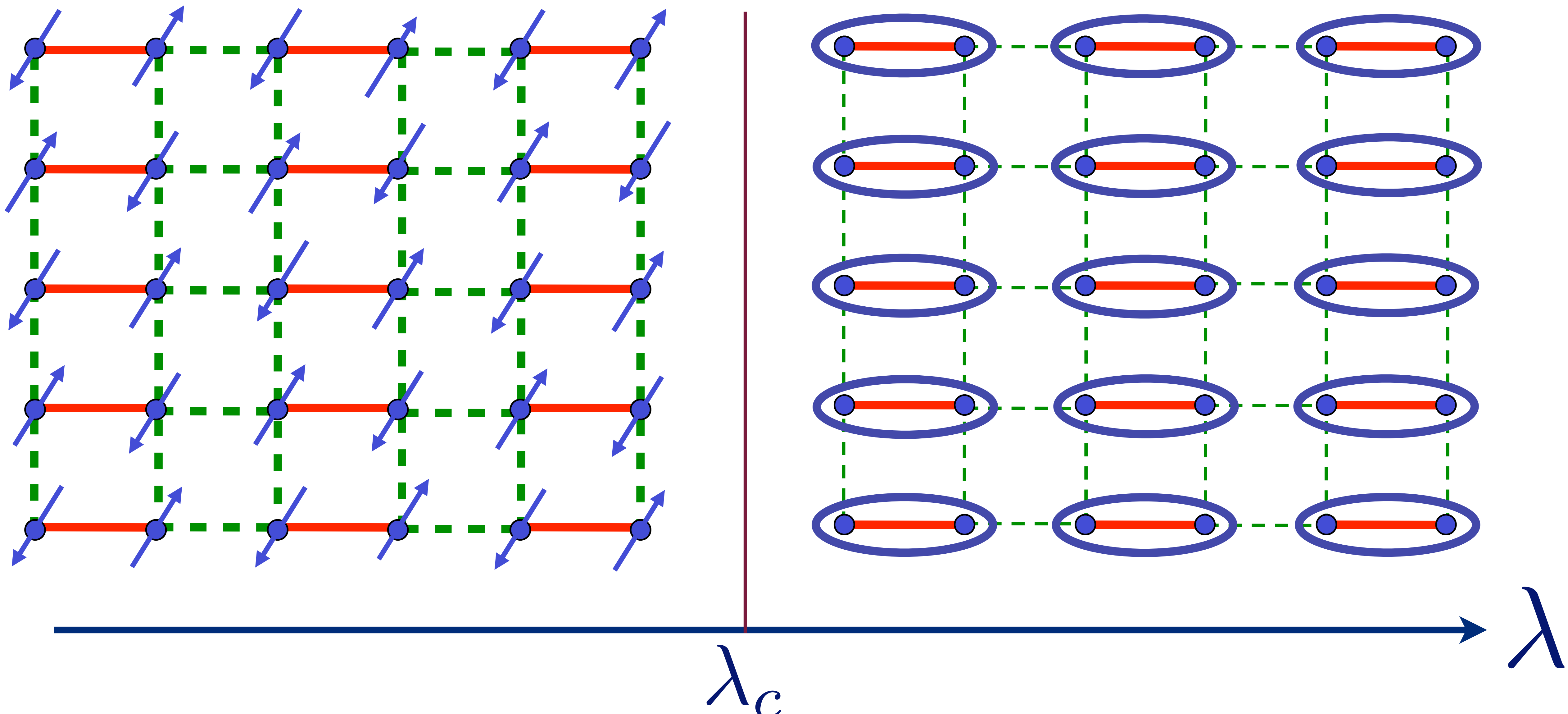


FIG. 1. Measured neutron profiles in the a^*c^* plane of $TlCuCl_3$ for $i=(1.35,0,0)$, $ii=(0,0,3.15)$ [r.l.u]. The spectrum at $T=1.5$ K

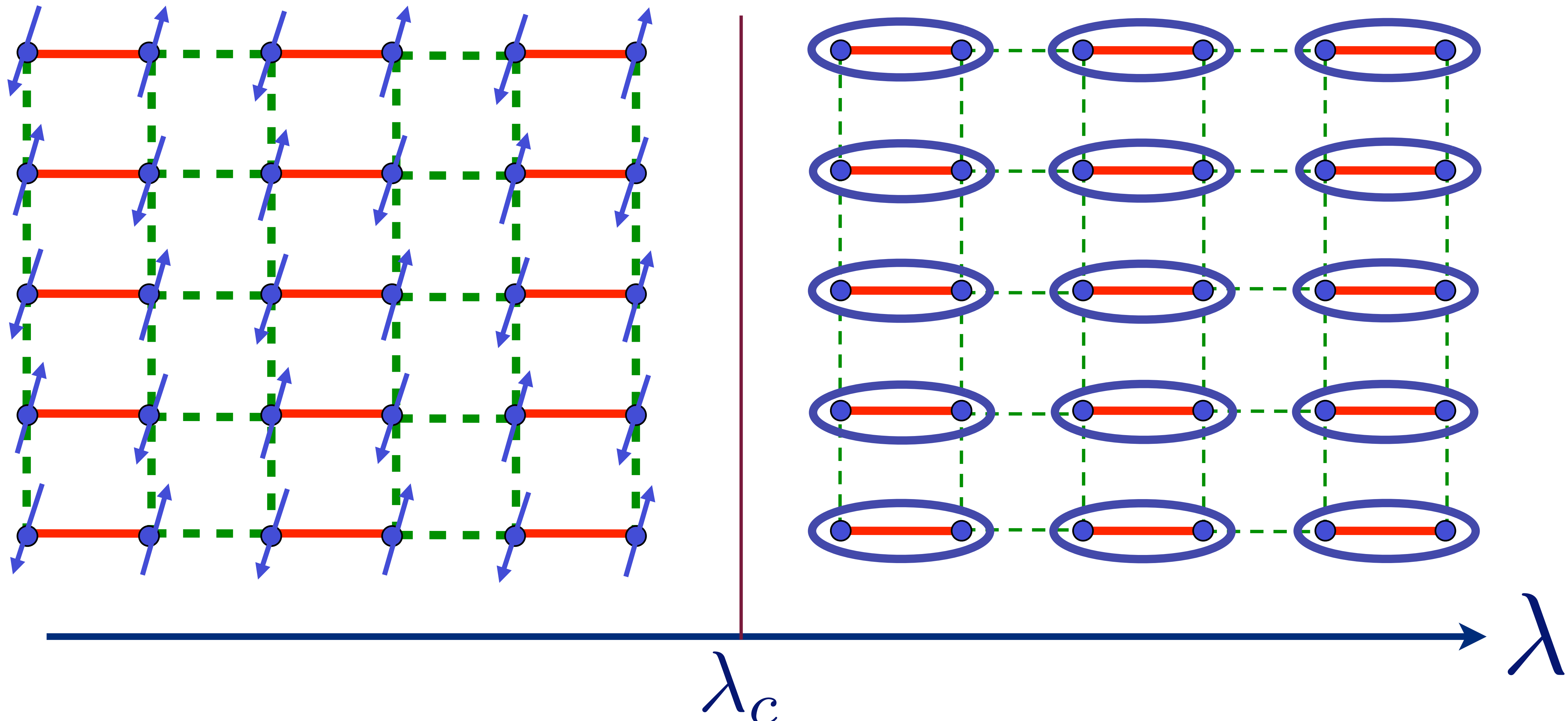
N. Cavadini, G. Heigold, W. Henggeler, A. Furrer, H.-U. Güdel, K. Krämer and H. Mutka, *Phys. Rev.* B 63 172414 (2001).

Excitation spectrum in the Néel phase



Spin waves
(Goldstone bosons)

Excitation spectrum in the Néel phase



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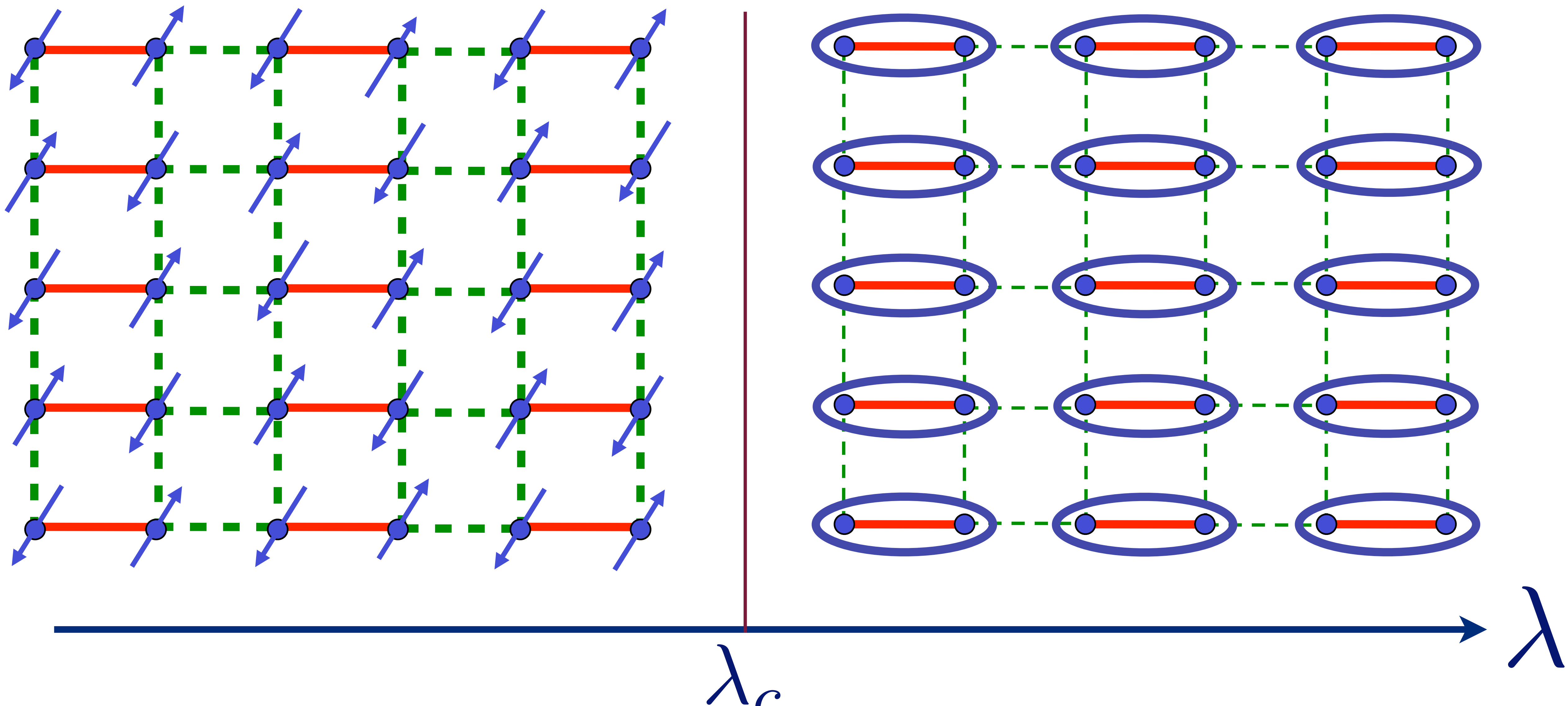

Spin waves
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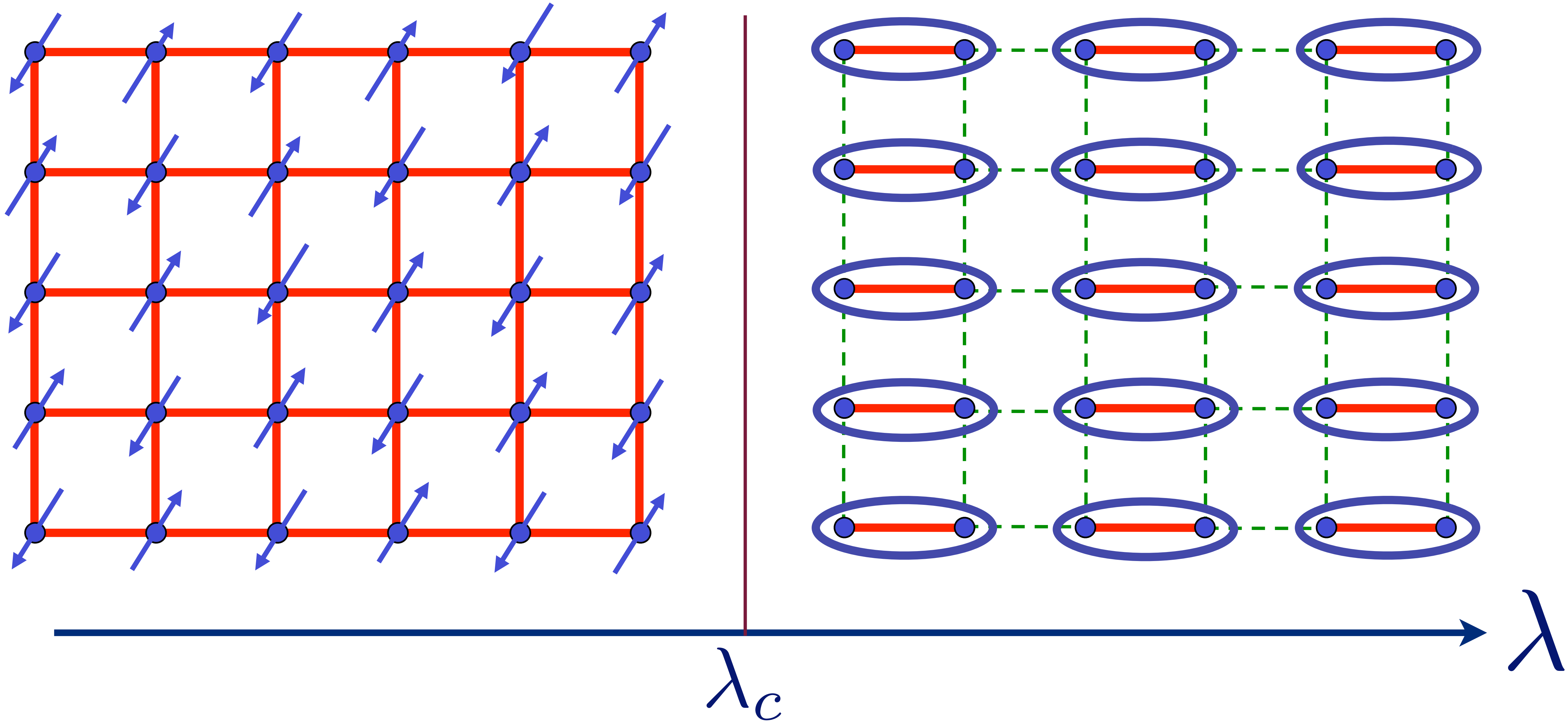
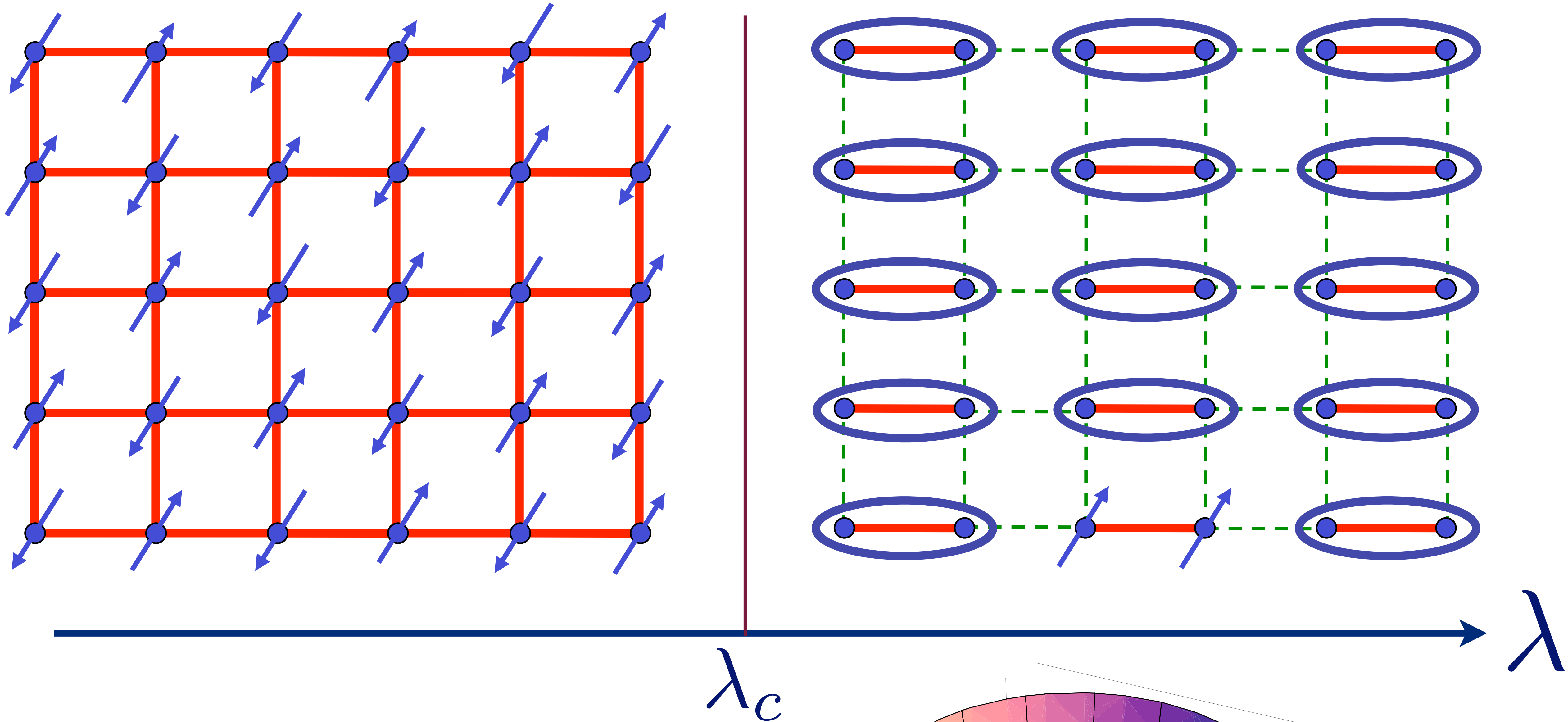
Description using Landau-Ginzburg field theory



Field theory for the quantum phase transition in terms of a O(3) order parameter $\vec{\varphi}$

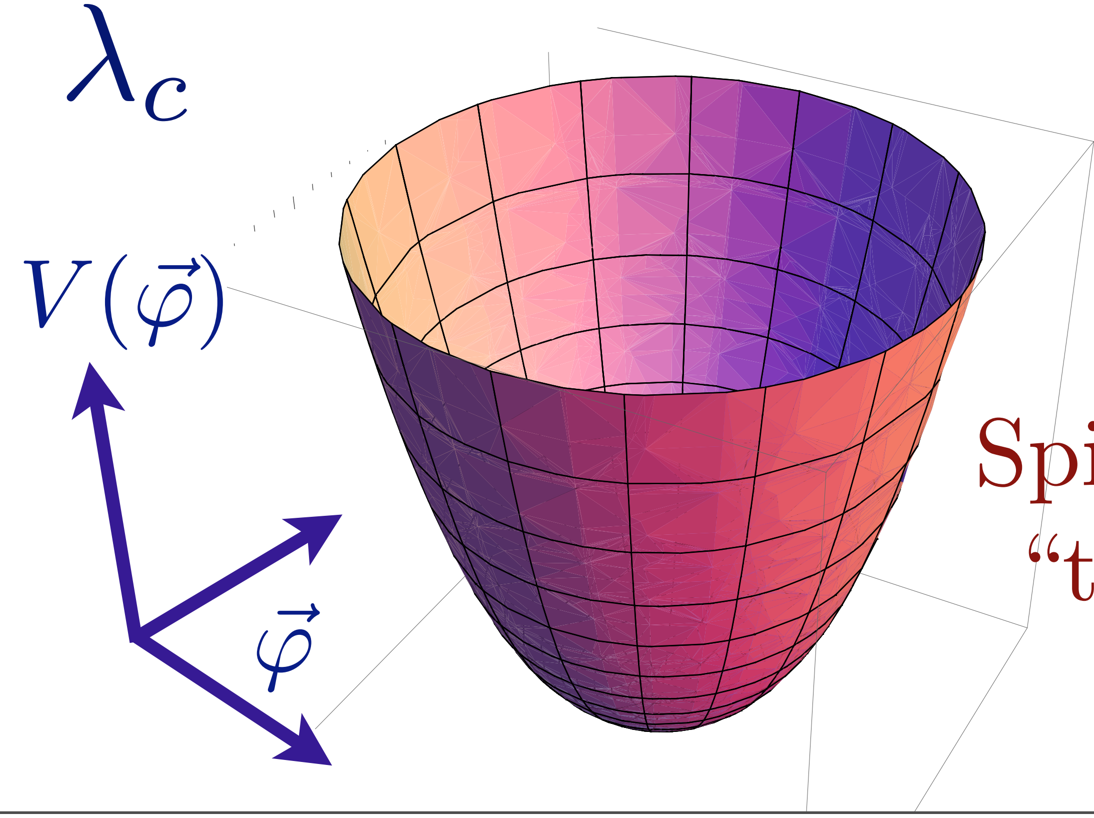
$$\mathcal{S} = \int d^3 r d\tau \left[(\partial_\tau \varphi)^2 + c^2 (\nabla_r \vec{\varphi})^2 + (\lambda - \lambda_c) \vec{\varphi}^2 + u \left(\vec{\varphi}^2 \right)^2 \right]$$

Excitation spectrum in the paramagnetic phase



$$V(\vec{\varphi}) = (\lambda - \lambda_c)\vec{\varphi}^2 + u\left(\vec{\varphi}^2\right)^2$$

$$\lambda > \lambda_c$$



Spin $S = 1$
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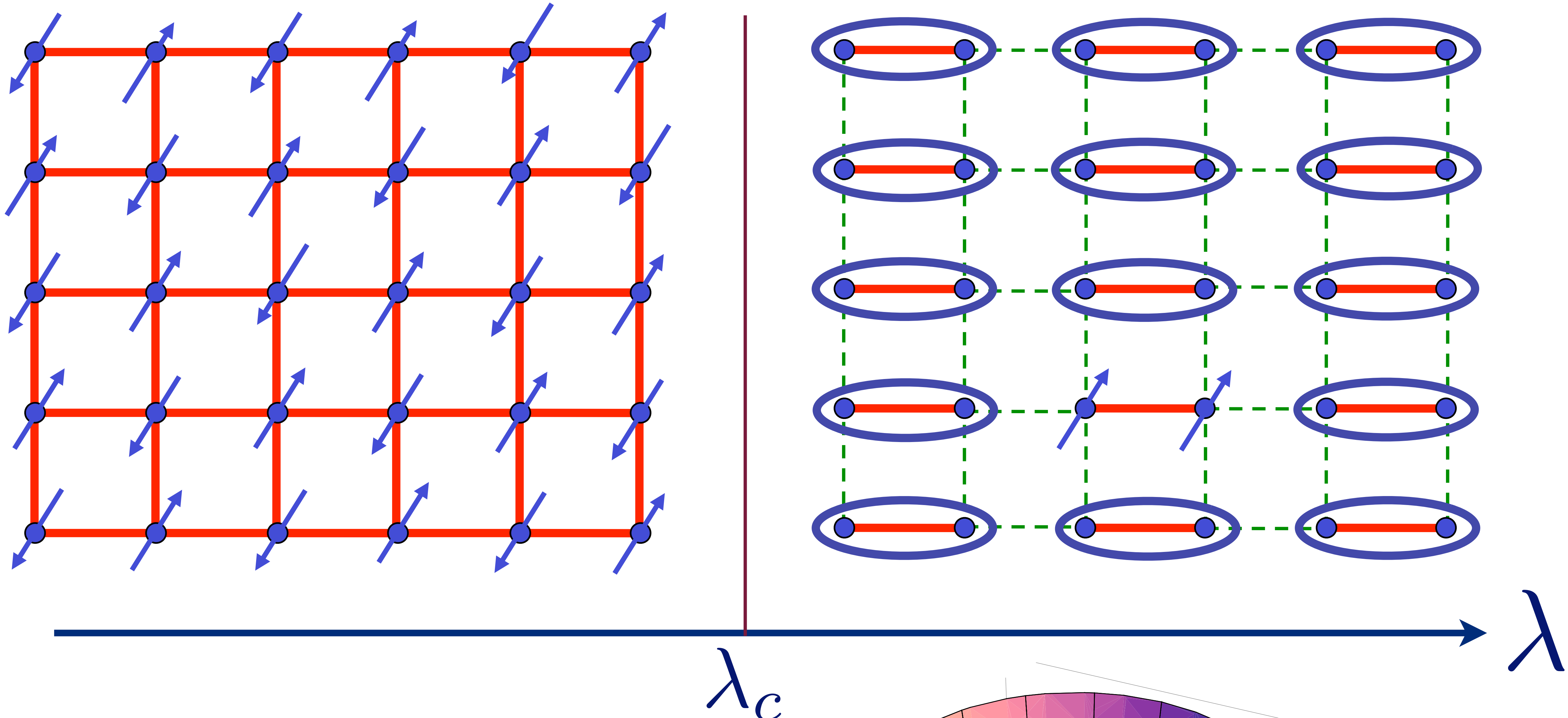

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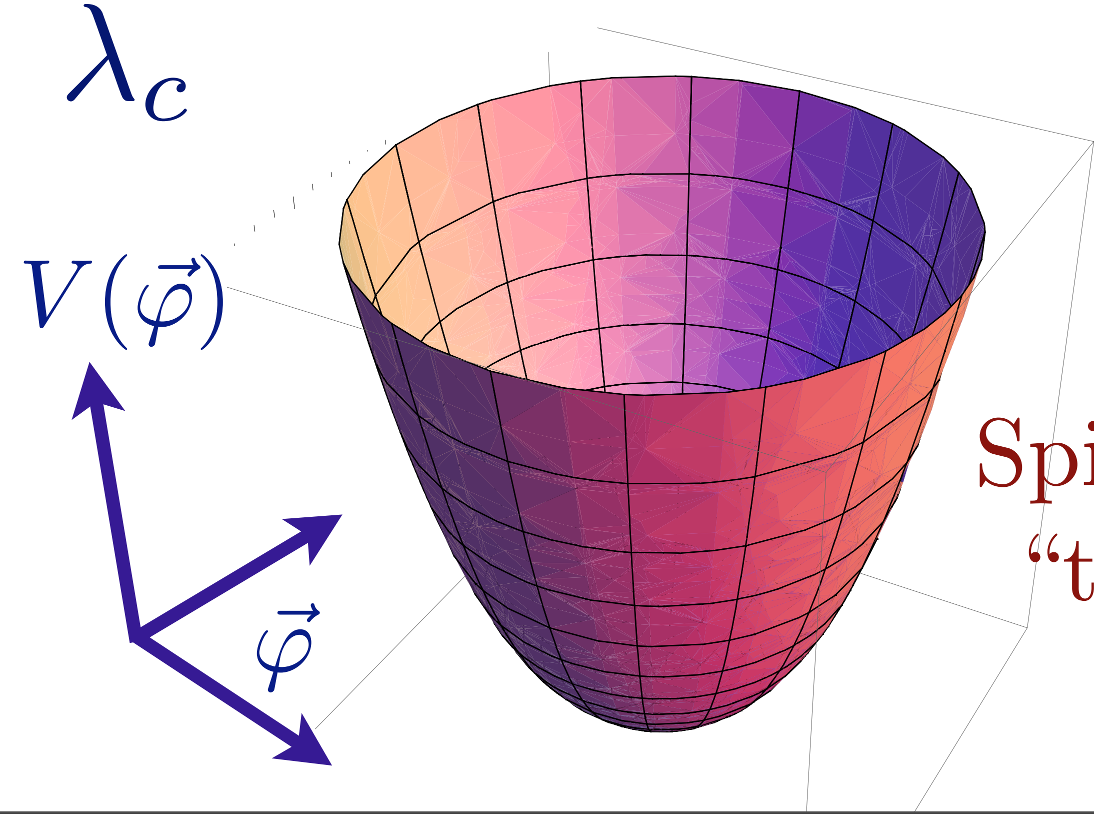

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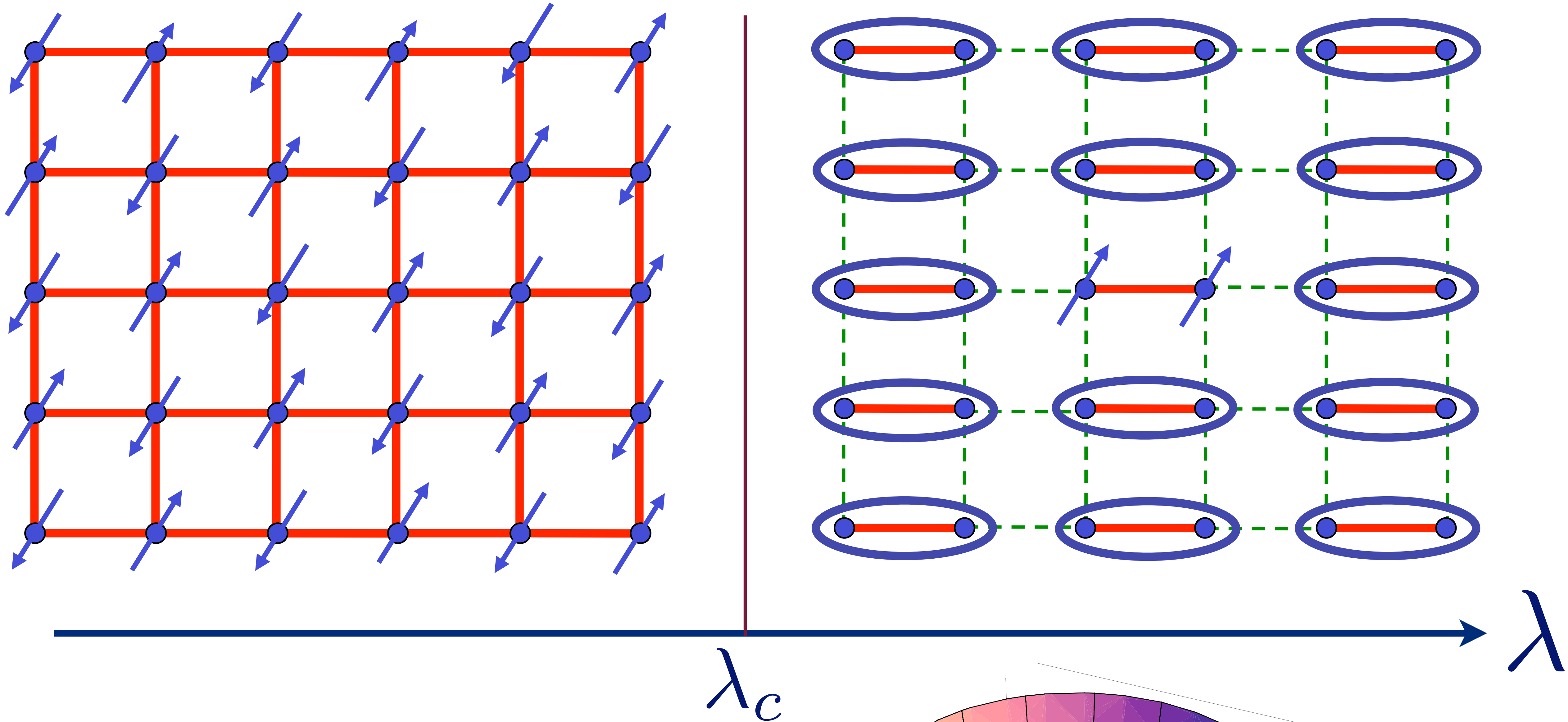

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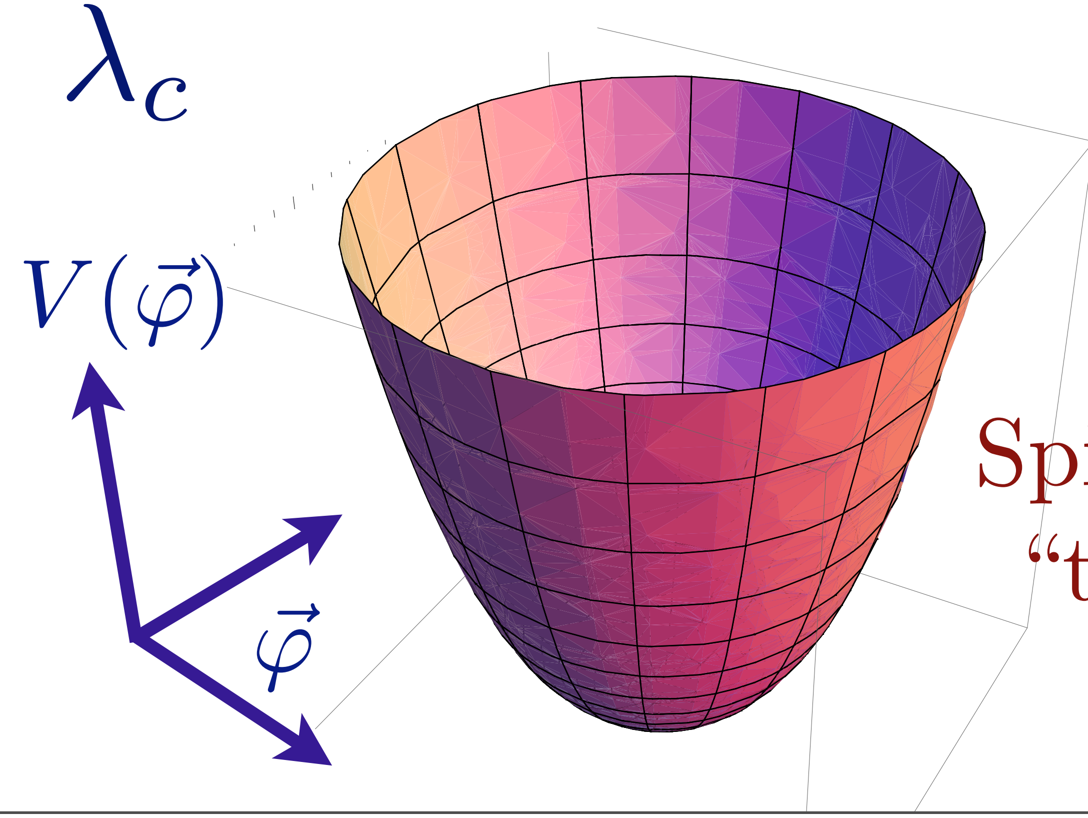

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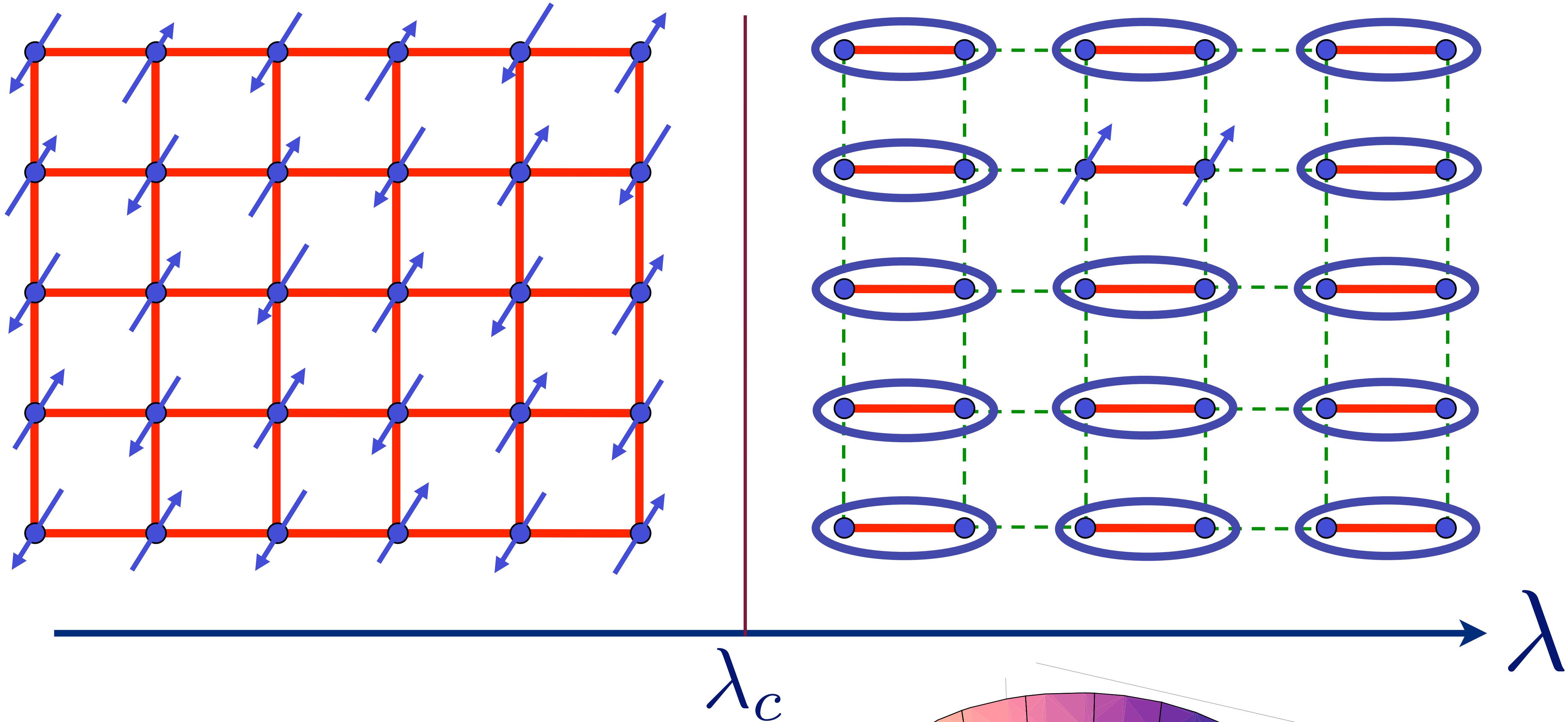

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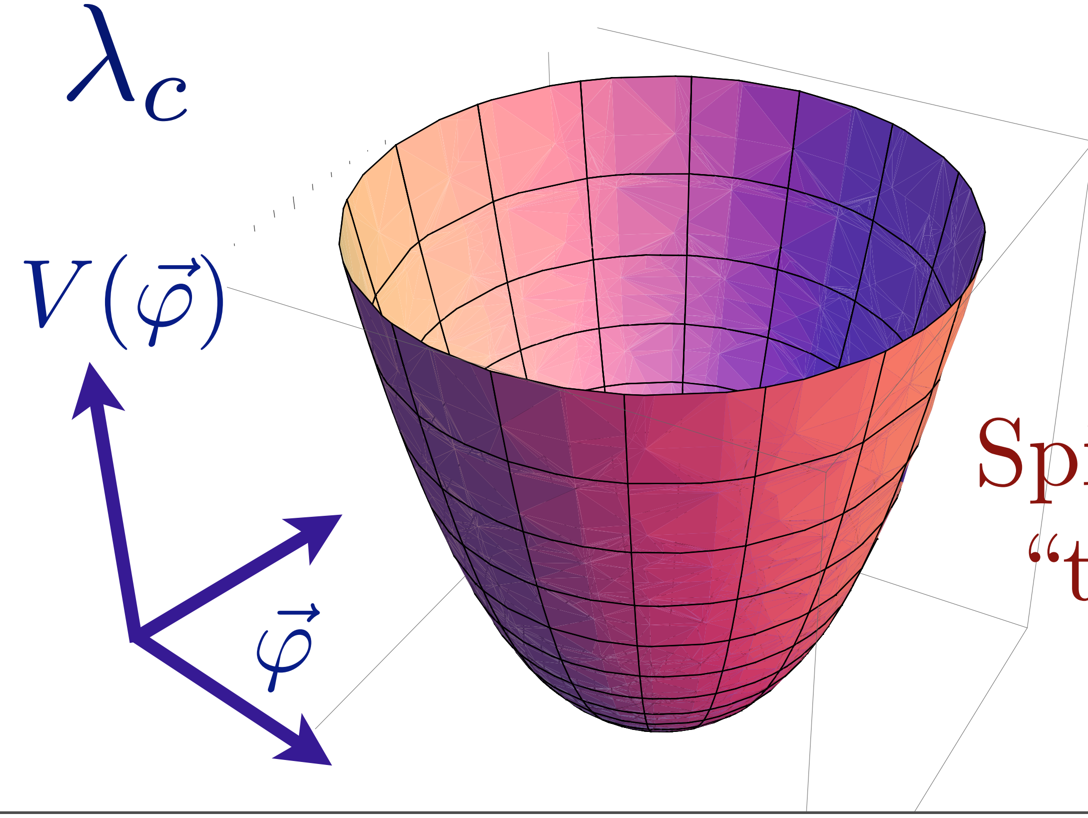

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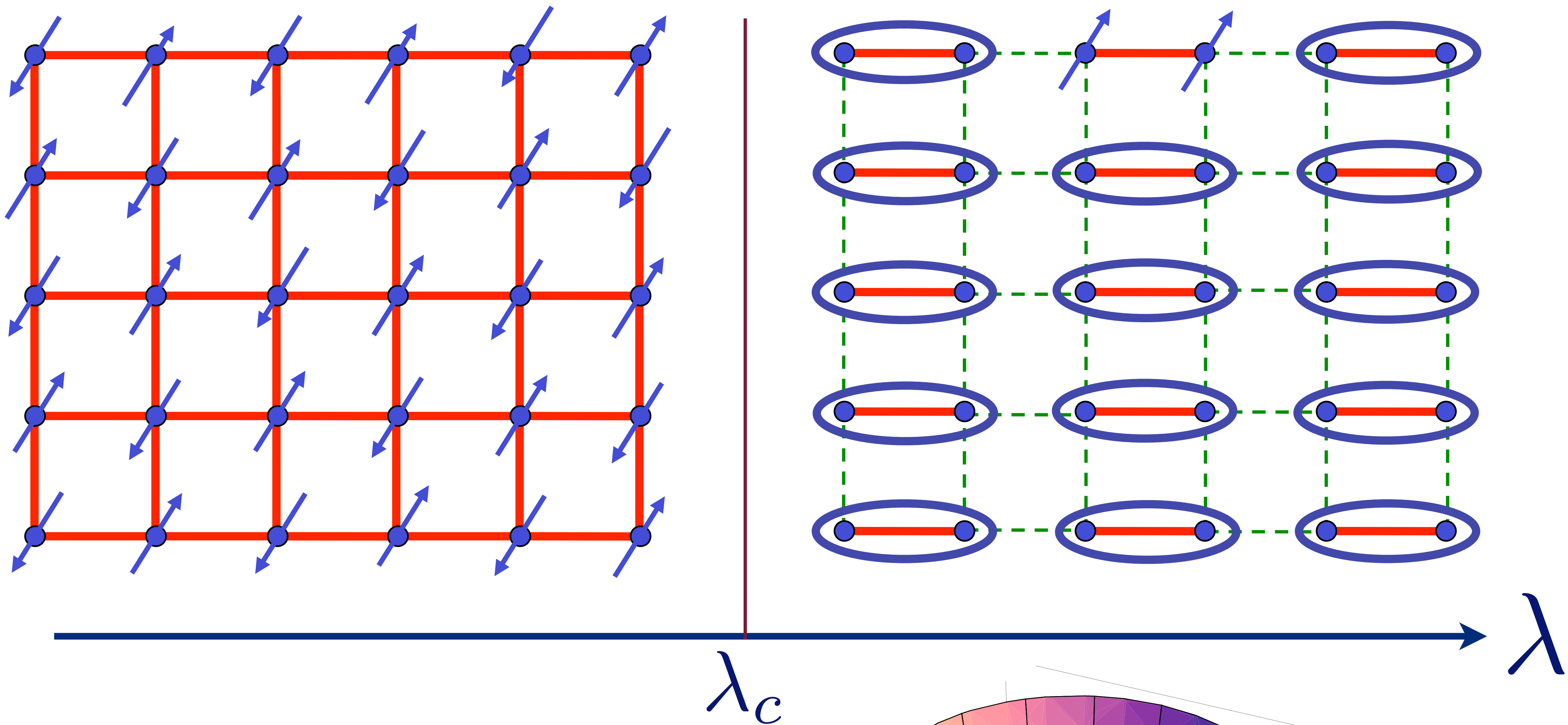

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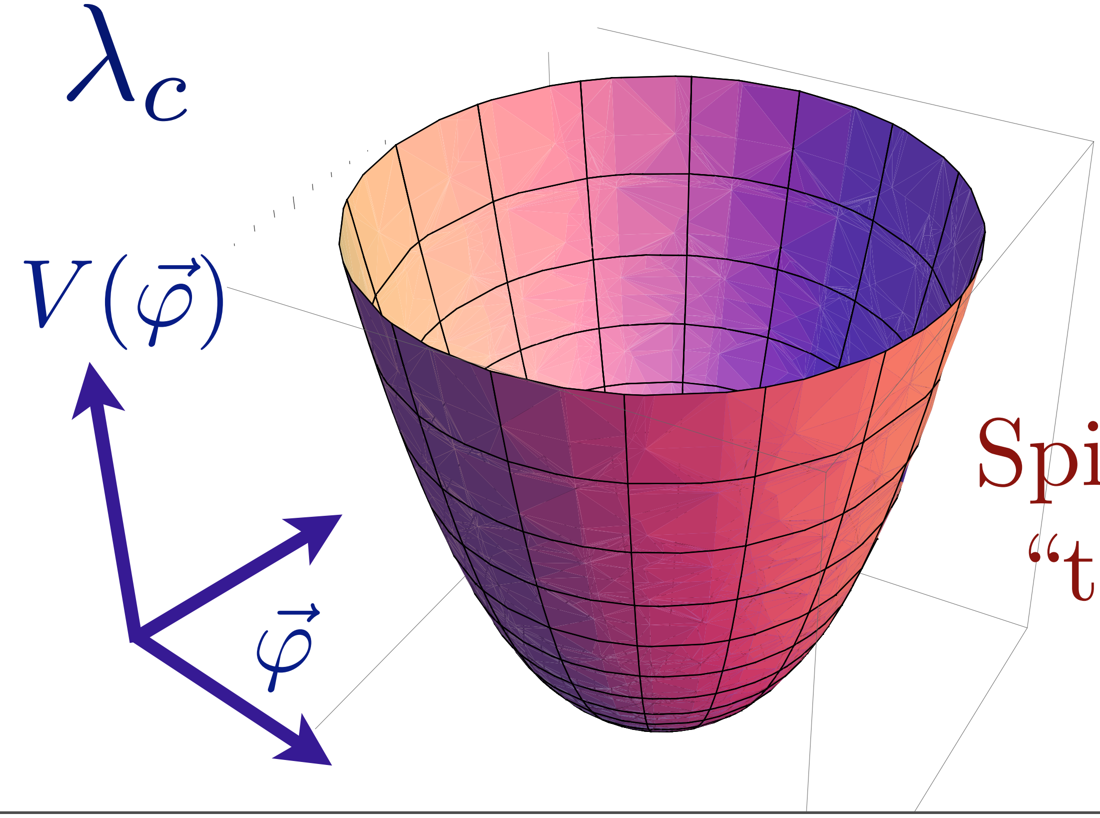

Spin $S = 1$
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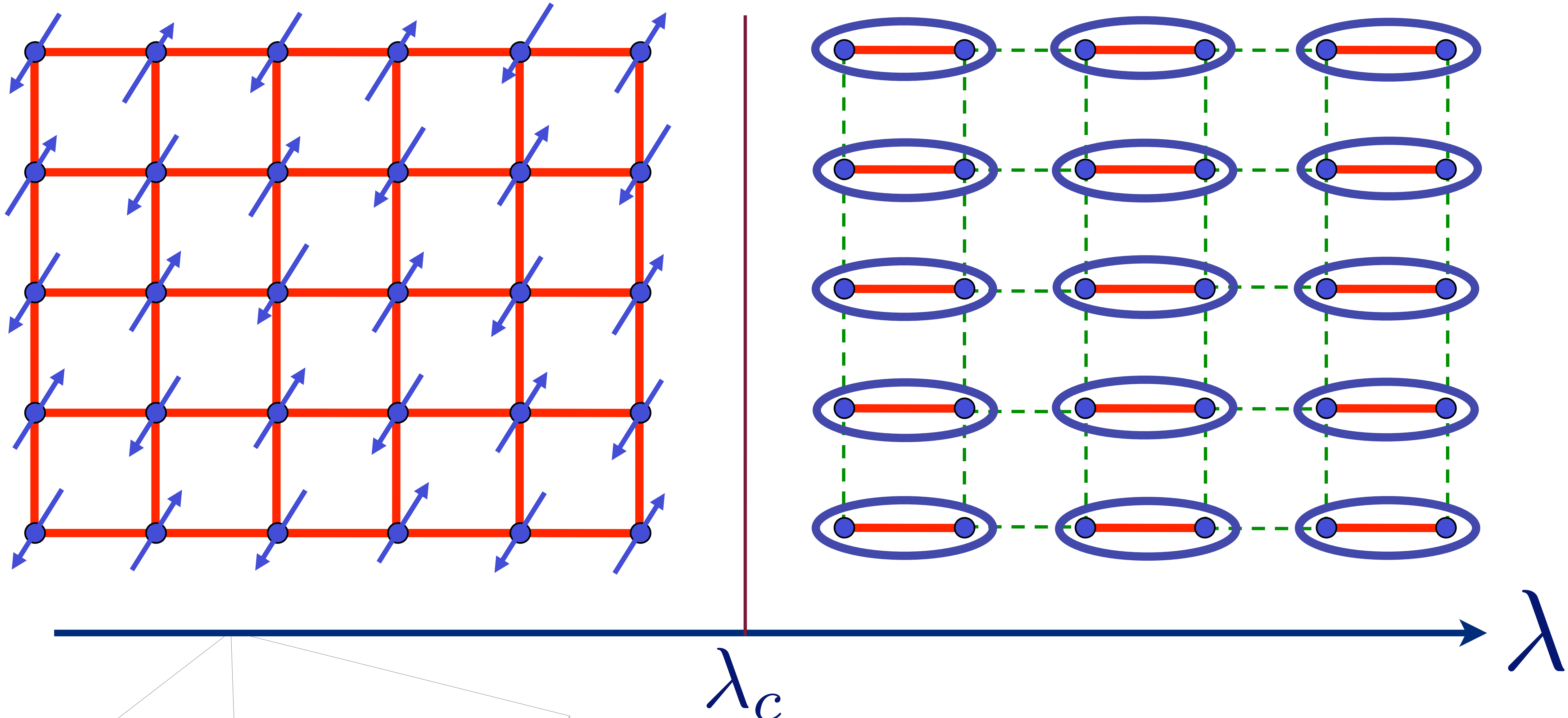
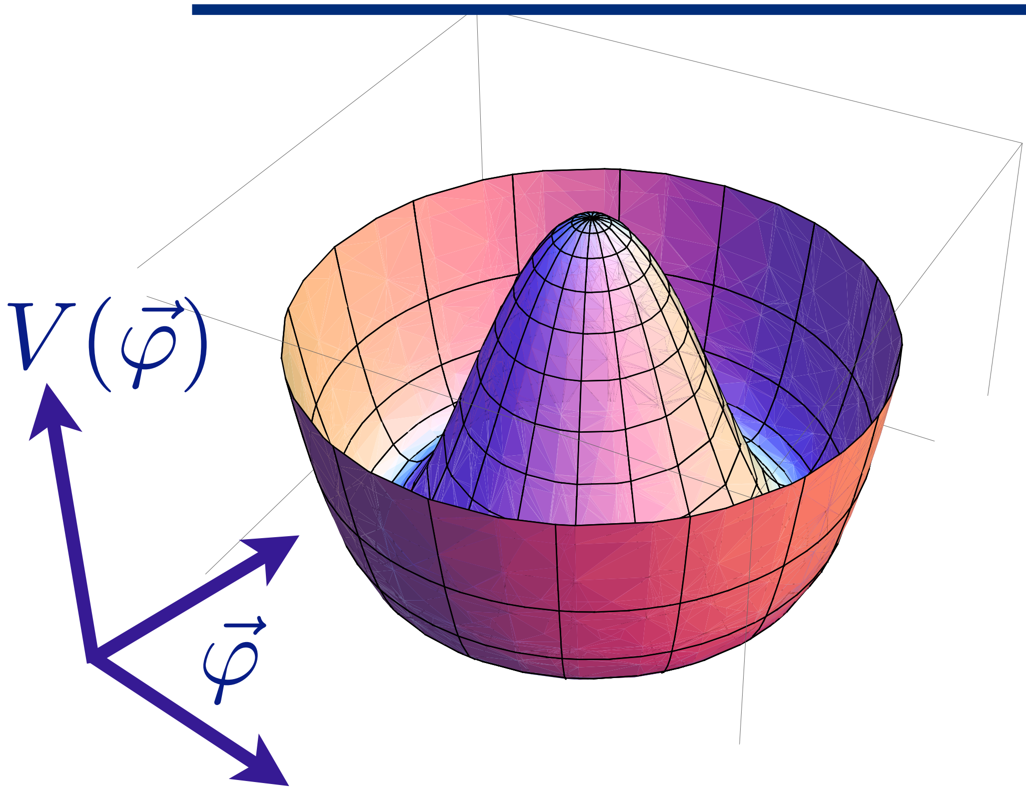
Excitation spectrum in the Néel phase





$$V(\vec{\varphi}) = (\lambda - \lambda_c)\vec{\varphi}^2 + u\left(\vec{\varphi}^2\right)^2$$

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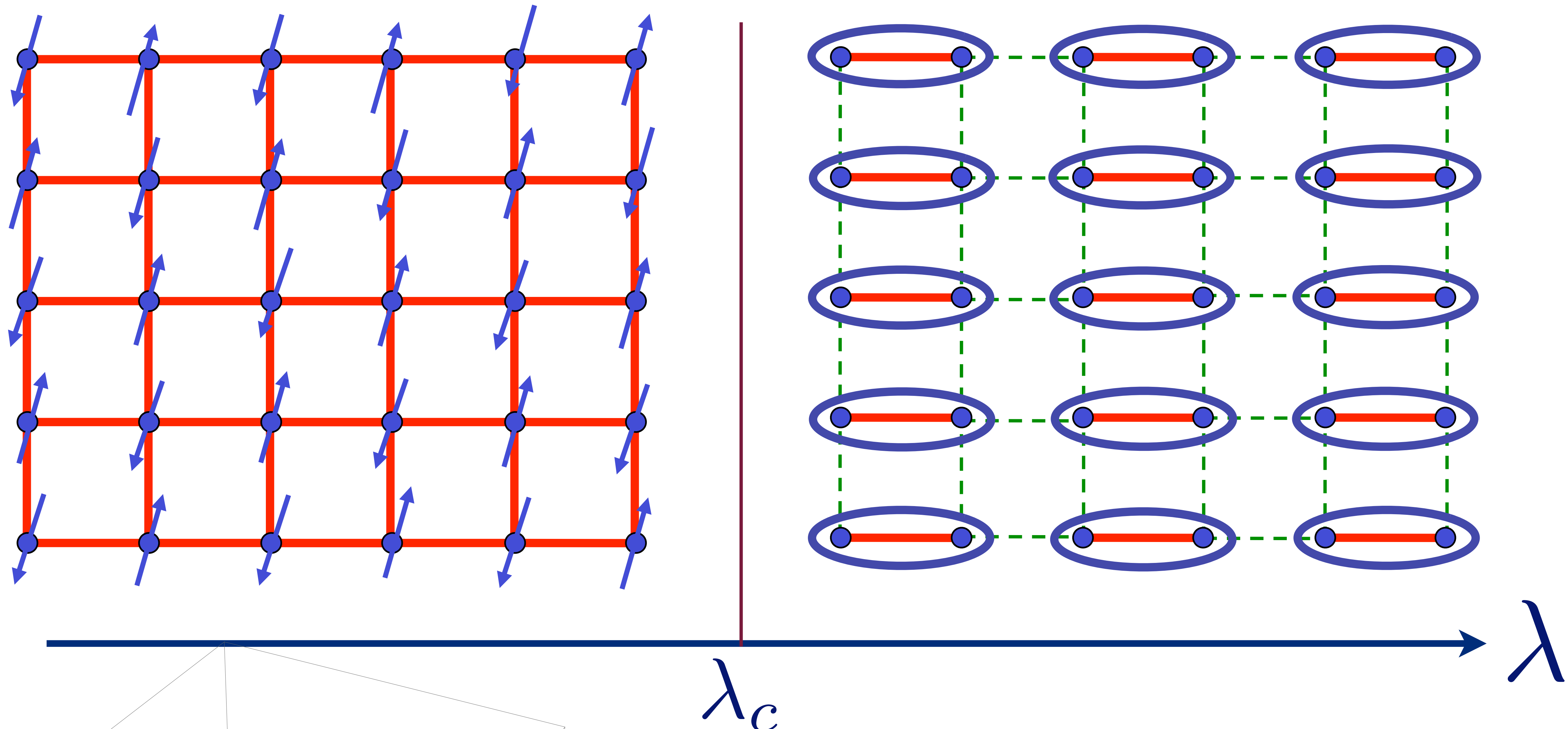
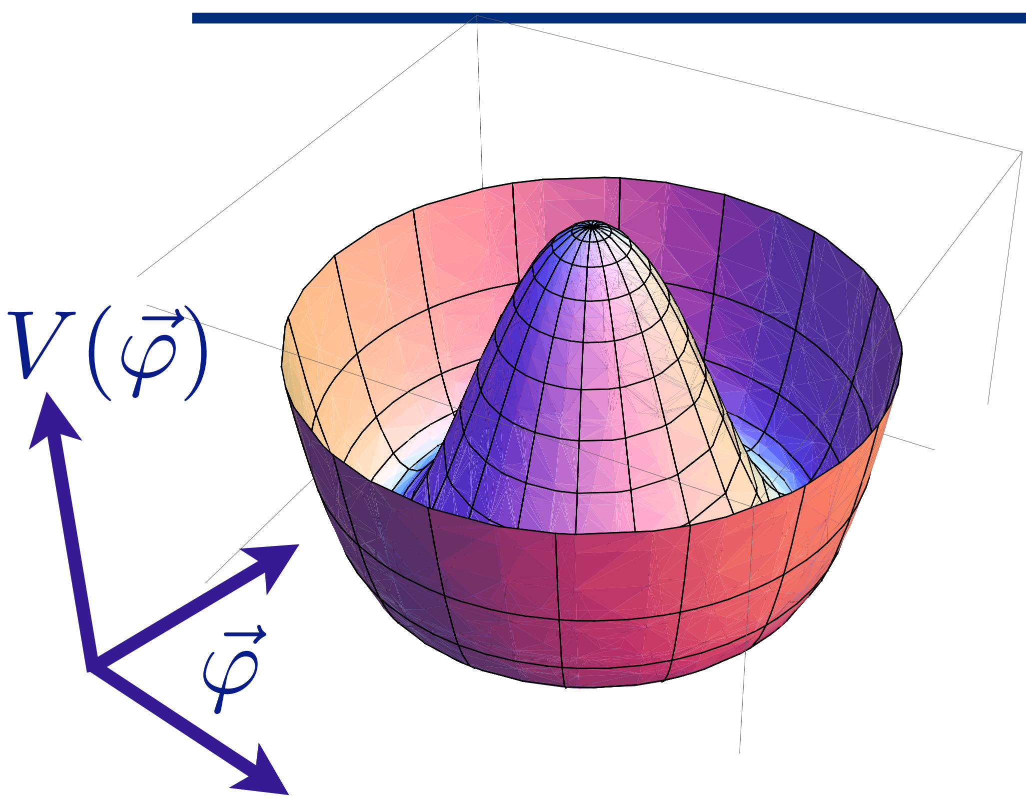
Spin waves (Goldstone boson)
and a longitudinal Higgs boson

Excitation spectrum in the Néel phase





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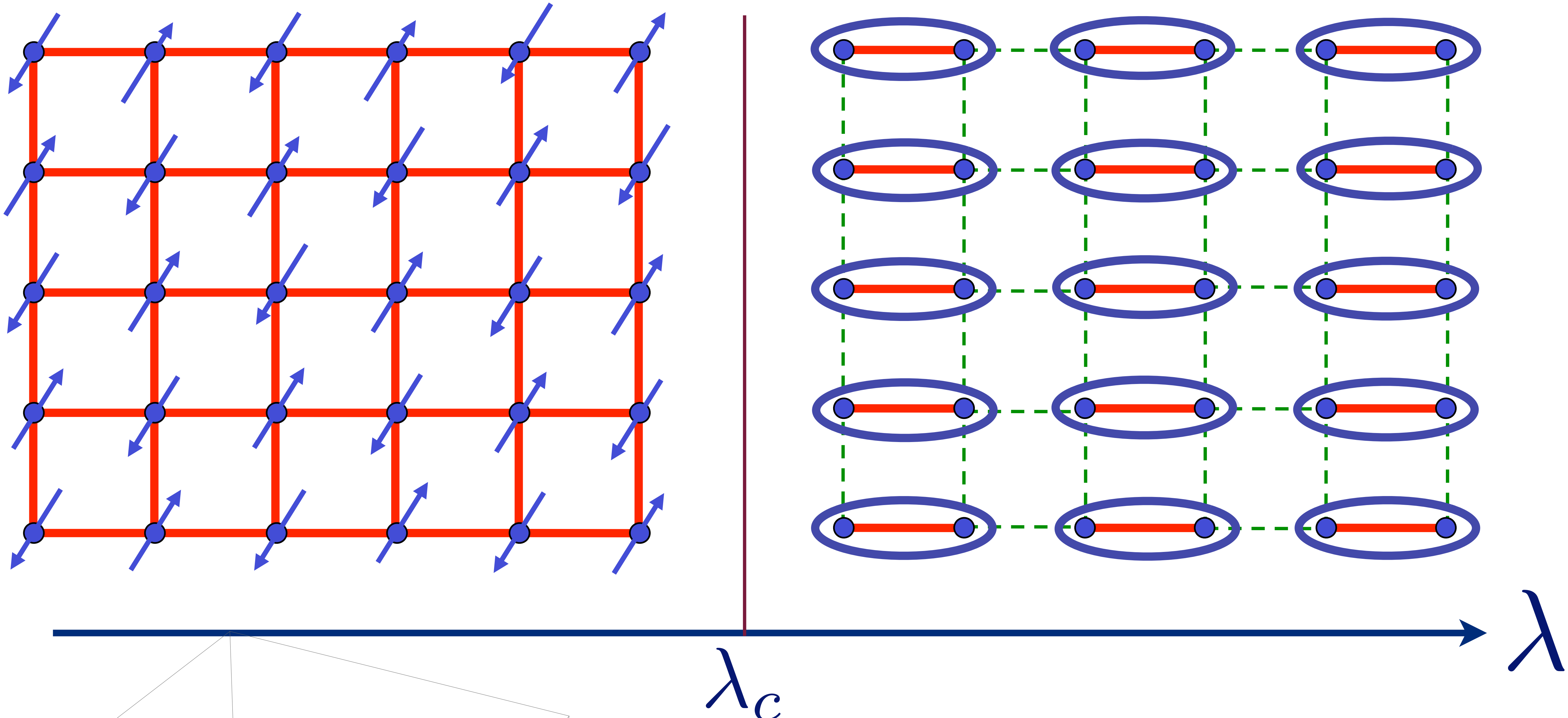

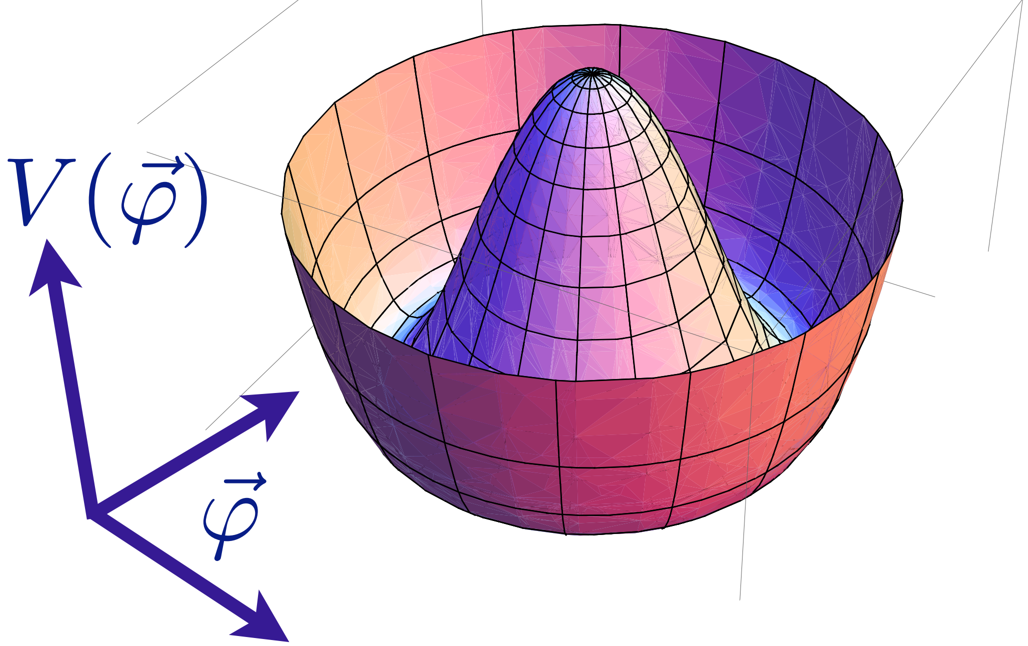

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Spin waves (Goldstone boson)
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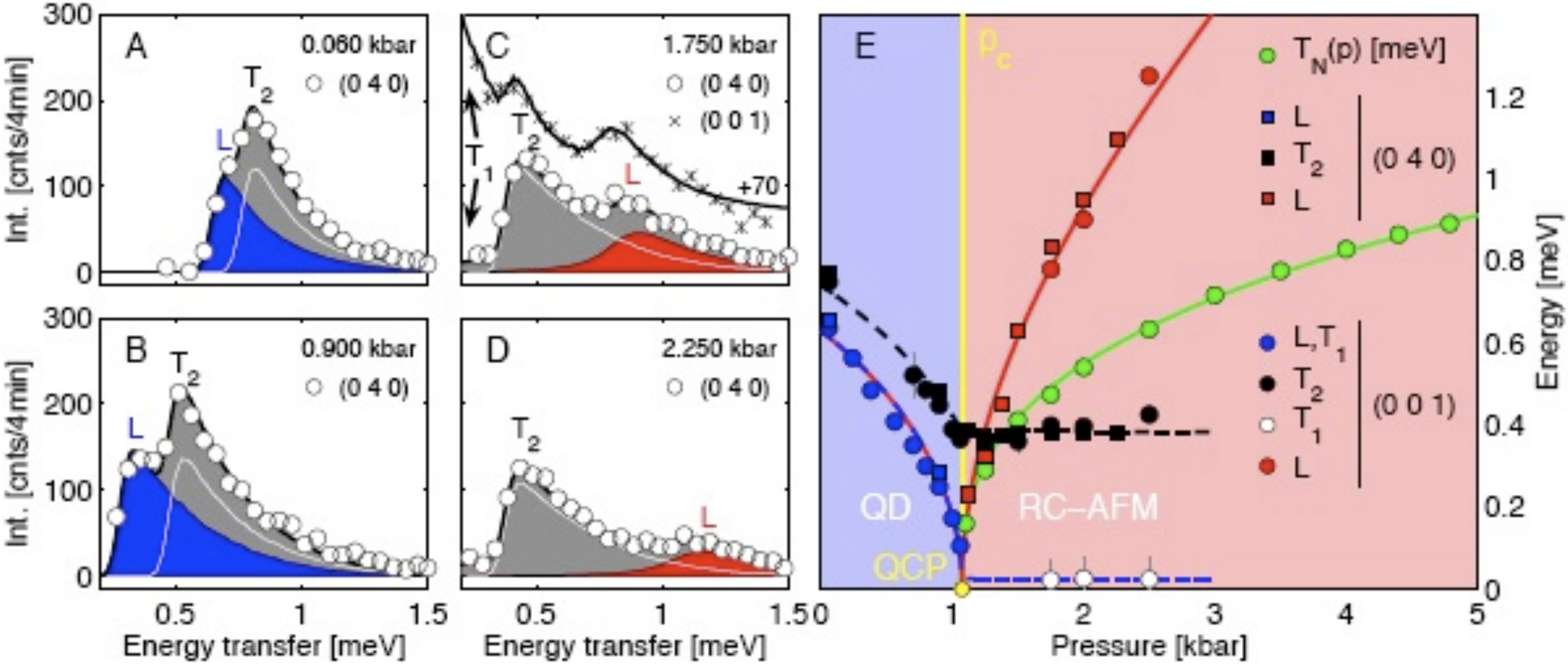
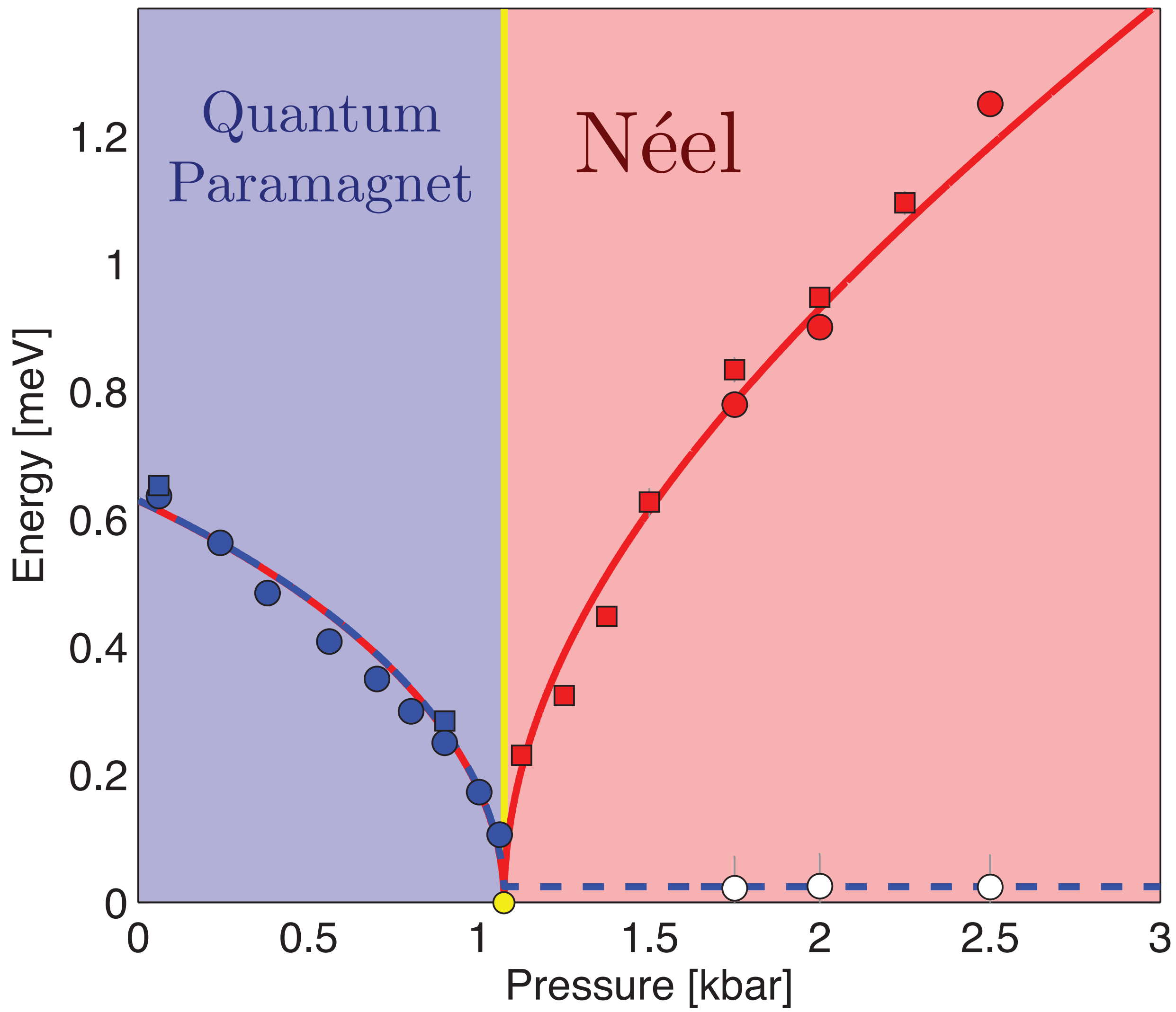
$TlCuCl_3$ with varying pressure



Observation of 3 → 2 low energy modes, emergence of new longitudinal mode (the "Higgs boson") in Néel phase, and vanishing of Néel temperature at quantum critical point

Christian Ruegg, Bruce Normand, Masashige Matsumoto, Albert Furrer, Desmond McMorrow, Karl Kramer, Hans-Ulrich Gudel, Severian Gvasaliya, Hannu Mutka, and Martin Boehm, *Phys. Rev. Lett.* **100**, 205701 (2008)

Excitations of $TlCuCl_3$ with varying pressure



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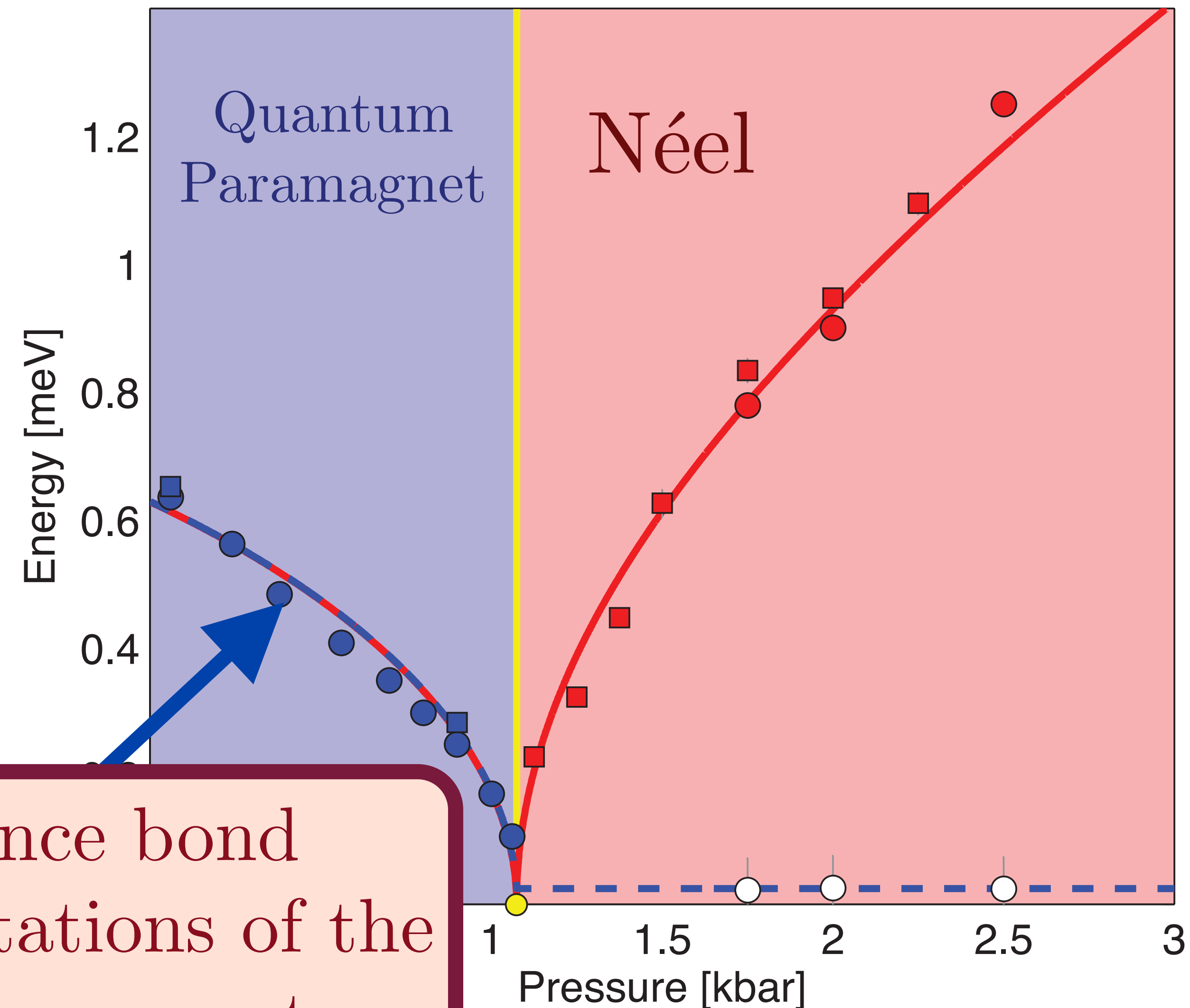

Broken valence bond
(“triplon”) excitations of the
quantum paramagnet

Christian Ruegg, Bruce Normand, Masashige Matsumoto, Albert Furrer,
Desmond McMorrow, Karl Kramer, Hans–Ulrich Gudel, Severian Gvasaliya,
Hannu Mutka, and Martin Boehm, *Phys. Rev. Lett.* **100**, 205701 (2008)

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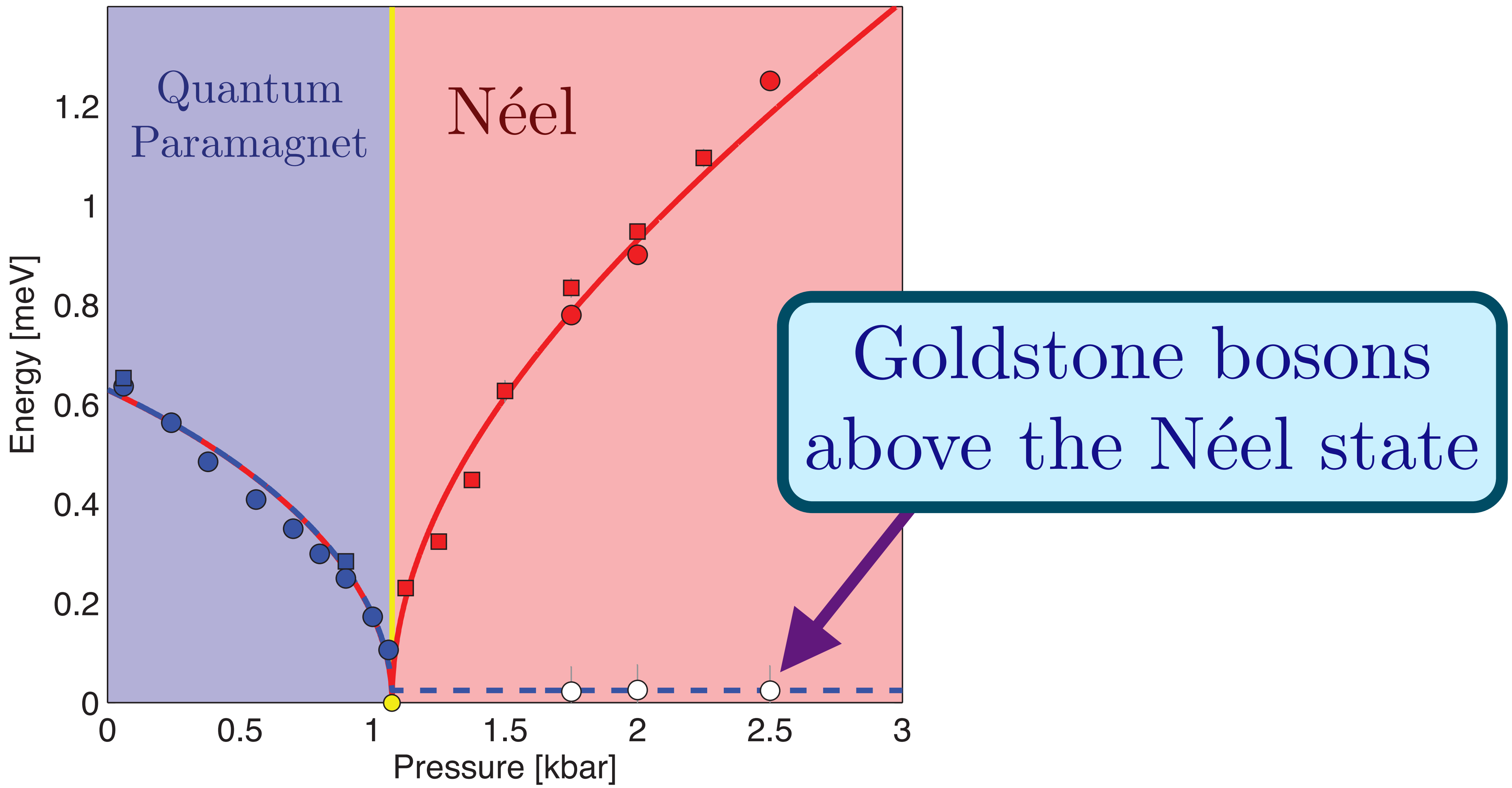

Christian Ruegg, Bruce Normand, Masashige Matsumoto, Albert Furrer, Desmond McMorrow, Karl Kramer, Hans-Ulrich Gudel, Severian Gvasaliya, Hannu Mutka, and Martin Boehm, *Phys. Rev. Lett.* **100**, 205701 (2008)

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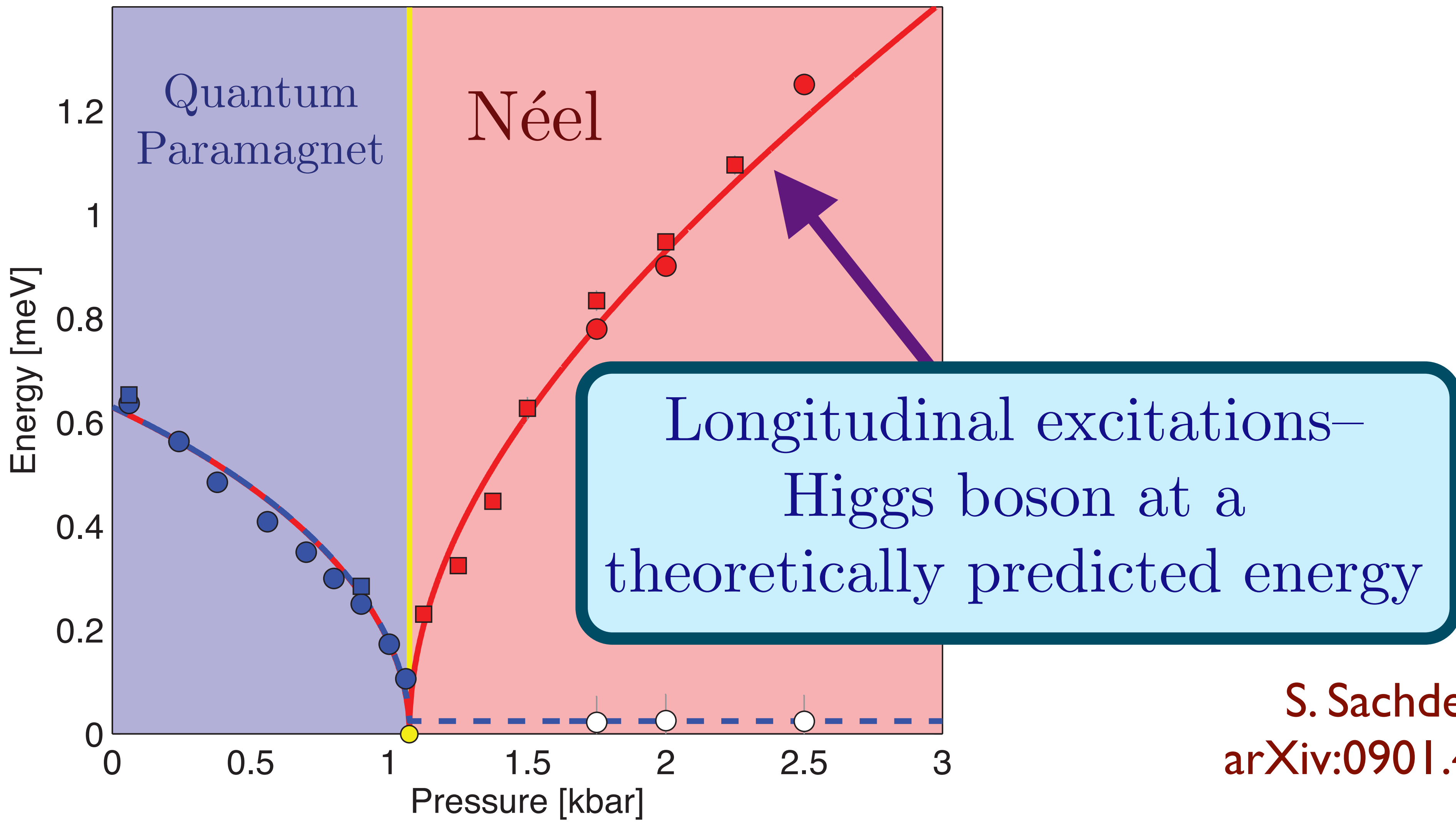


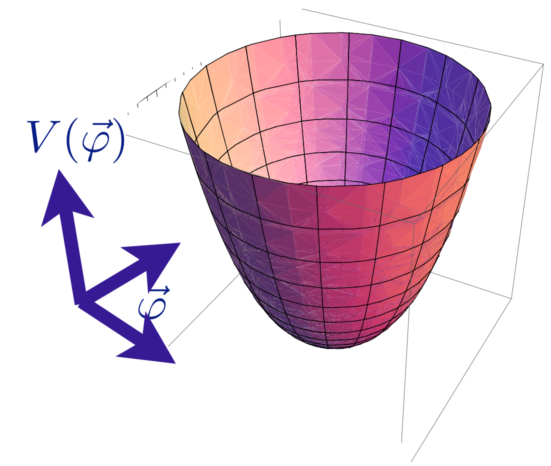
S. Sachdev,
arXiv:0901.4103

Christian Ruegg, Bruce Normand, Masashige Matsumoto, Albert Furrer, Desmond McMorrow, Karl Kramer, Hans–Ulrich Gudel, Severian Gvasaliya, Hannu Mutka, and Martin Boehm, *Phys. Rev. Lett.* **100**, 205701 (2008)

Prediction of quantum field theory

Potential for $\vec{\varphi}$ fluctuations: $V(\vec{\varphi}) = (\lambda - \lambda_c)\vec{\varphi}^2 + u\left(\vec{\varphi}^2\right)^2$

Paramagnetic phase, $\lambda > \lambda_c$



Expand about $\vec{\varphi} = 0$:

$$V(\vec{\varphi}) \approx (\lambda - \lambda_c)\vec{\varphi}^2$$

Yields 3 particles with energy gap $\sim \sqrt{(\lambda - \lambda_c)}$

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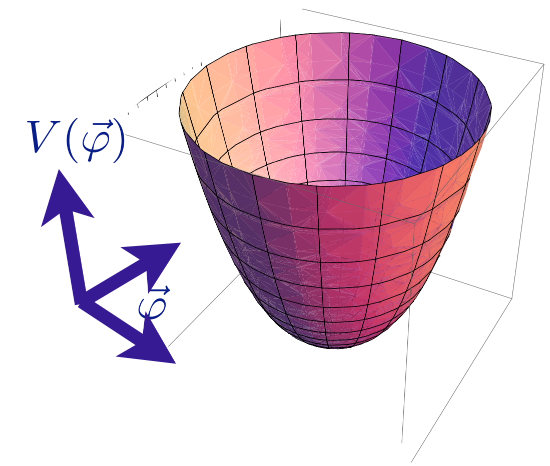

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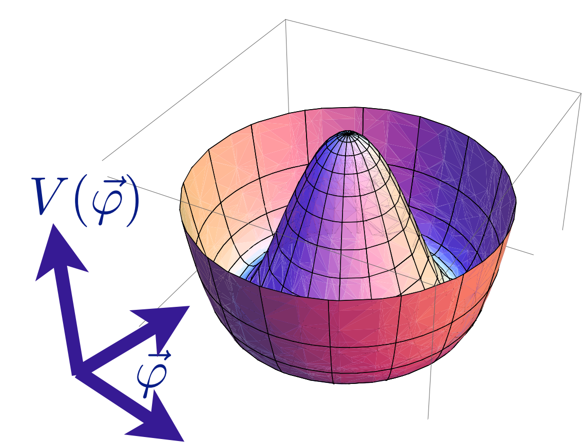

Expand $\vec{\varphi} = \left(0, 0, \sqrt{(\lambda_c - \lambda)/(2u)}\right) + \vec{\varphi}_1$:

$$V(\vec{\varphi}) \approx 2(\lambda_c - \lambda)\varphi_{1z}^2$$

Yields 2 gapless spin waves and one Higgs particle with energy gap $\sim \sqrt{2(\lambda_c - \lambda)}$

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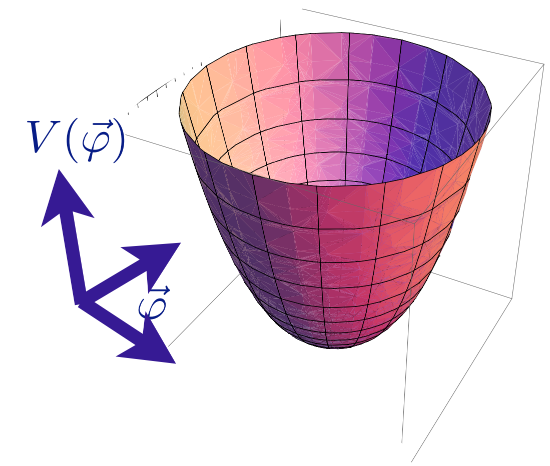

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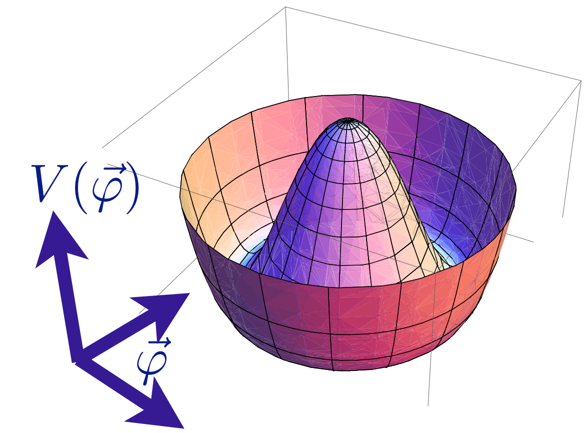
Yields 3 particles with energy gap $\sim \sqrt{(\lambda - \lambda_c)}$

Néel phase, $\lambda < \lambda_c$

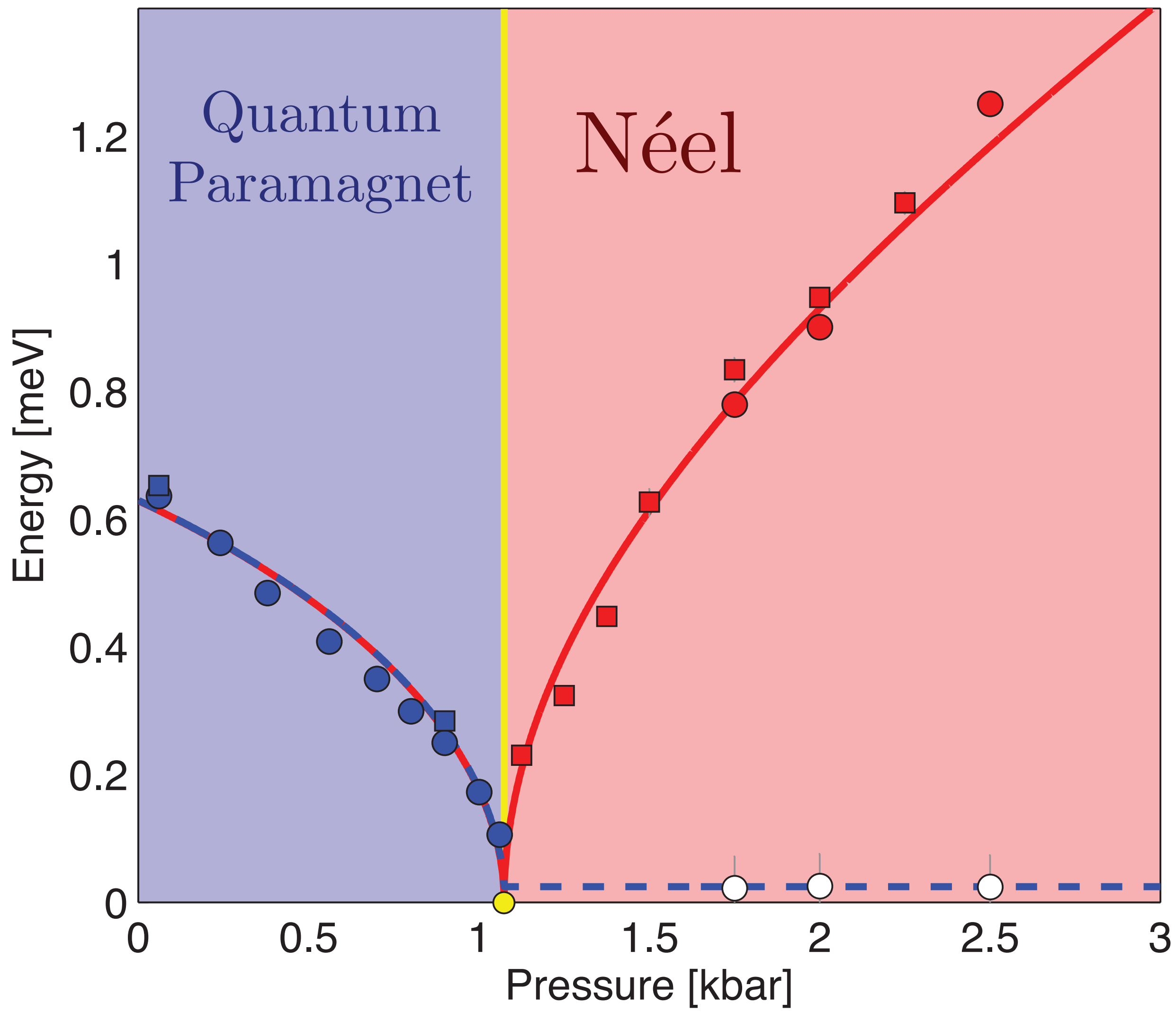


Expand $\vec{\varphi} = \left(0, 0, \sqrt{(\lambda_c - \lambda)/(2u)}\right) + \vec{\varphi}_1$:

$$V(\vec{\varphi}) \approx 2(\lambda_c - \lambda)\varphi_{1z}^2$$

Yields 2 gapless spin waves and one Higgs particle with energy gap $\sim \boxed{\sqrt{2}}\sqrt{(\lambda_c - \lambda)}$

Excitations of $TlCuCl_3$ with varying pressure



Christian Ruegg, Bruce Normand, Masashige Matsumoto, Albert Furrer,
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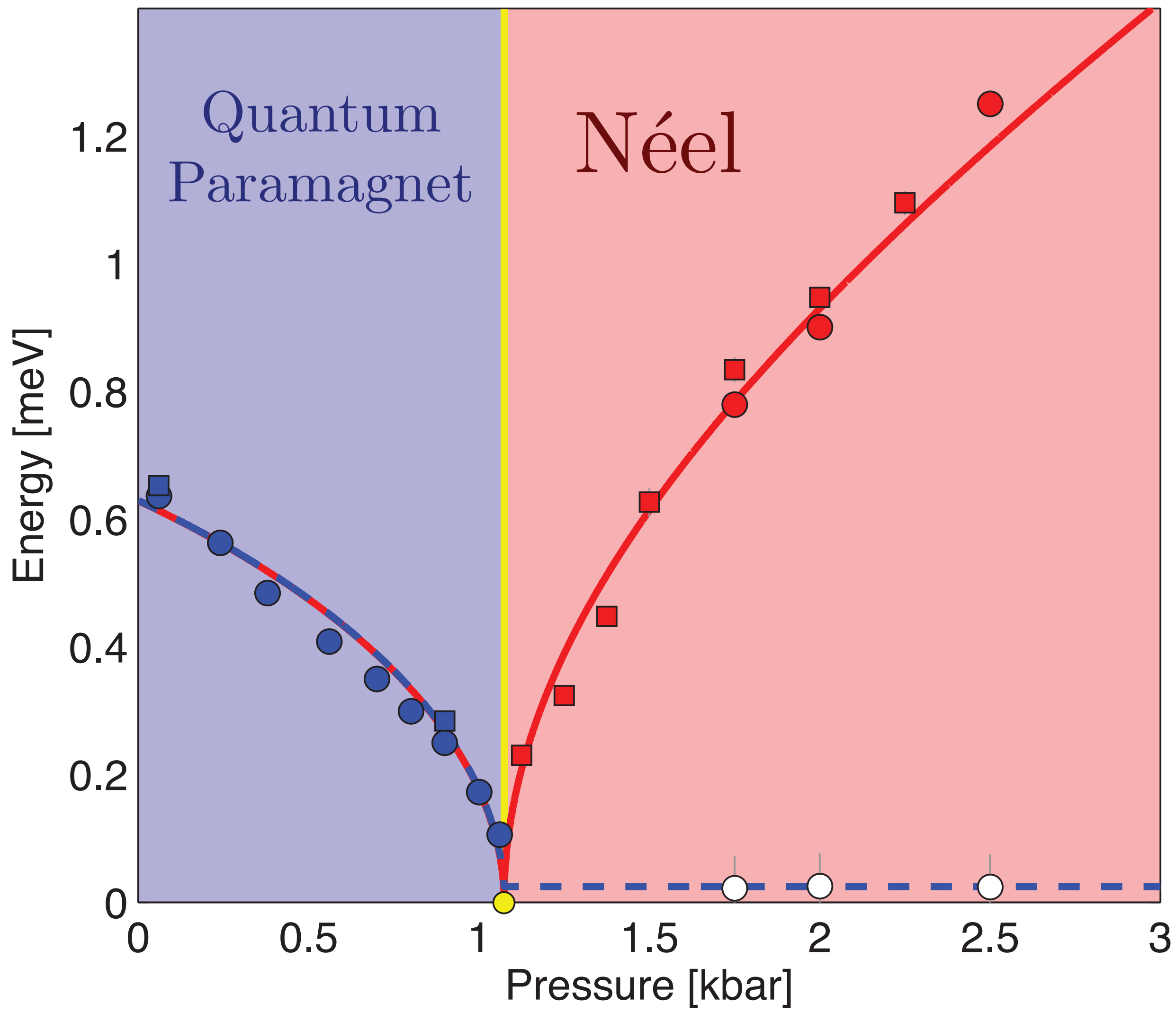

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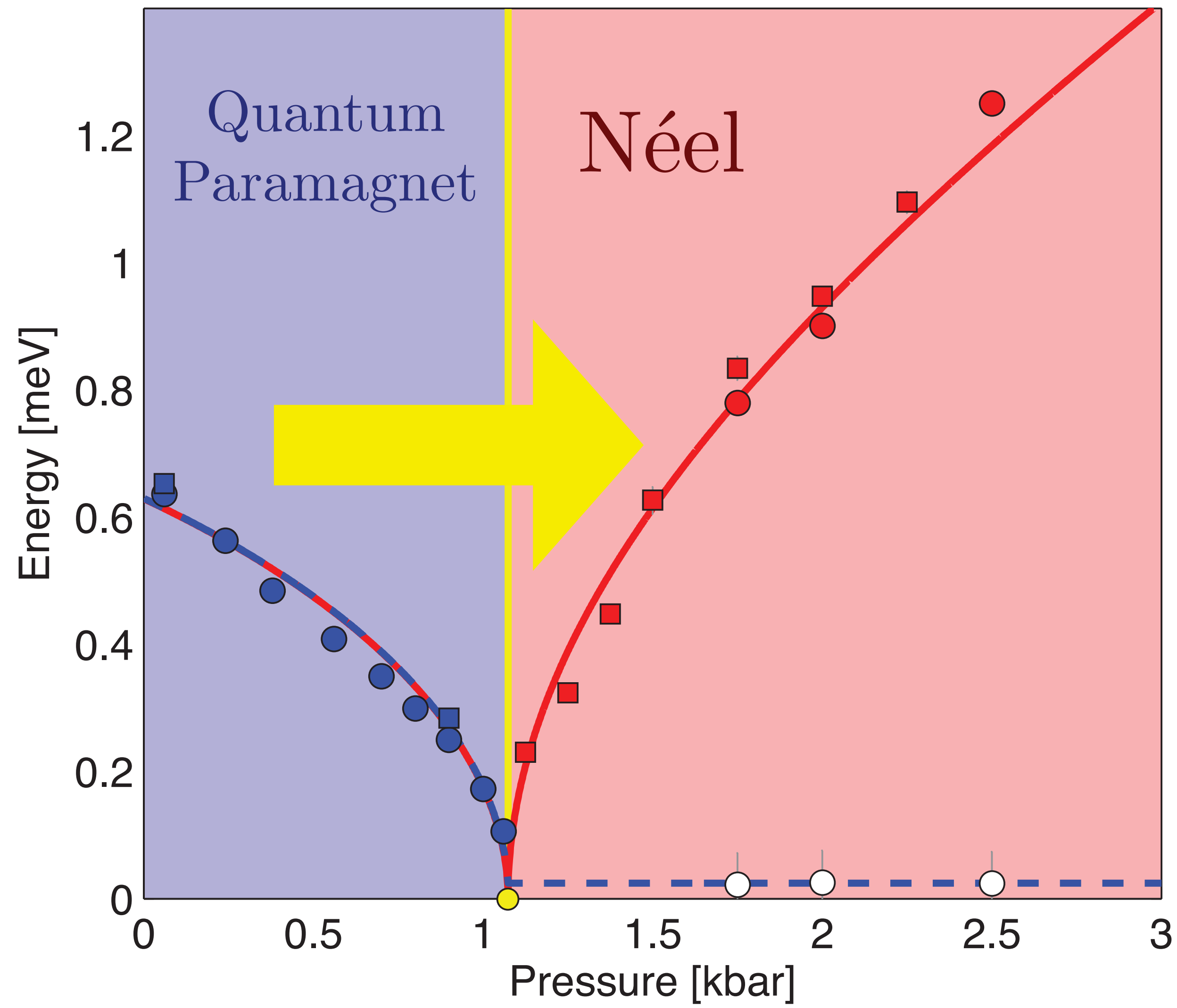

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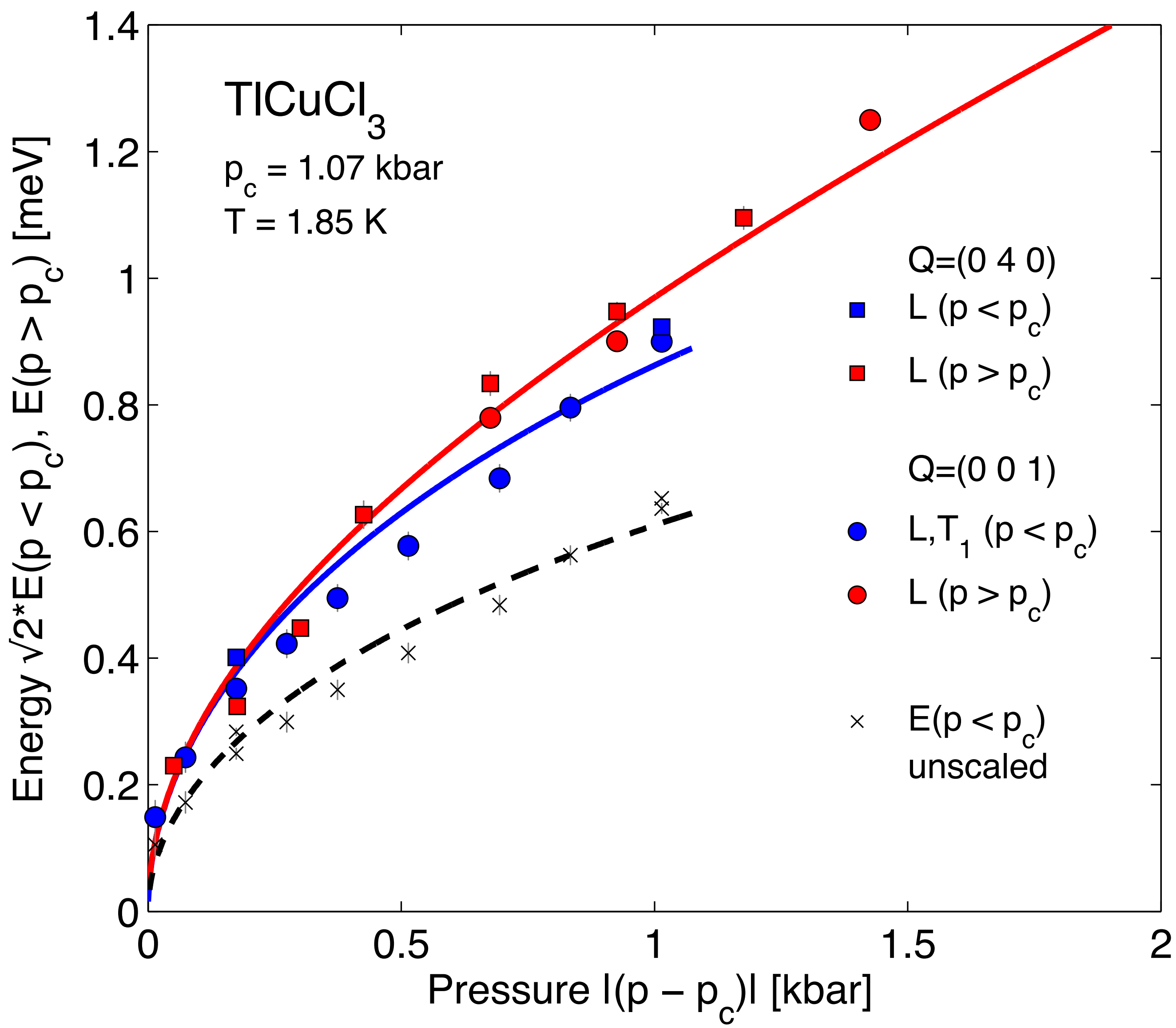
Prediction of quantum field theory

$$\frac{\text{Energy of Higgs boson}}{\text{Energy of triplon}} = \sqrt{2}$$

$$V(\vec{\varphi}) = (\lambda - \lambda_c)\vec{\varphi}^2 + u\left(\vec{\varphi}^2\right)^2$$



S. Sachdev, arXiv:0901.4103

Outline

1. Higgs boson in a 3-dimensional antiferromagnet

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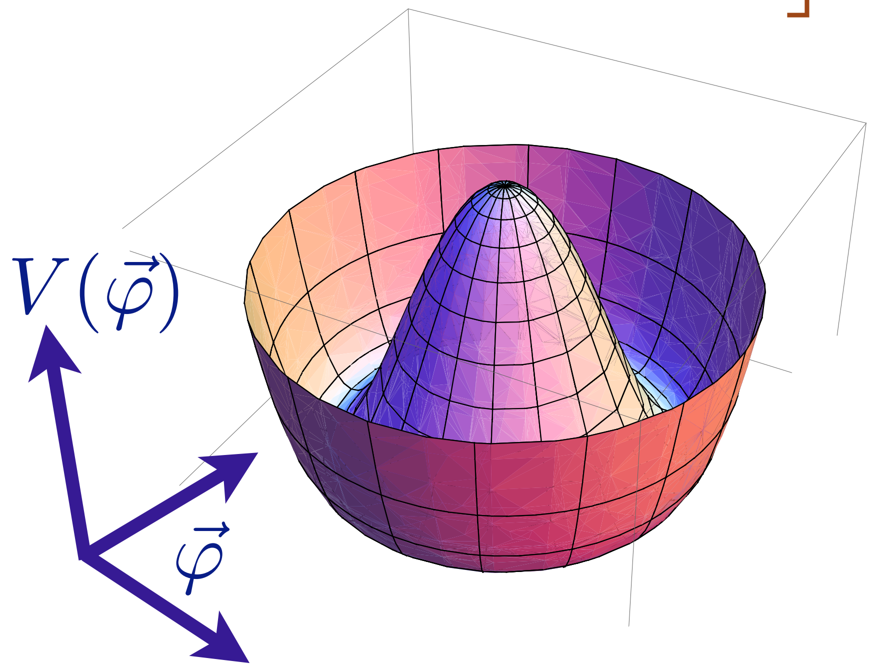
$$\mathcal{S} = \int d^d r d\tau \left[(\partial_\tau \varphi)^2 + c^2 (\nabla_r \vec{\varphi})^2 + (\lambda - \lambda_c) \vec{\varphi}^2 + u \left(\vec{\varphi}^2 \right)^2 \right]$$

- It is possible for the Higgs boson to decay into pairs of Goldstone bosons. In $d = 3$, this process is (marginally) subdominant, and the Higgs boson is well-defined.



A. V. Chubukov, S. Sachdev, and J. Ye, Phys. Rev. B **49**, 11919 (1994).
S. Sachdev, Phys. Rev. B **59**, 14054 (1999).
W. Zwerger, Phys. Rev. Lett. **92**, 027203 (2004).
D. Podolsky, A. Auerbach, and D. P. Arovas, Phys. Rev. B **84**, 174522 (2011).
D. Podolsky and S. Sachdev, Phy. Rev. B **86**, 054508 (2012).

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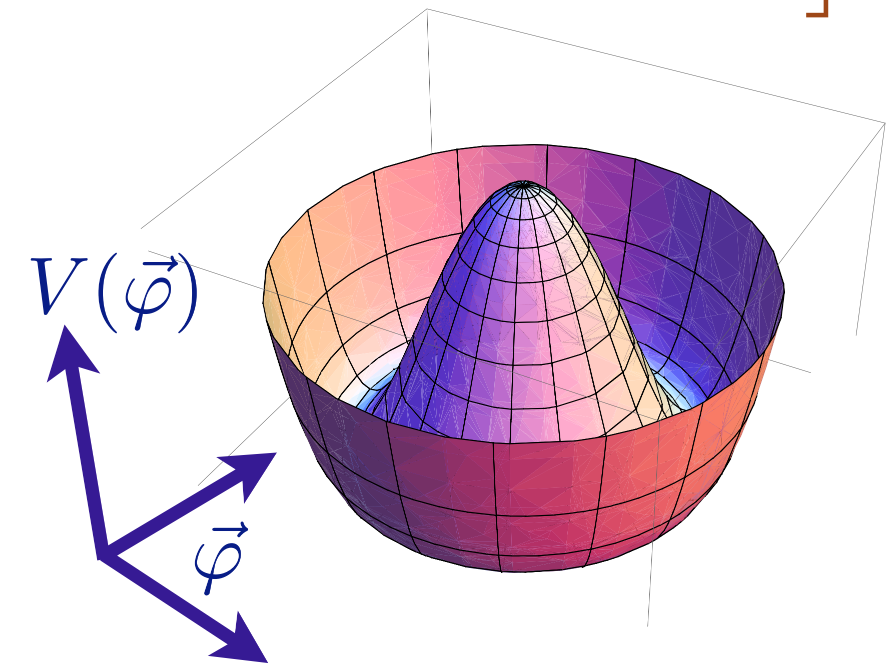

- It is possible for the Higgs boson to decay into pairs of Goldstone bosons. In $d = 3$, this process is (marginally) subdominant, and the Higgs boson is well-defined.

- In $d = 2$ the decay into Goldstone bosons is dominant, and there is no Higgs boson. Nevertheless, there is a Higgs "mode" linked to a pole of response functions in the lower-half of the complex frequency plane.

A. V. Chubukov, S. Sachdev, and J. Ye, Phys. Rev. B **49**, 11919 (1994).
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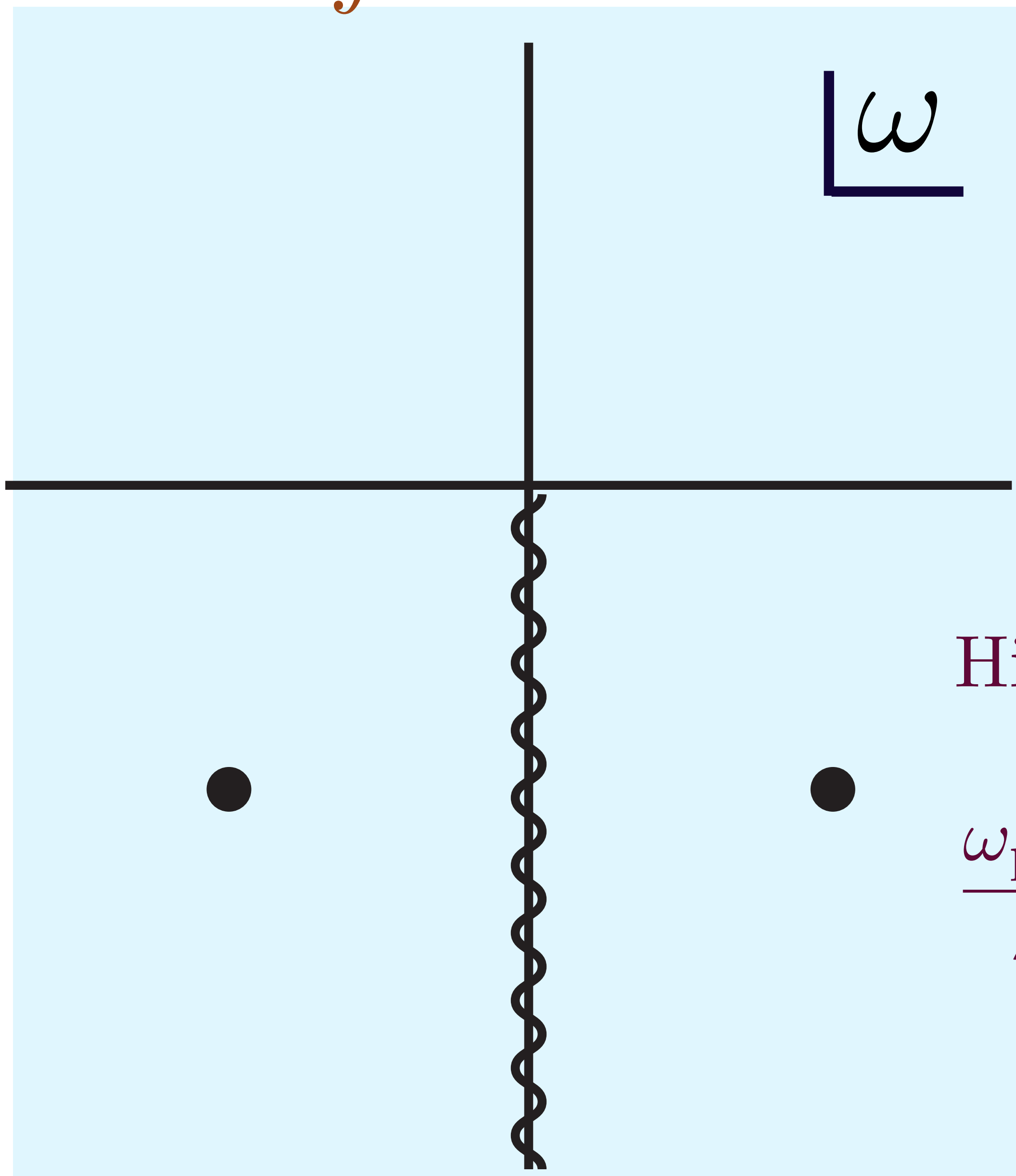
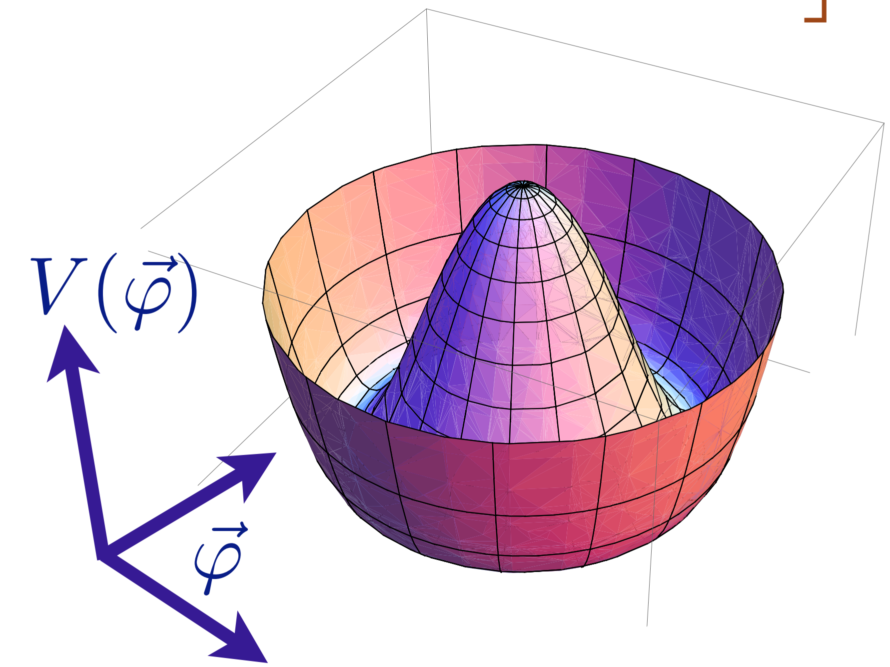




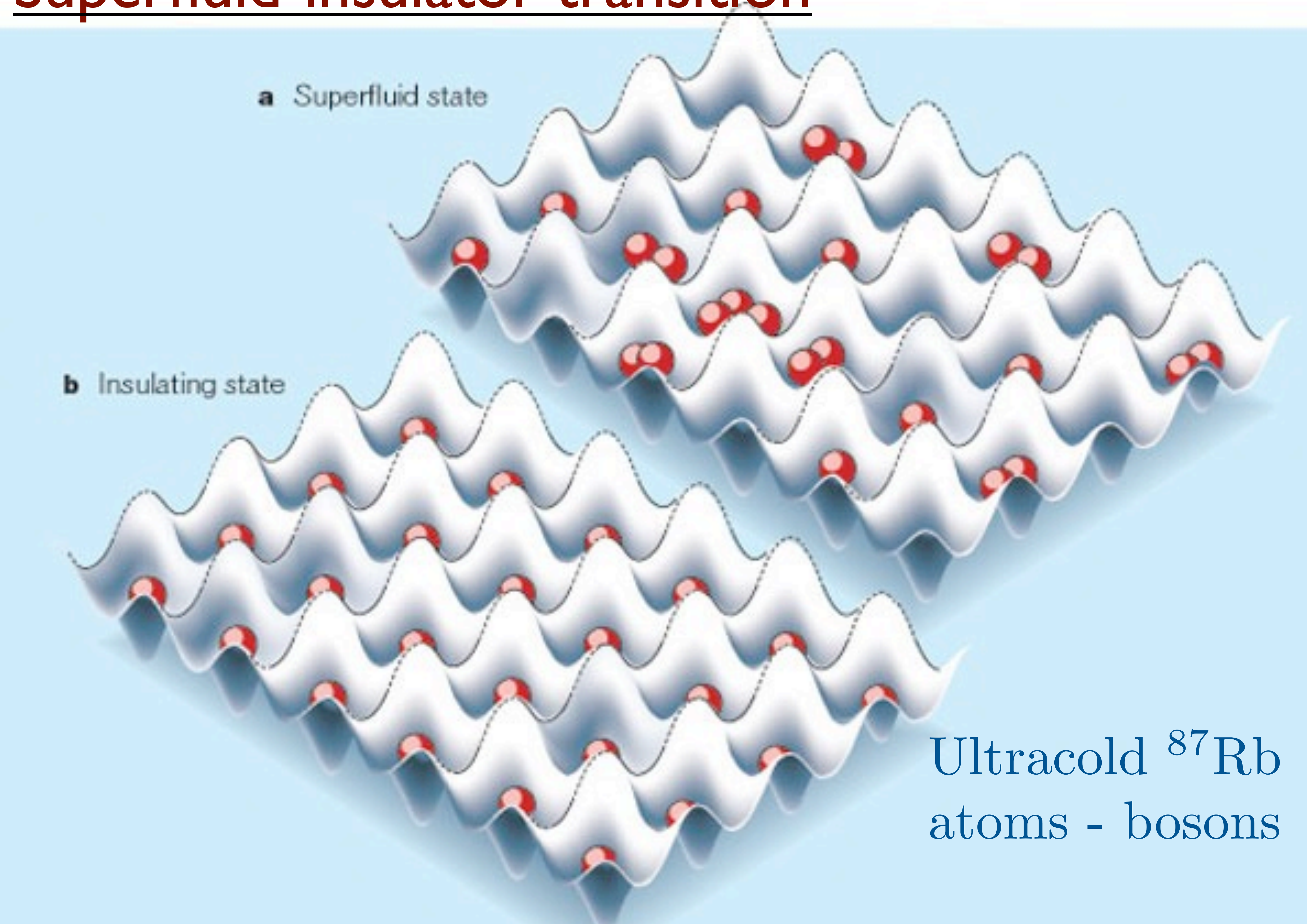
Higgs pole in $d = 2$ at

$$\frac{\omega_{\rm pole}}{\Delta} = -i\,\frac{4}{\pi} + \frac{1}{N}\left(\frac{16\left(4 + \sqrt{2}\log\left(3 - 2\sqrt{2}\right)\right)}{\pi^2} + 2.46531203396\, i \right) + \mathcal{O}\left(\frac{1}{N^2}\right)$$

where Δ is the particle gap at the complementary point in the "paramagnetic" state.

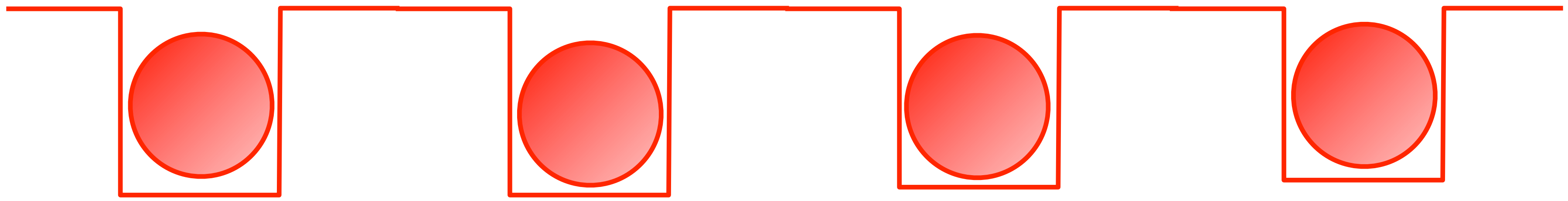
D. Podolsky and S. Sachdev, Phy. Rev. B **86**, 054508 (2012).

Superfluid-insulator transition



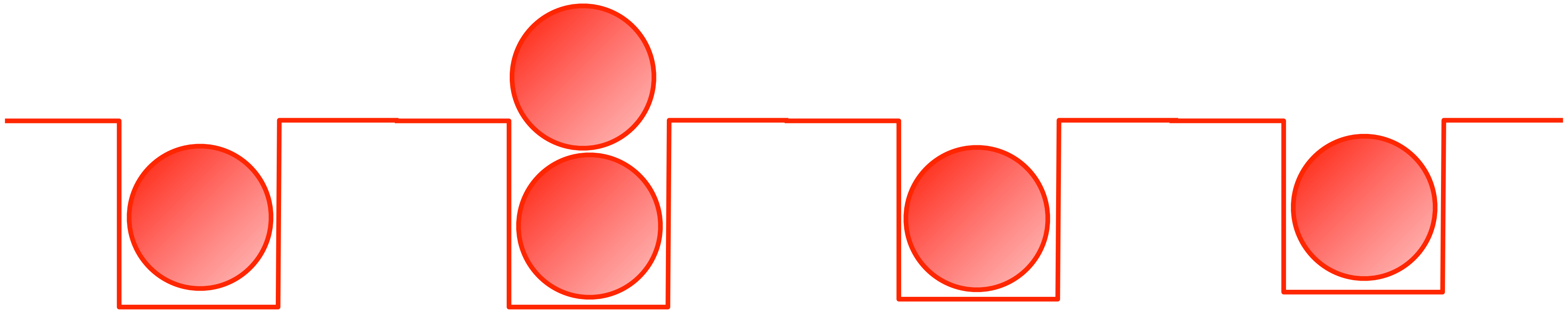
Ultracold ^{87}Rb atoms - bosons

M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).



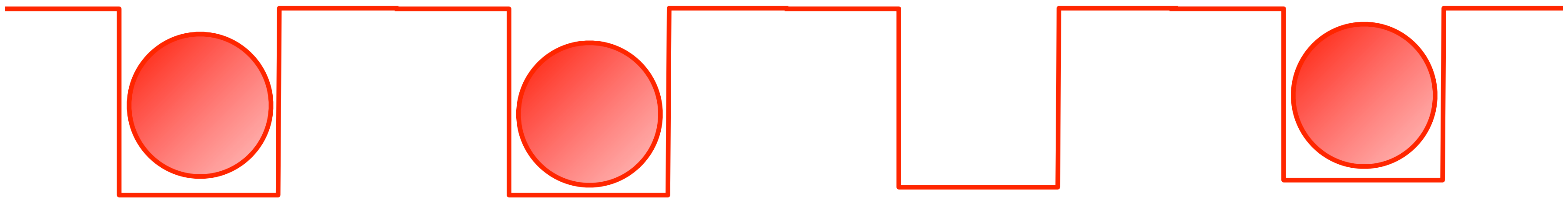
Insulator (the vacuum)
at large repulsion between bosons

Excitations of the insulator:



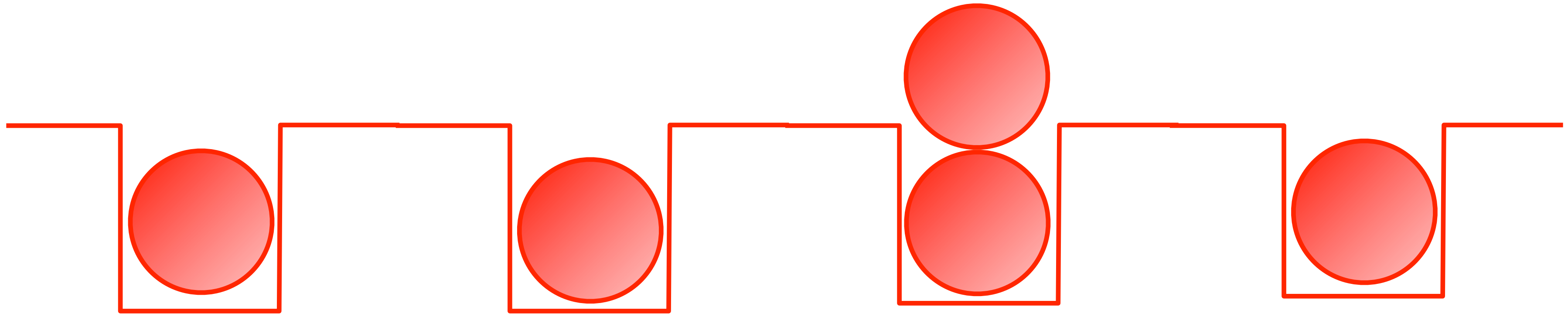
$$\text{Particles} \sim \psi^\dagger = \varphi_1 - i\varphi_2$$

Excitations of the insulator:



Holes $\sim \psi = \varphi_1 + i\varphi_2$

Excitations of the insulator:



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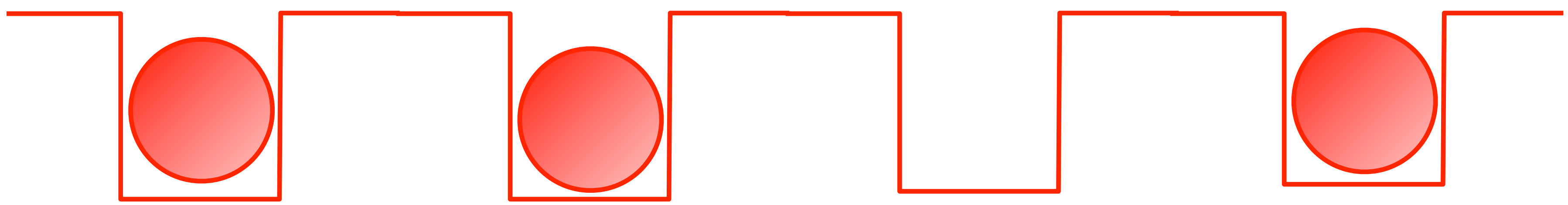

Holes $\sim \psi = \varphi_1 + i\varphi_2$

Density of particles $=$ density of holes $\Rightarrow$
"Relativistic" field theory for $\vec{\varphi}$:

$$\mathcal{S} = \int d^2 r d\tau \left[(\partial_\tau \varphi)^2 + c^2 (\nabla_r \vec{\varphi})^2 + (\lambda - \lambda_c) \vec{\varphi}^2 + u \left(\vec{\varphi}^2 \right)^2 \right]$$

M.P. A. Fisher, P.B. Weichmann, G. Grinstein, and D.S. Fisher, *Phys. Rev. B* **40**, 546 (1989).

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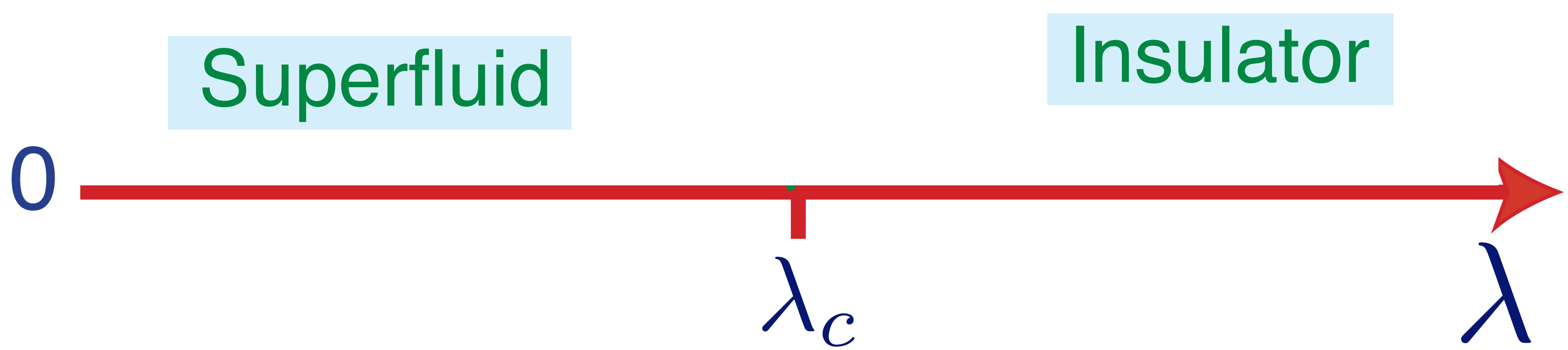
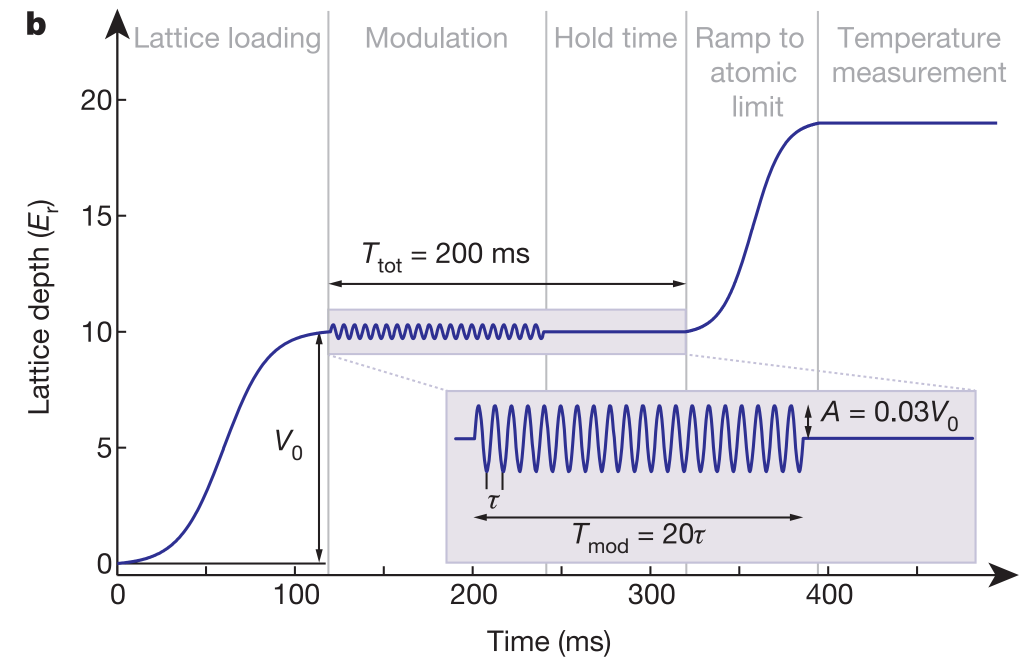
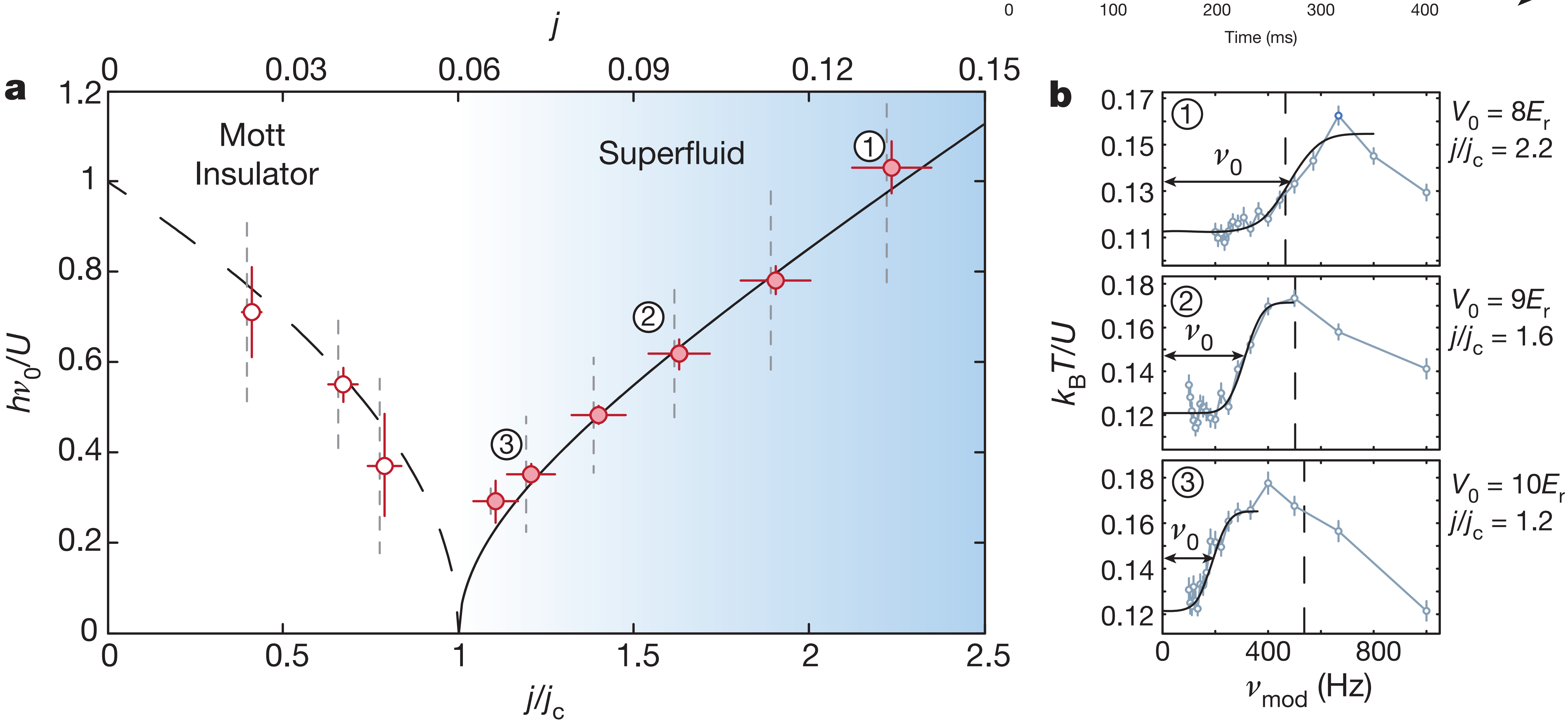
Superfluid-insulator transition described by $\vec{\varphi}$ with $N = 2$ components

$\langle \vec{\varphi} \rangle \neq 0$ $\qquad$ $\langle \vec{\varphi} \rangle = 0$



Observation of Higgs mode across the superfluid-insulator transition of ultracold atoms in a 2-dimensional optical lattice:
Response to modulation of lattice depth scales as expected from the LHP pole





Manuel Endres, Takeshi Fukuhara, David Pekker, Marc Cheneau, Peter Schaub, Christian Gross, Eugene Demler, Stefan Kuhr, and Immanuel Bloch, *Nature* **487**, 454 (2012).

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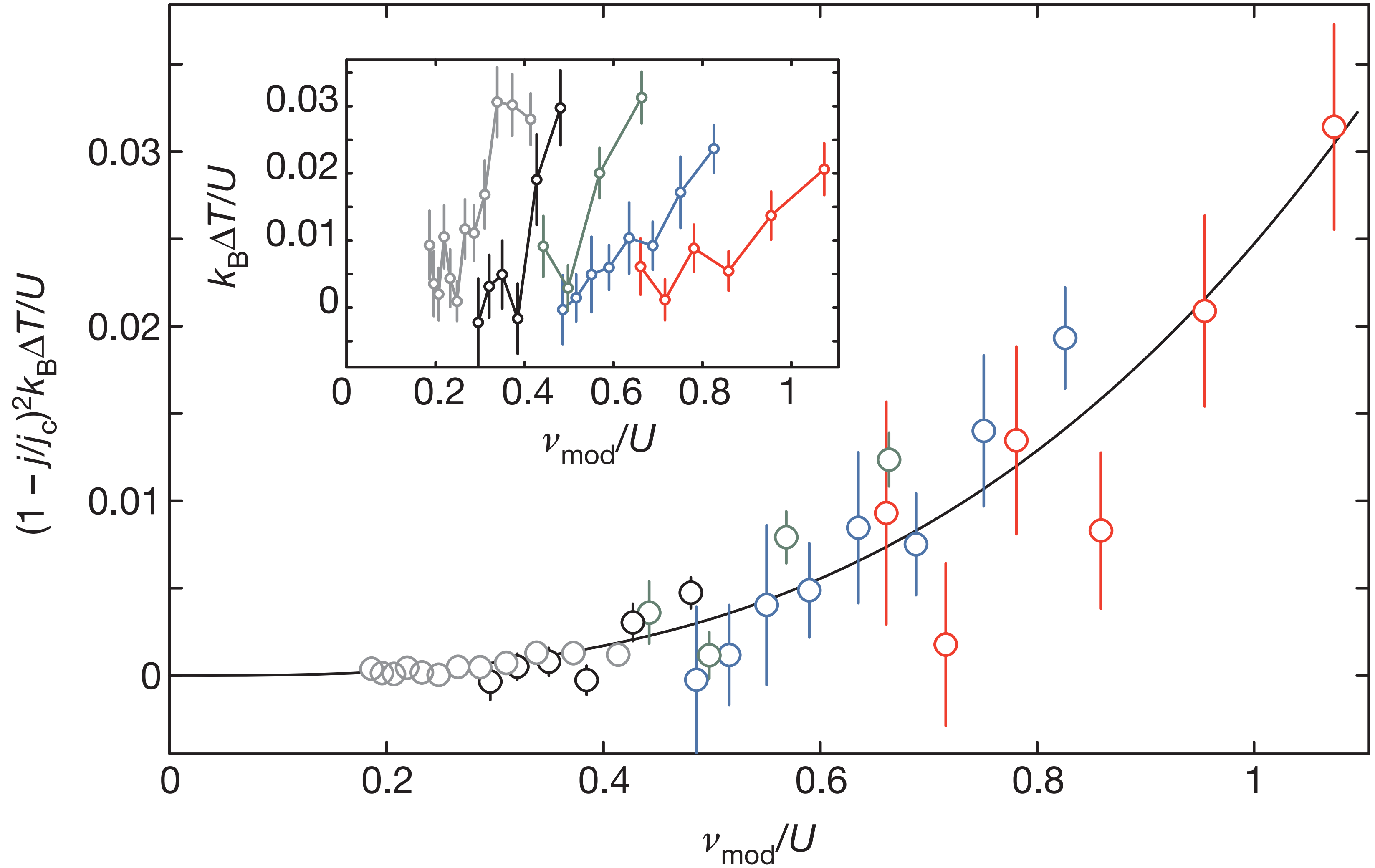

Figure 4 | Scaling of the low-frequency response. The low-frequency response in the superfluid regime shows a scaling compatible with the prediction $(1-j/j_c)^{-2}\nu^3$ (Methods). Shown is the temperature response rescaled with $(1-j/j_c)^2$ for $V_0 = 10E_r$ (grey), $9.5E_r$ (black), $9E_r$ (green), $8.5E_r$ (blue) and $8E_r$ (red) as a function of the modulation frequency. The black line is a fit of the form $a\nu^b$ with a fitted exponent $b = 2.9(5)$. The inset shows the same data points without rescaling, for comparison. Error bars, s.e.m.

D. Podolsky and S. Sachdev, Phy. Rev. B **86**, 054508 (2012).

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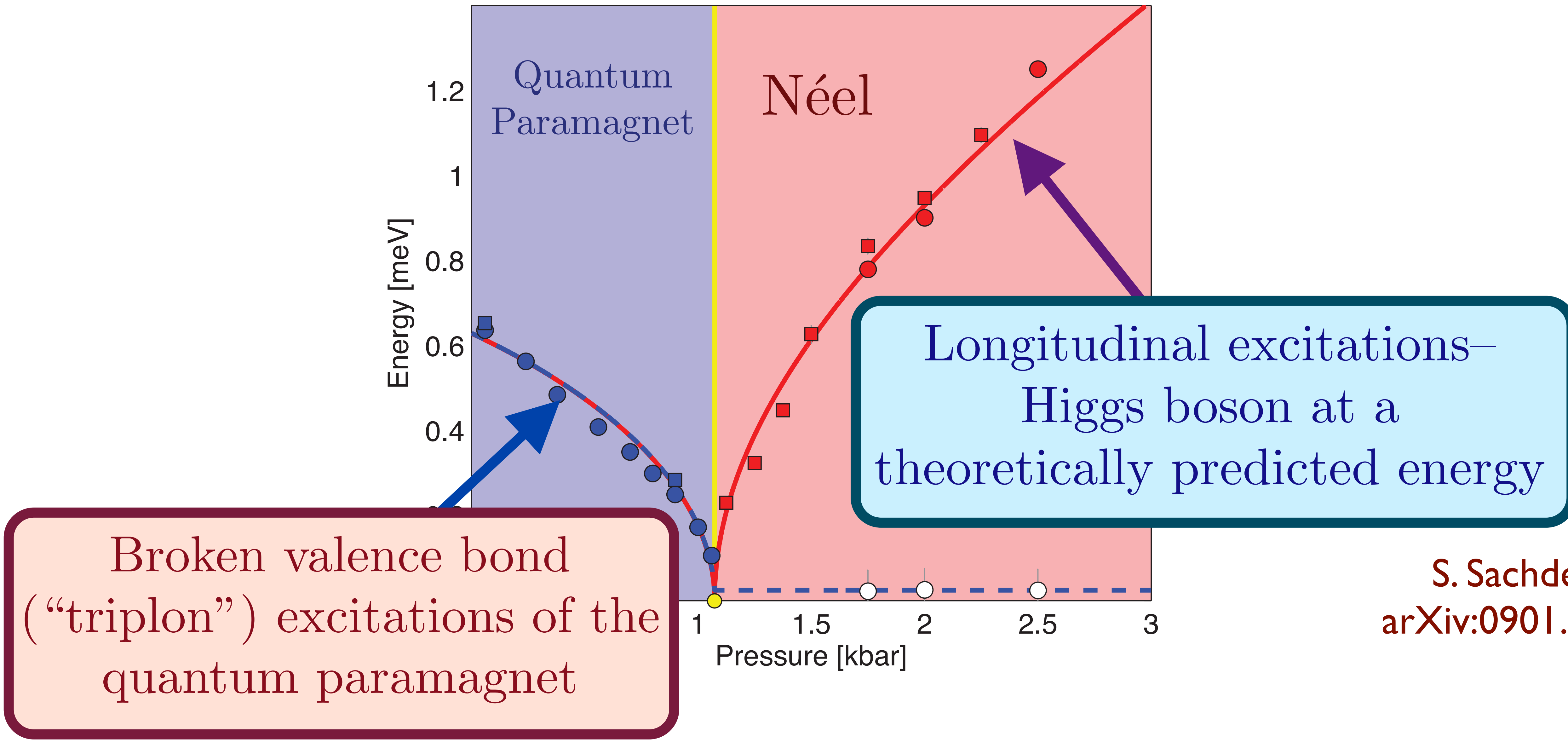

S. Sachdev, arXiv:0901.4103

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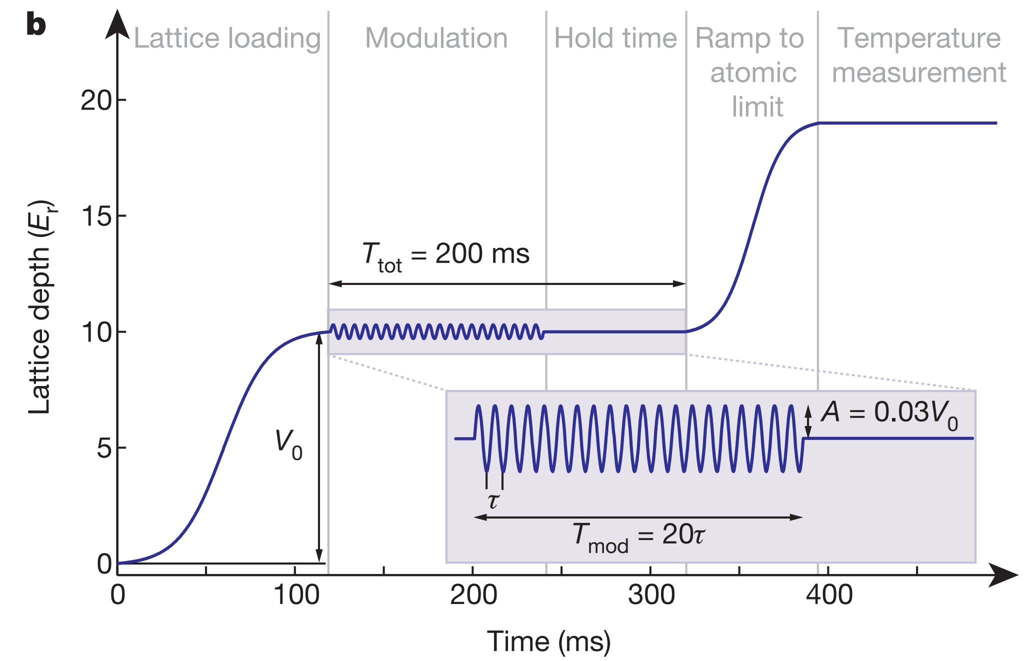

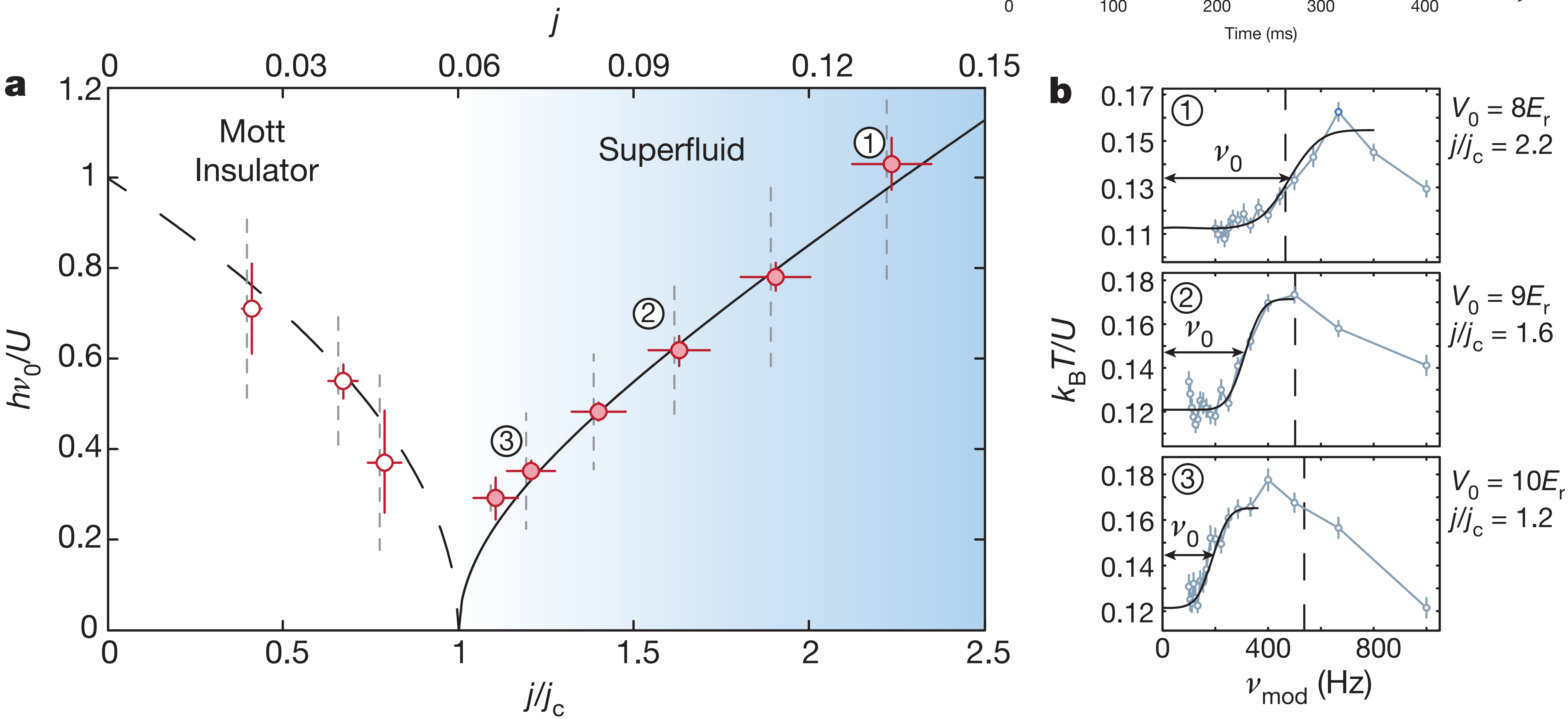

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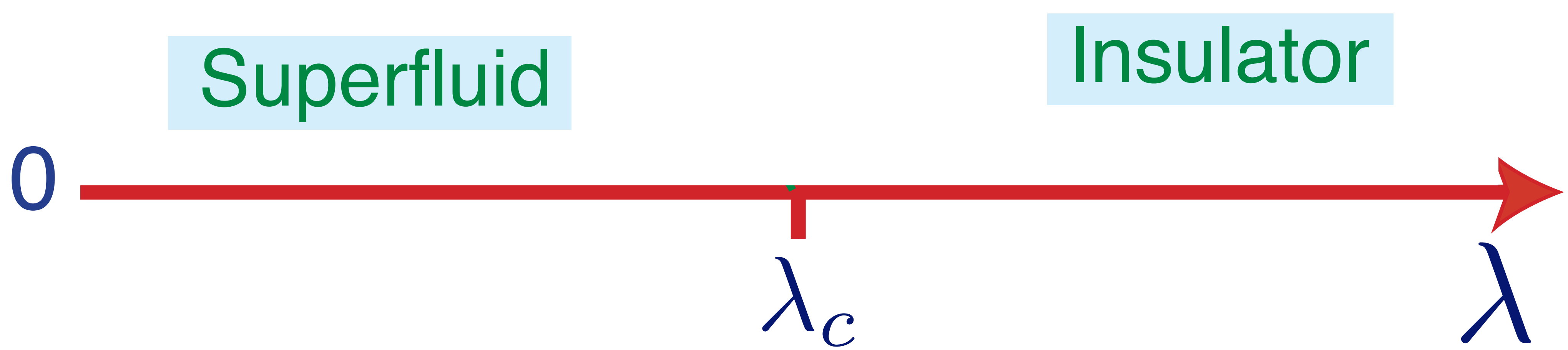

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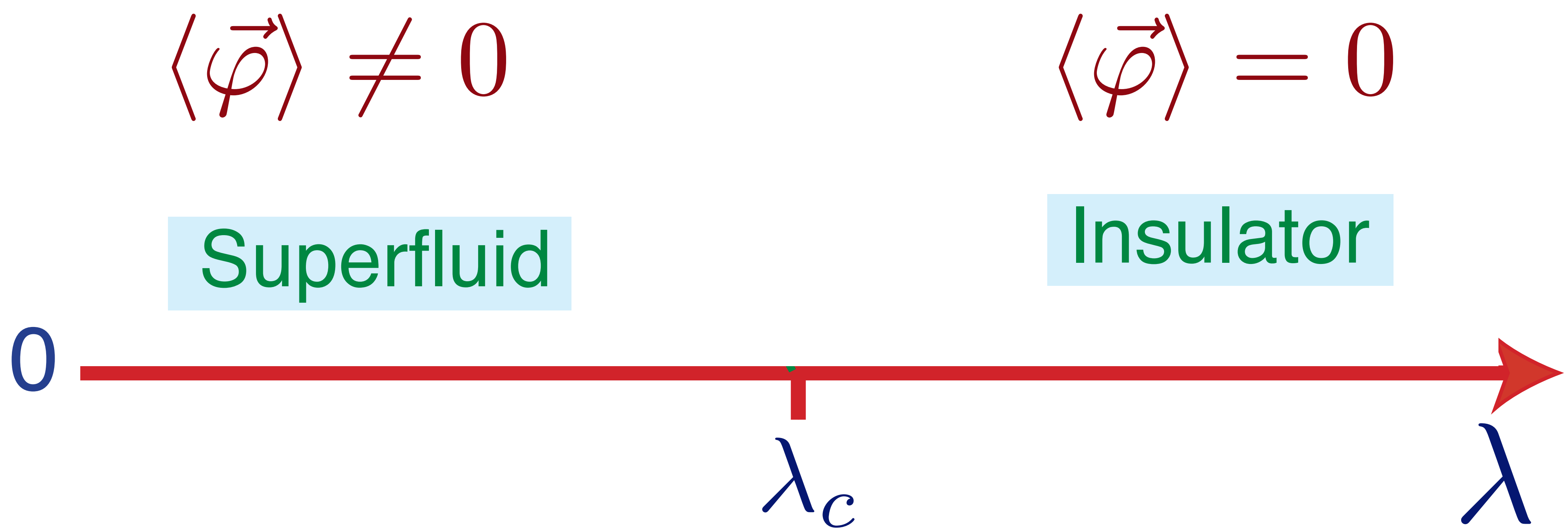

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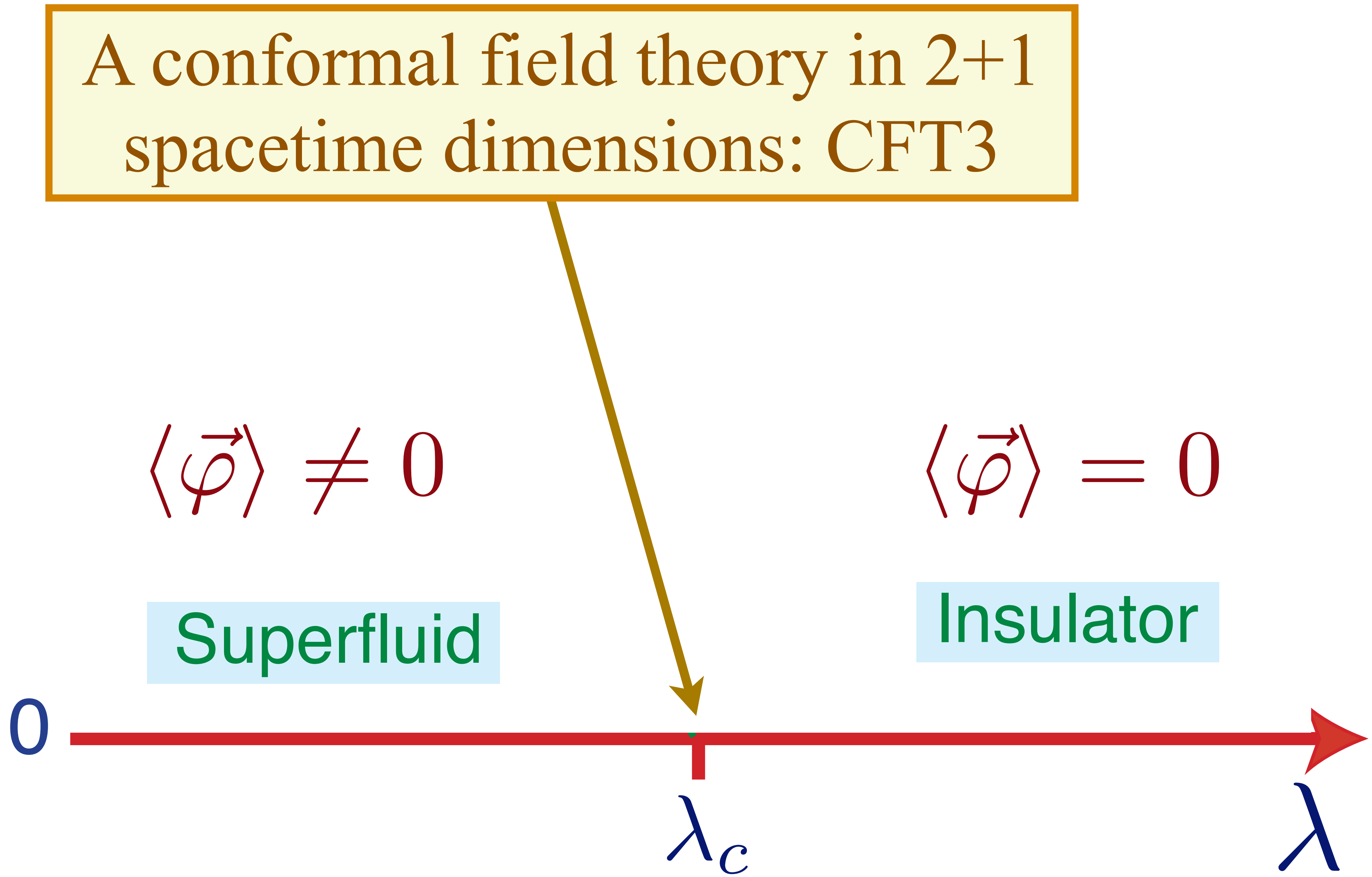
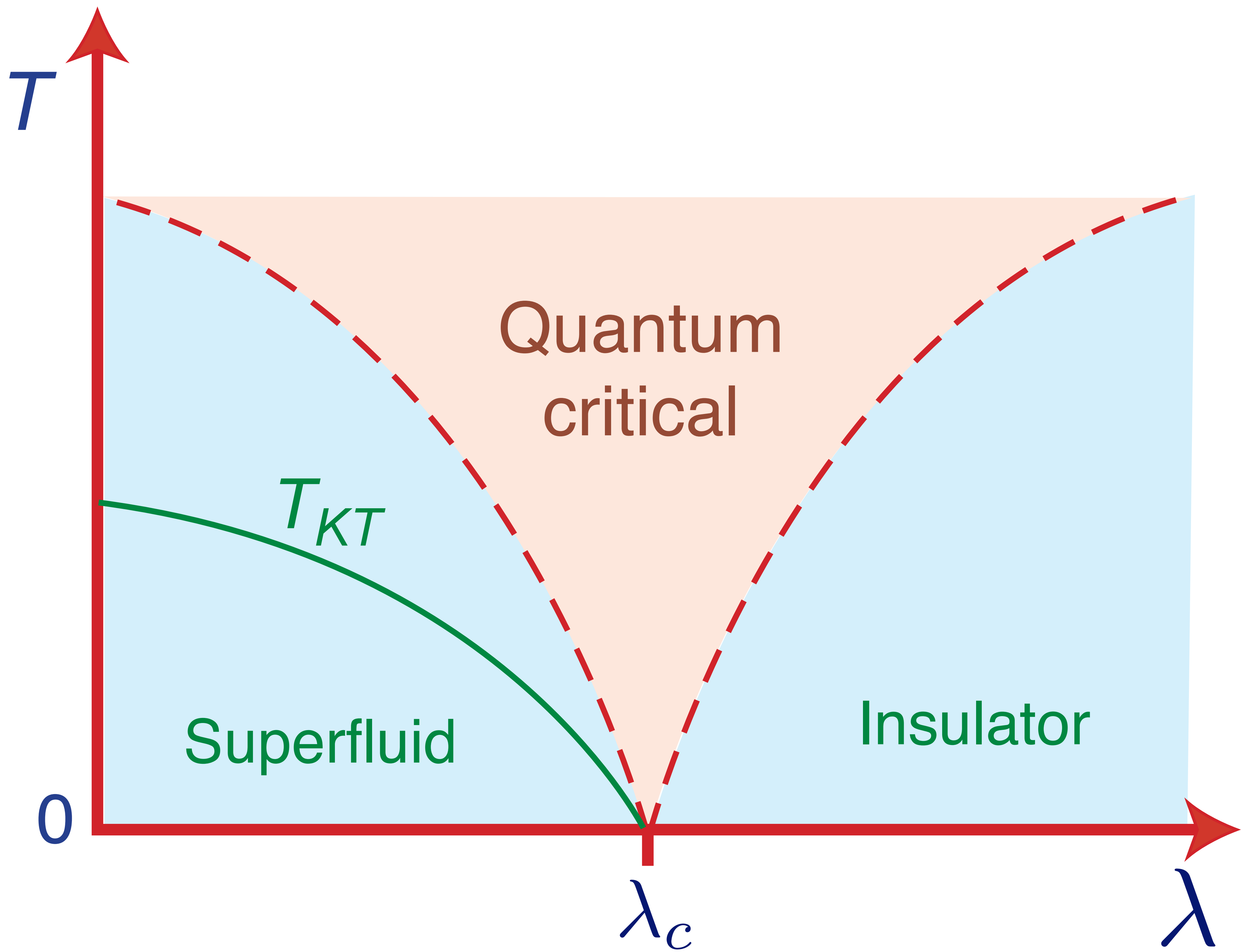
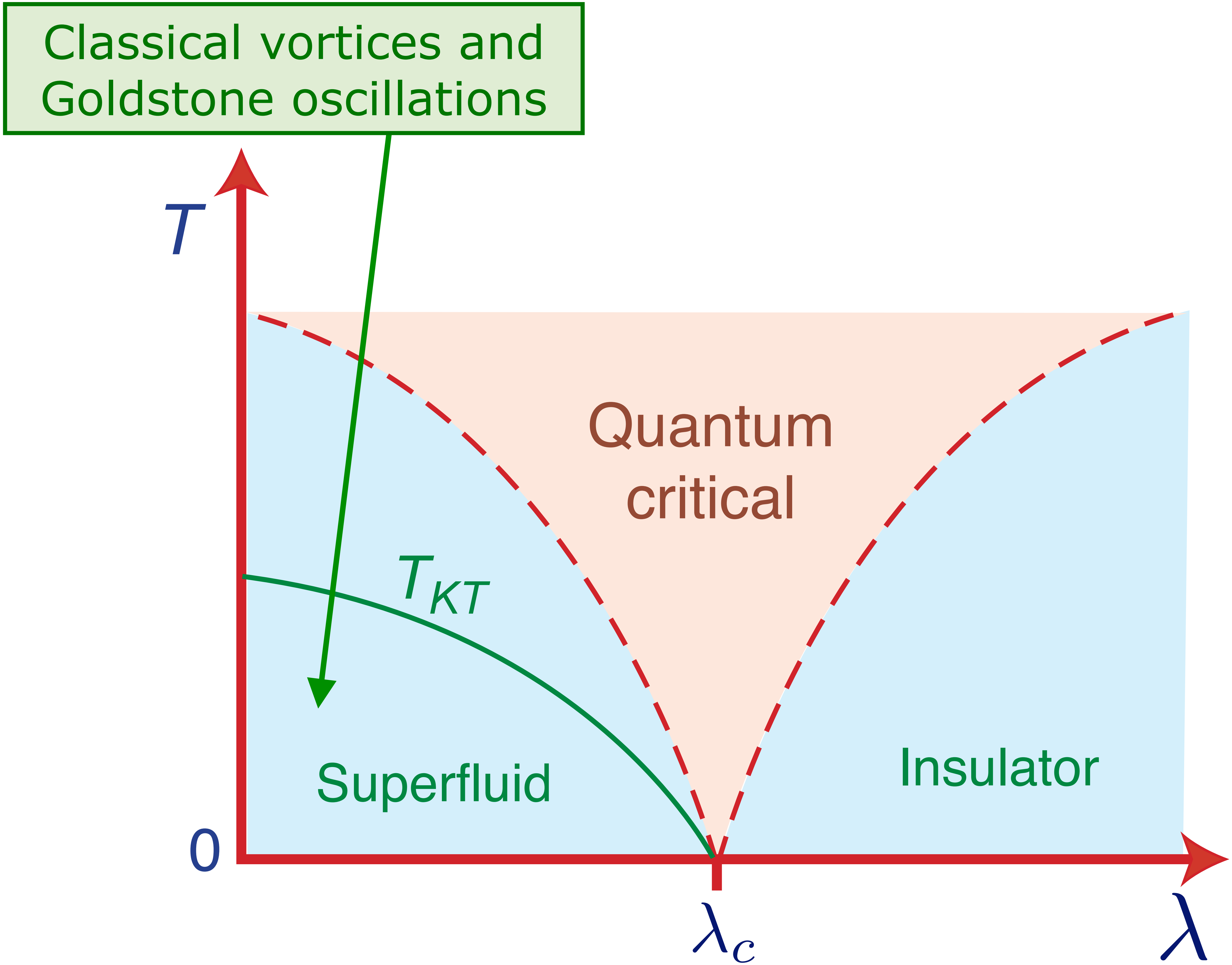
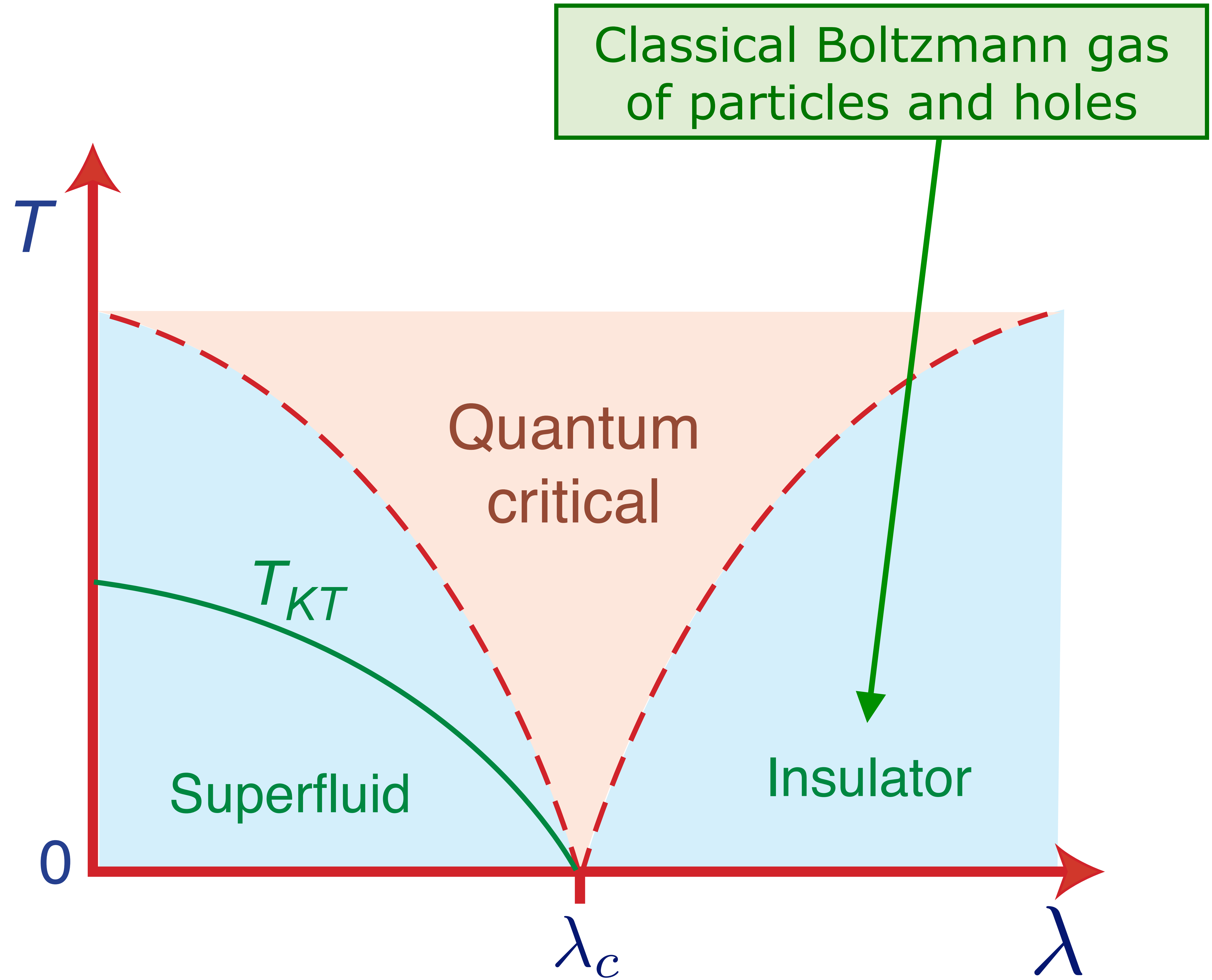
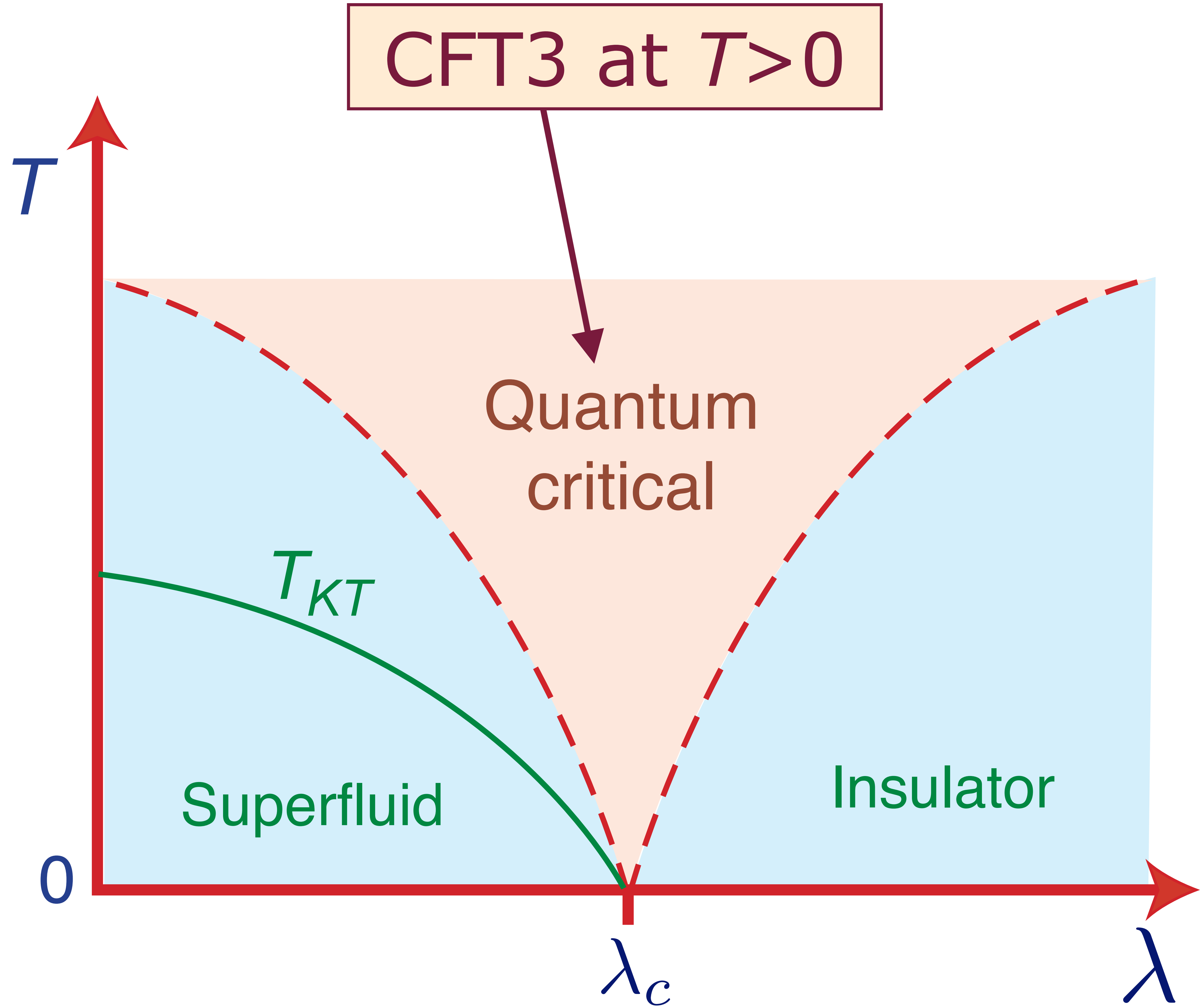




T
Quantum
critical
T_{KT}
Superfluid
Insulator
0
λ_c
λ


Classical vortices and
Goldstone oscillations
T
Quantum
critical
T_{KT}
Superfluid
Insulator
0
λ_c
λ


Classical Boltzmann gas
of particles and holes
T
Quantum
critical
T_{KT}
Superfluid
Insulator
0
λ_c
λ


CFT3 at T>0
T
Quantum
critical
T_KT
Superfluid
Insulator
0
λ_c
λ

Quantum critical dynamics

Quantum *"nearly perfect fluid"*
with shortest possible local equilibration time, $\tau_{\rm eq}$

$$\boxed{\tau_{\rm eq} = \mathcal{C} \frac{\hbar}{k_B T}}$$

where $\mathcal{C}$ is a *universal* constant.

Response functions are characterized by poles in LHP
with $\omega \sim k_B T / \hbar$
(analogs of Higgs pole)

S. Sachdev, *Quantum Phase Transitions,* Cambridge (1999).

Quantum critical dynamics

Transport co-oefficients not determined by collision rate, but by universal constants of nature

Conductivity

$$\sigma = \frac{Q^2}{h} \times [\text{Universal constant } \mathcal{O}(1)\ ]$$

(Q is the "charge" of one boson)

M.P.A. Fisher, G. Grinstein, and S.M. Girvin, *Phys. Rev. Lett*. **64**, 587 (1990)
K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Quantum critical dynamics

Transport co-oefficients not determined by collision rate, but by universal constants of nature

Momentum transport

$$\frac{\eta}{s} \equiv \frac{\text{viscosity}}{\text{entropy density}}$$

$$= \frac{\hbar}{k_B} \times [\text{Universal constant } \mathcal{O}(1)\ ]$$

P. Kovtun, D. T. Son, and A. Starinets, *Phys. Rev. Lett.* **94,** 11601 (2005)

Quantum critical dynamics

Describe charge transport using Boltzmann theory of interacting bosons:

$$\frac{dv}{dt} + \frac{v}{\tau_c} = F.$$

This gives a frequency (ω) dependent conductivity

$$\sigma(\omega) = \frac{\sigma_0}{1 - i\,\omega\,\tau_c}$$

where $\tau_c \sim \hbar/(k_B T)$ is the time between boson collisions.

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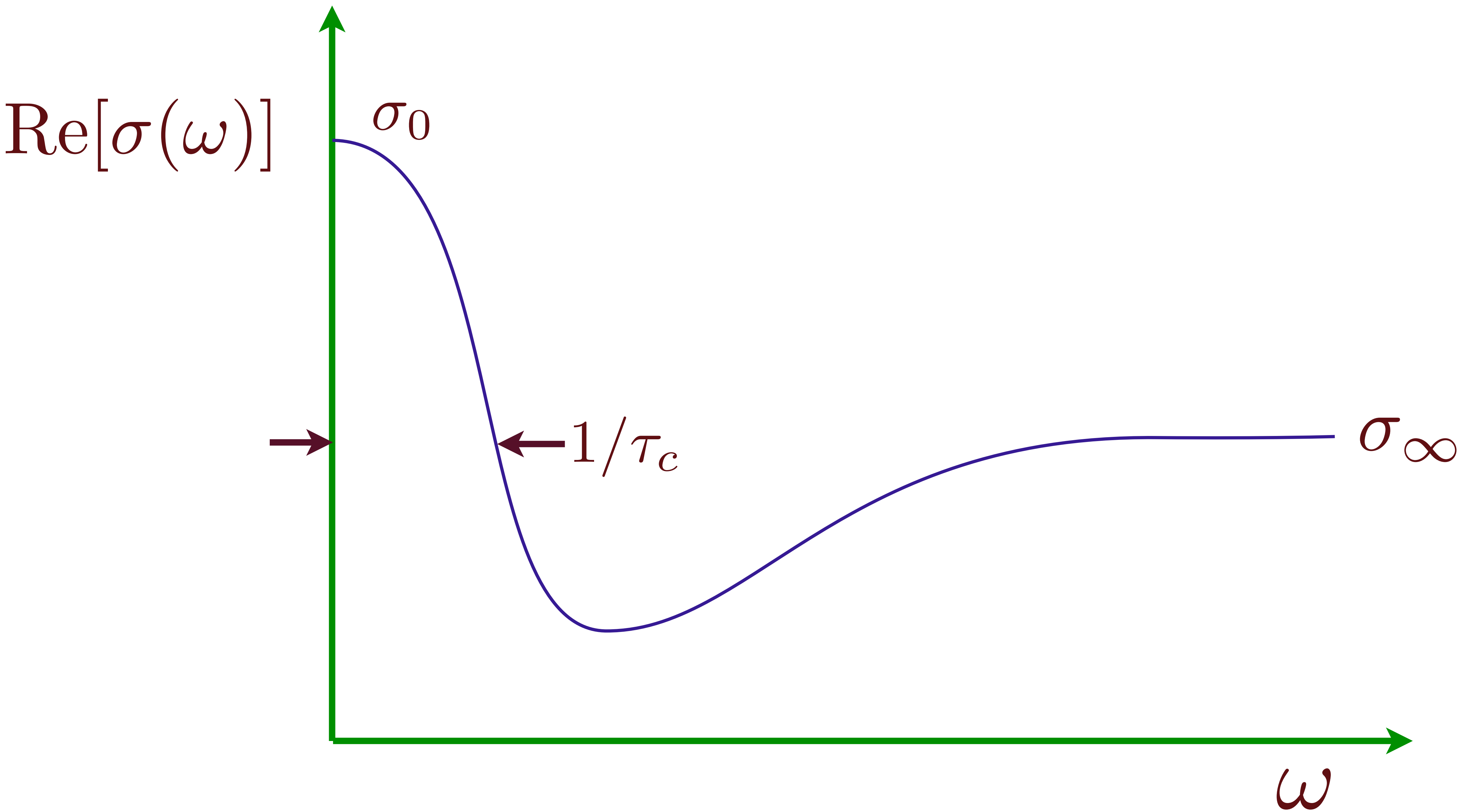
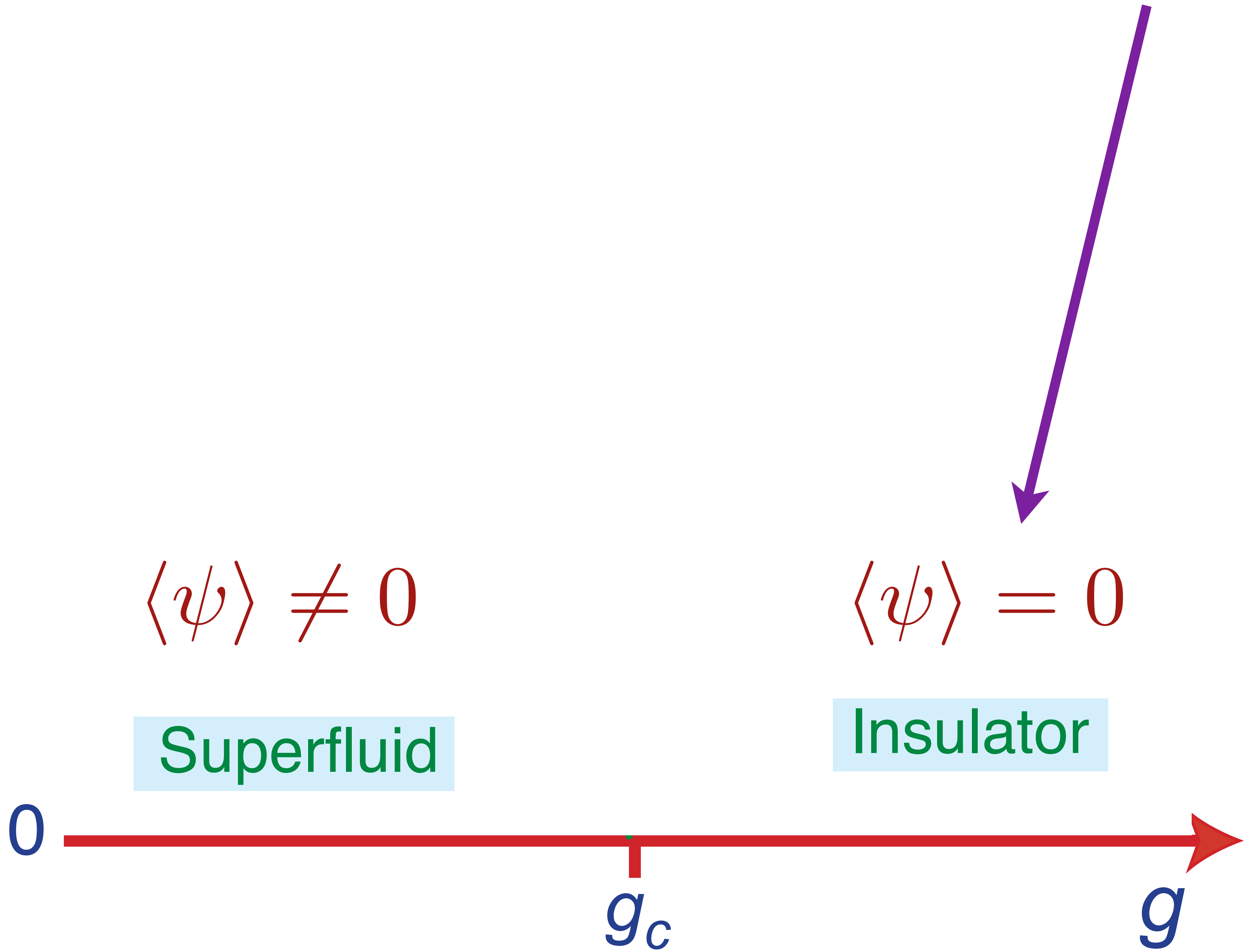
Also, we have $\sigma(\omega \to \infty) = \sigma_\infty$, associated with the density of states for particle-hole creation (the "optical conductivity") in the CFT3.

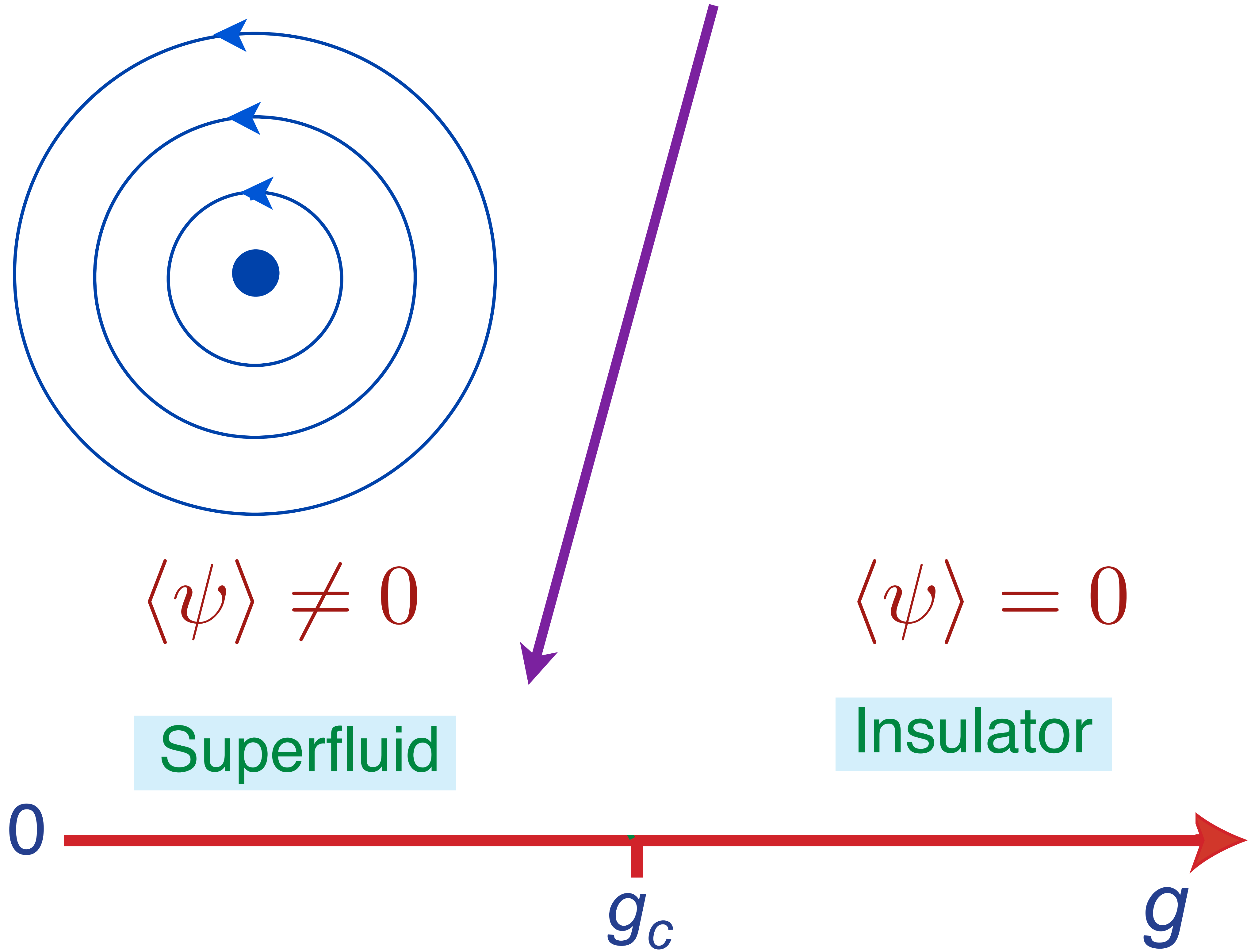
Boltzmann theory of bosons



So far, we have described the quantum critical point using the boson particle and hole excitations of the insulator.



However, we could equally well describe the conductivity using the excitations of the superfluid, which are *vortices*.



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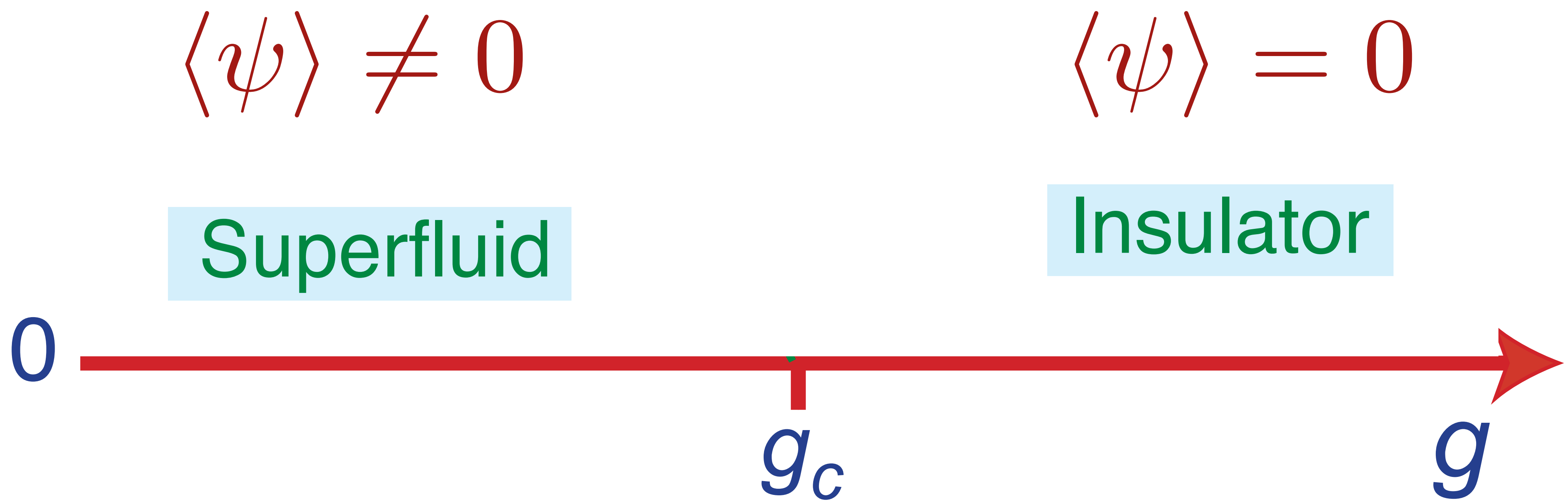
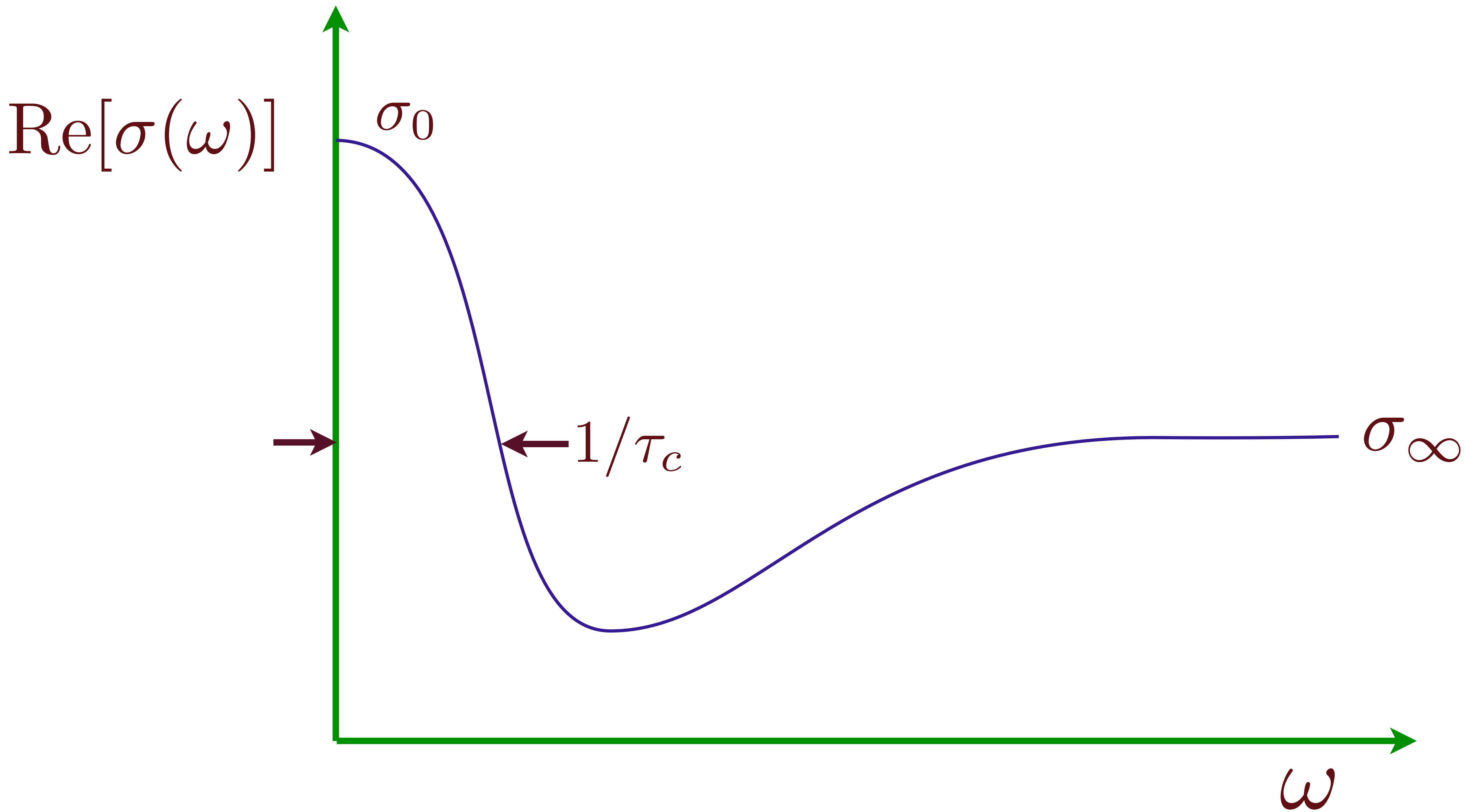
These are quantum particles (in 2+1 dimensions) which described by a (mirror/e.m.) “dual” CFT3 with an emergent U(1) gauge field. Their $T > 0$ dynamics can also be described by a Boltzmann equation:

$$\text{Conductivity} = \text{Resistivity of vortices}$$

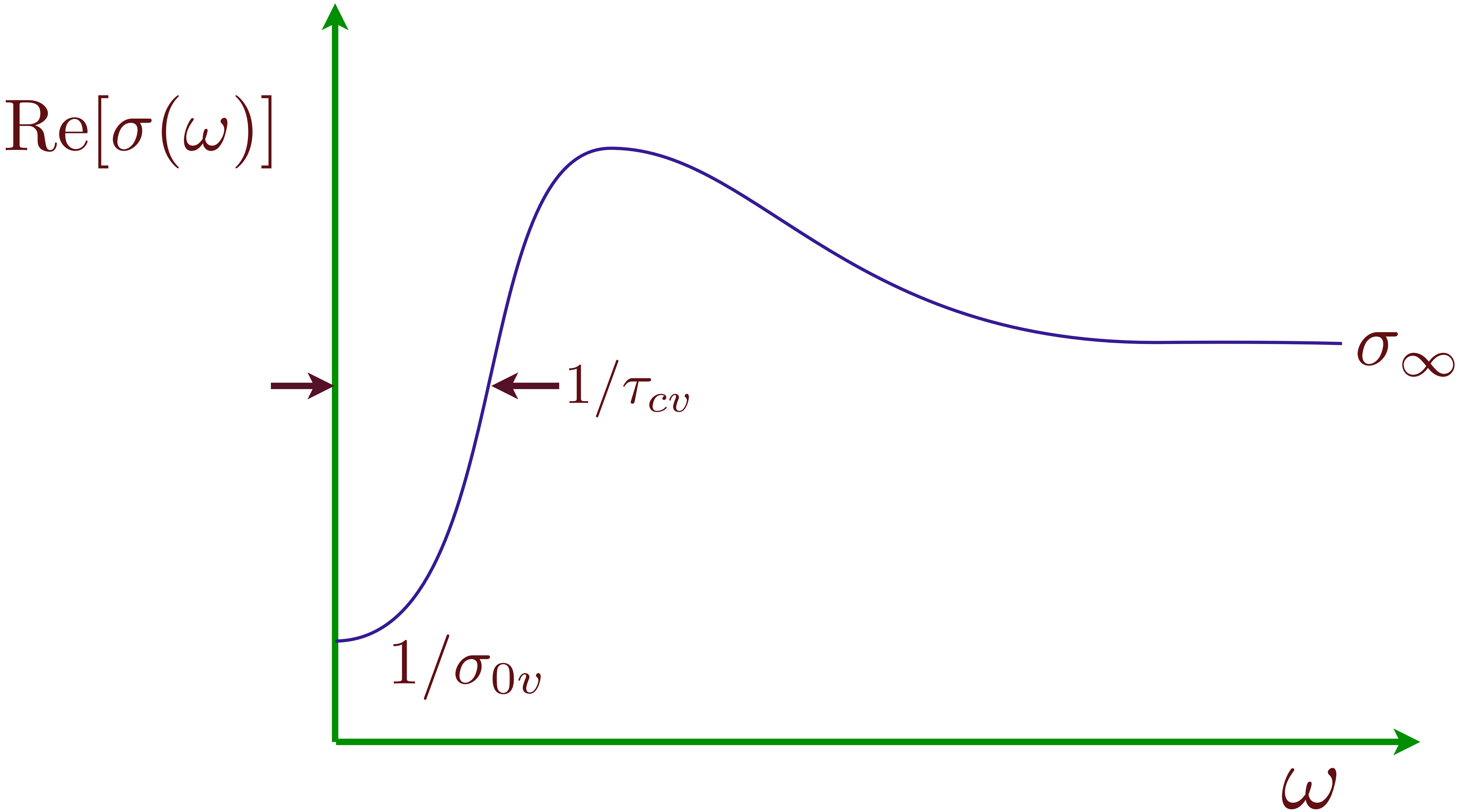


M.P.A. Fisher, *Physical Review Letters* **65**, 923 (1990)

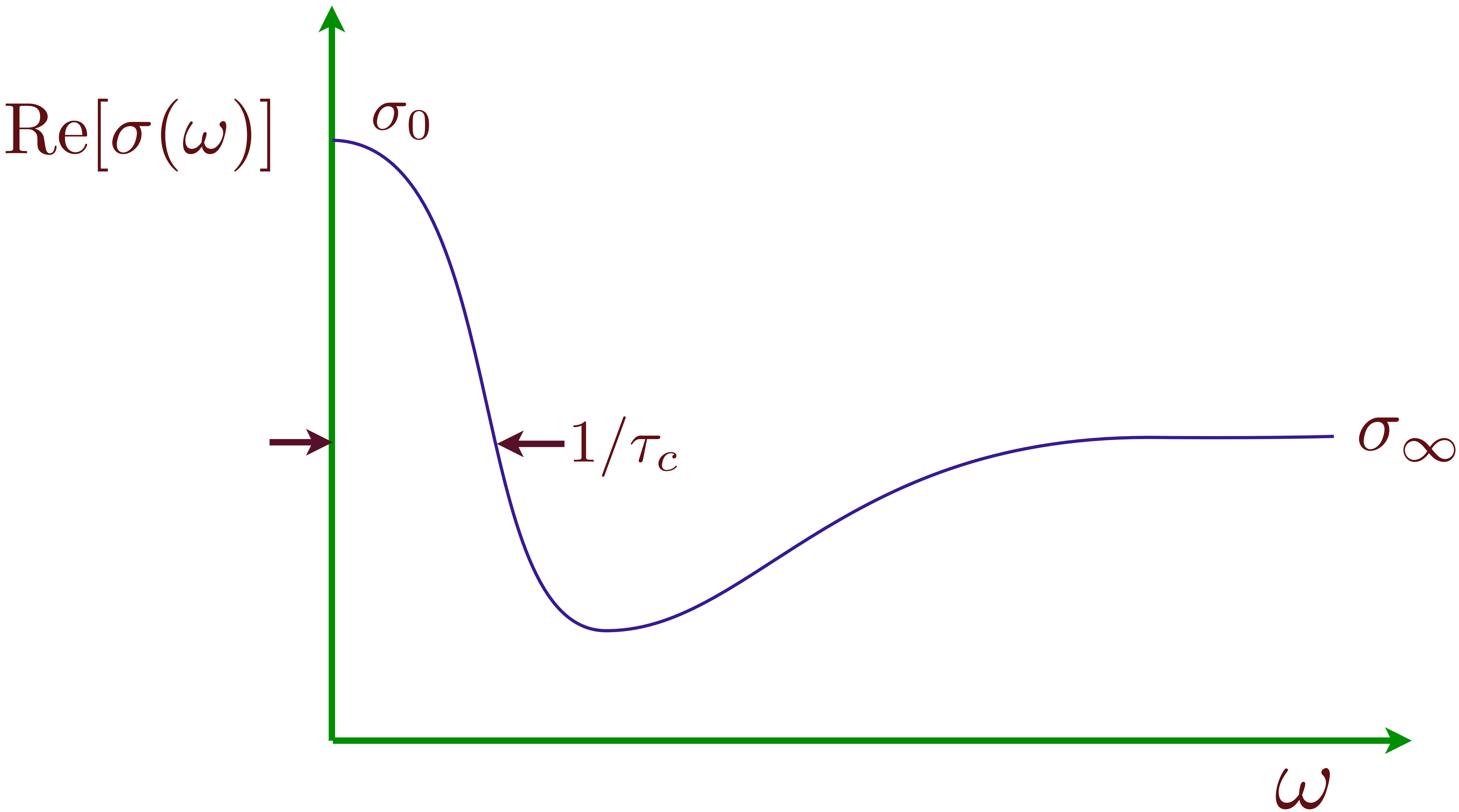
Boltzmann theory of bosons



Boltzmann theory of vortices

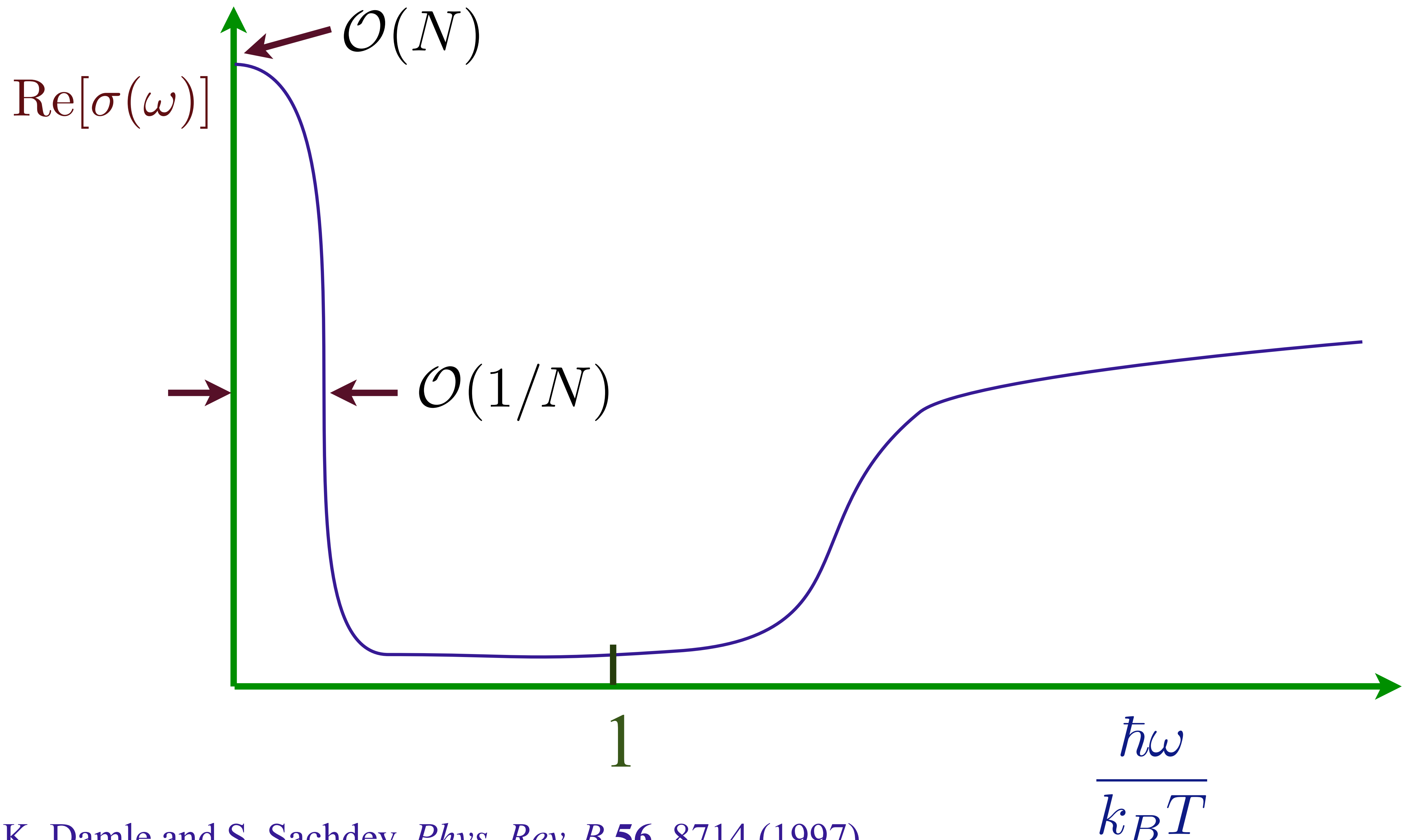


Boltzmann theory of bosons



Vector large *N* expansion for CFT3

$$\sigma = \frac{Q^2}{h} \Sigma \left(\frac{\hbar \omega}{k_B T} \right) ; \quad \Sigma \to \text{a universal function}$$



K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

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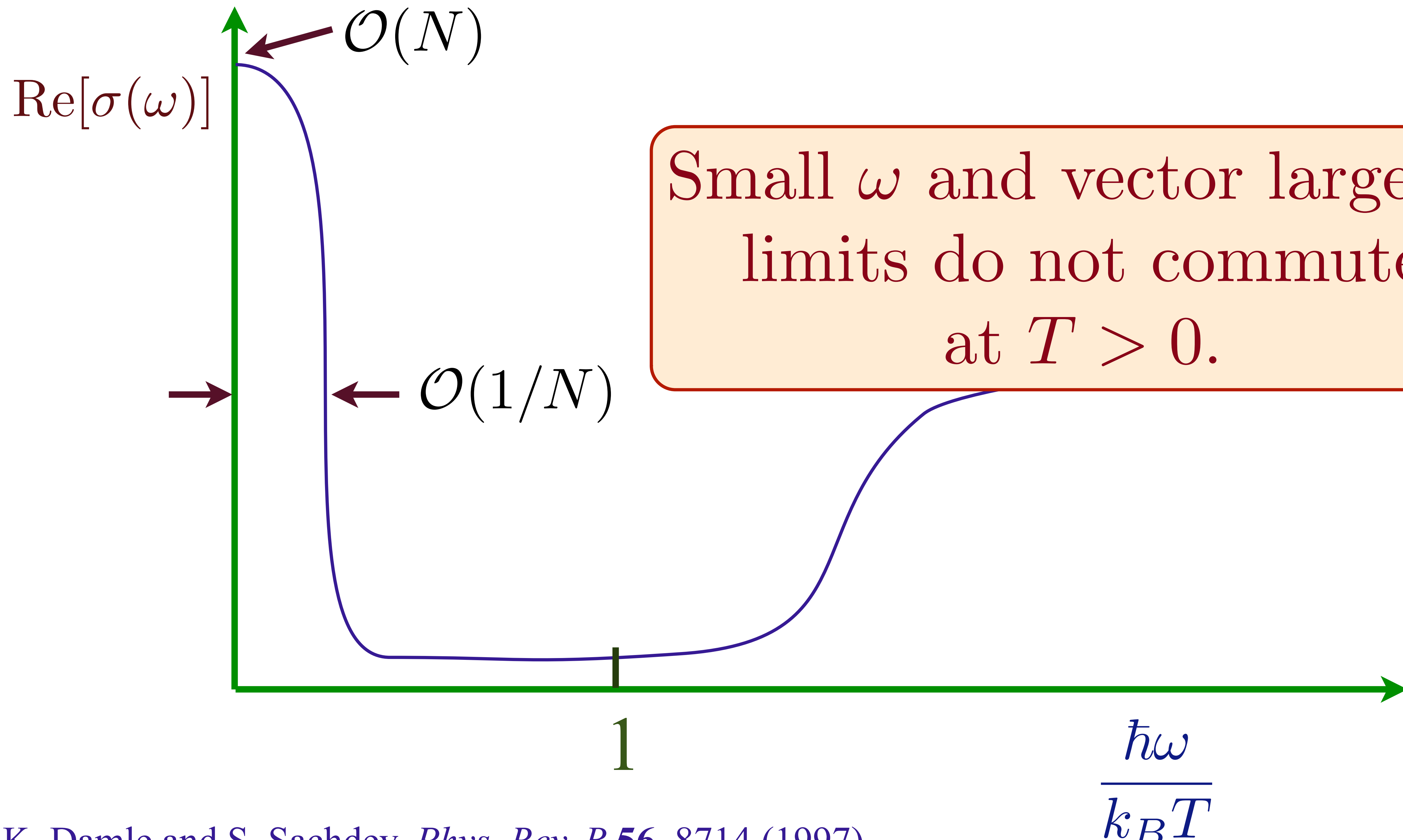

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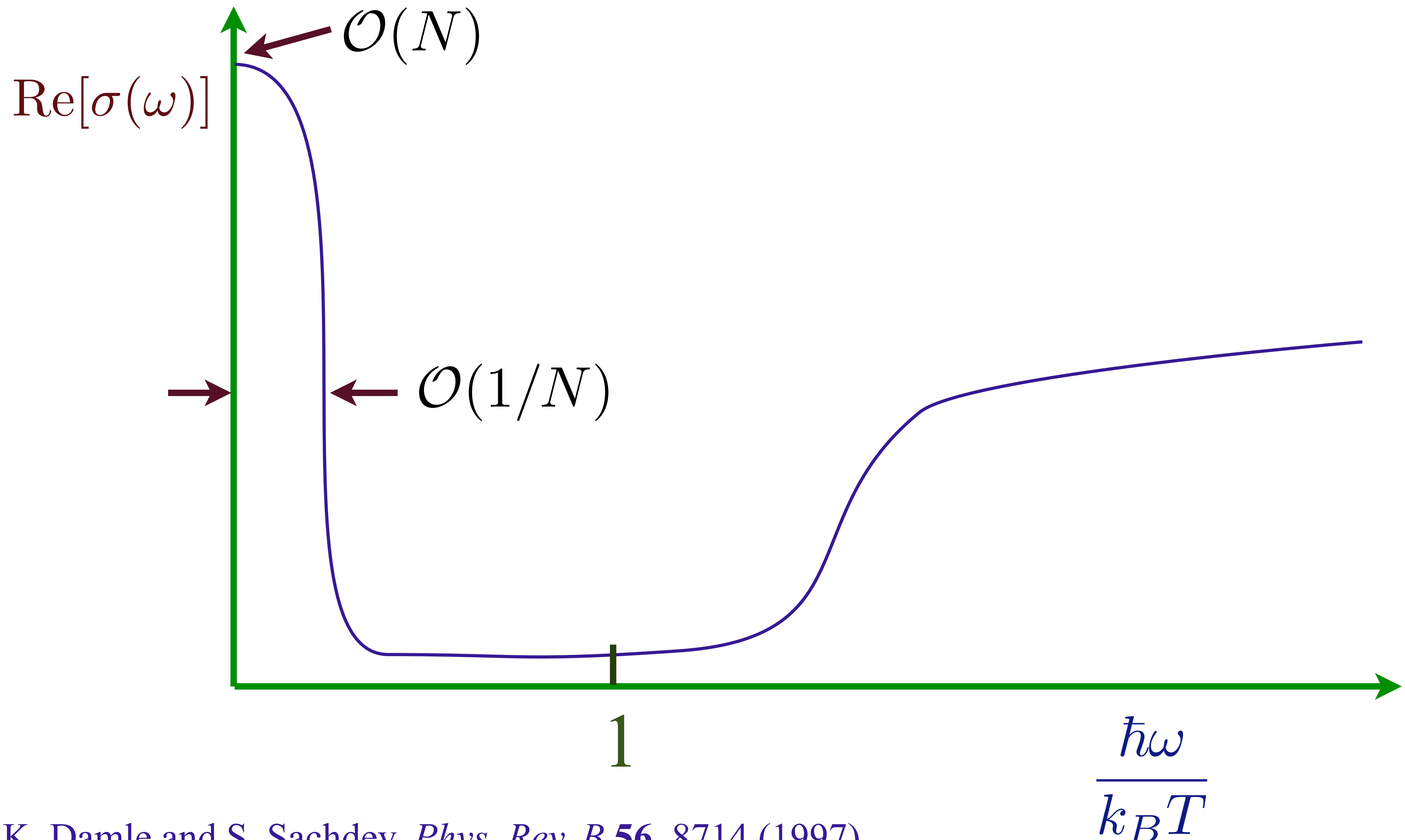

K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Vector large N expansion for CFT3

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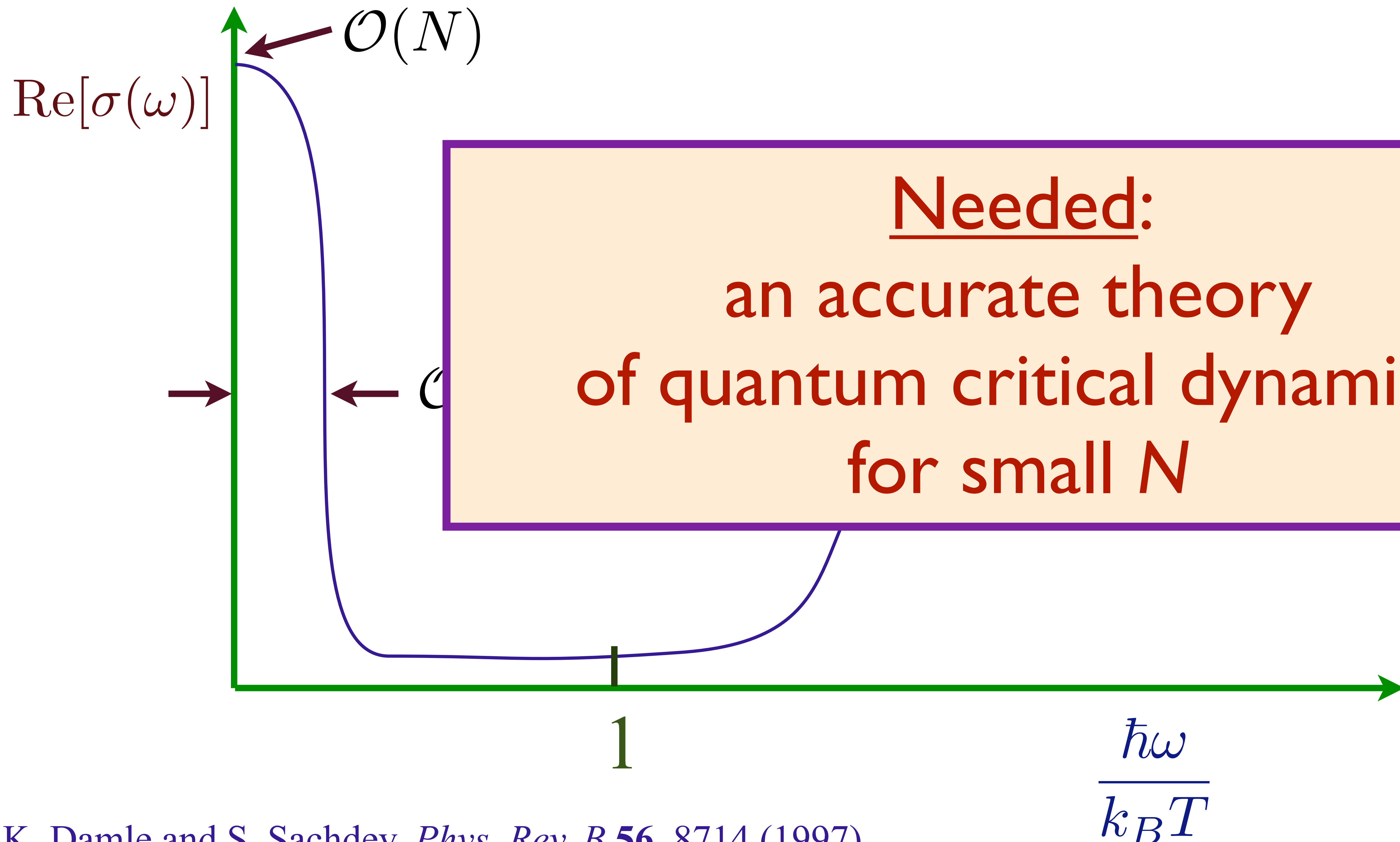

Needed:
an accurate theory
of quantum critical dynamics
for small N

K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Outline

1. Higgs boson in a 3-dimensional antiferromagnet

2. Higgs "pole" near the superfluid-insulator transition in 2 dimensions

3. Quantum criticality and conformal field theories

4. Holography and the quasi-normal modes of black-hole horizons

Outline

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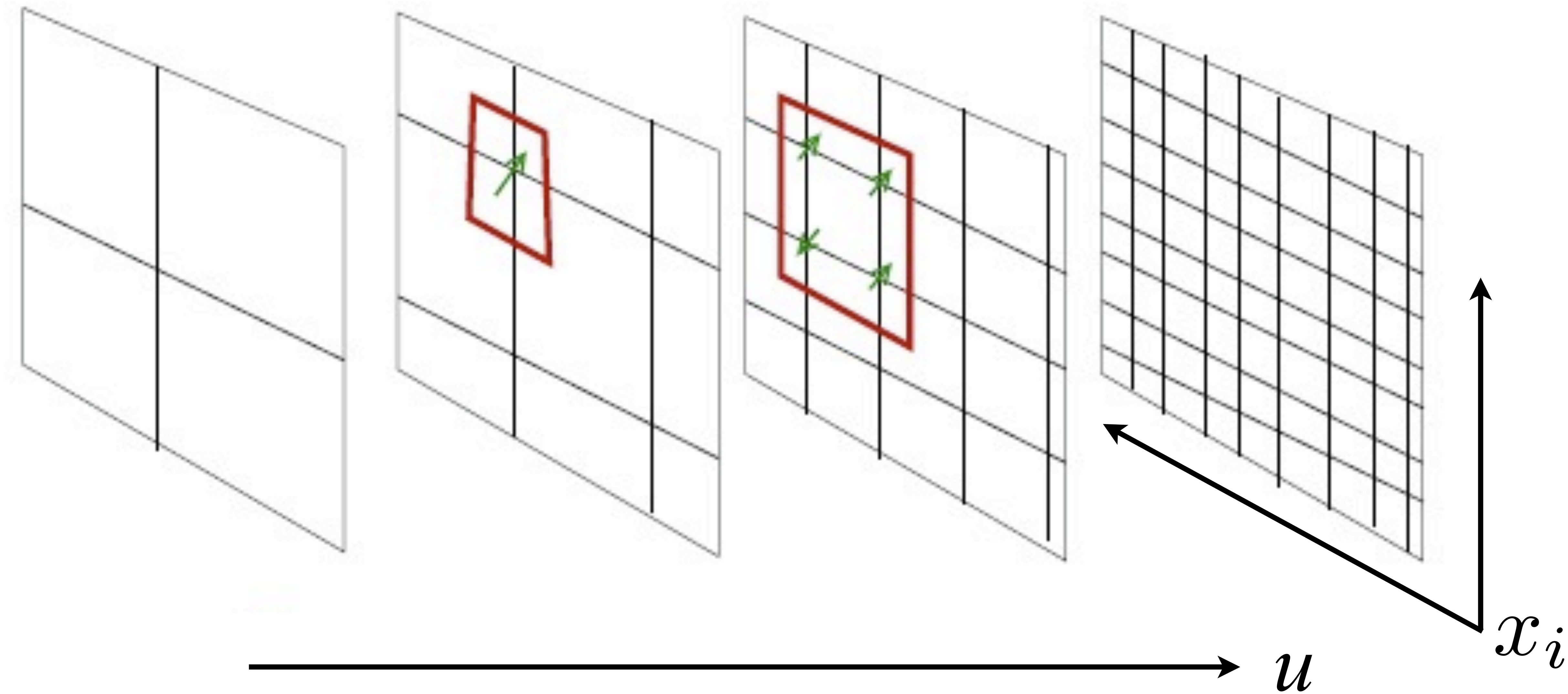
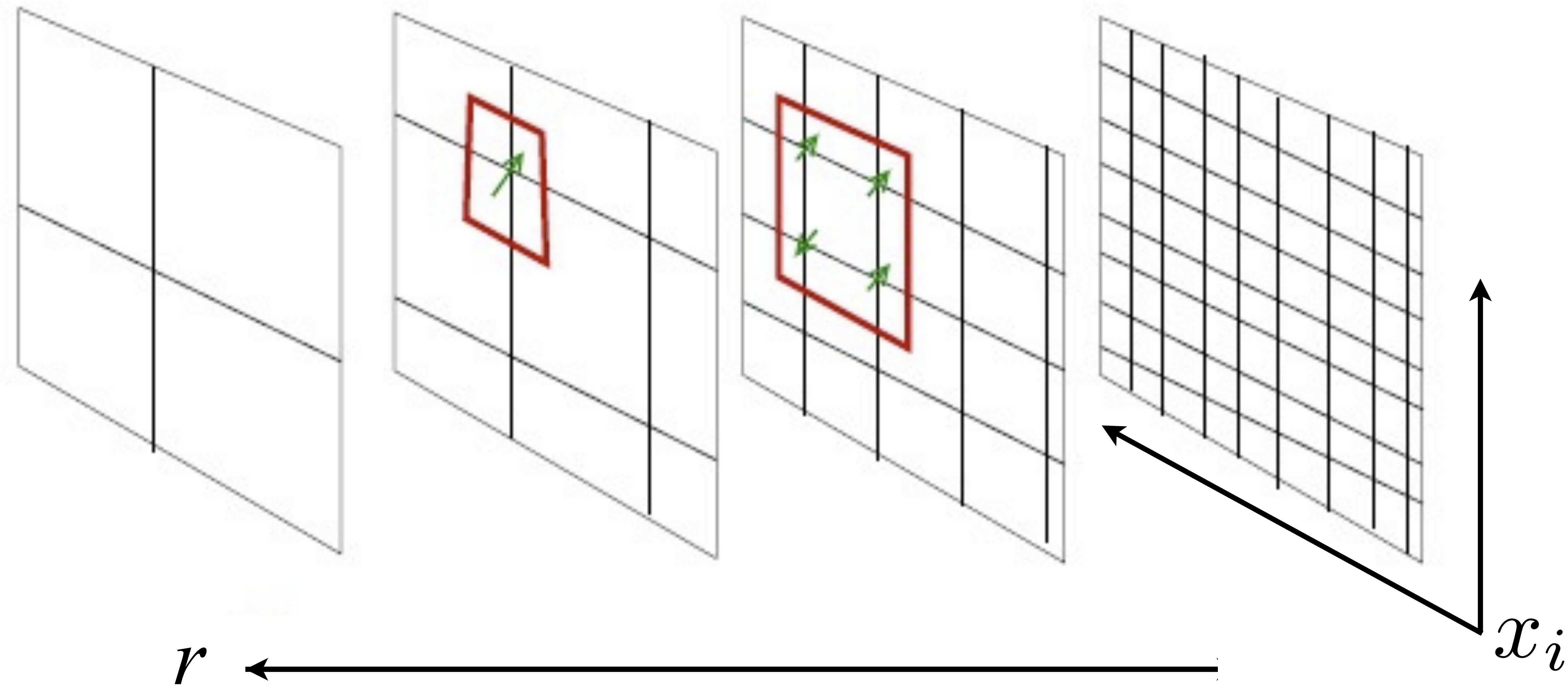
Field theories in $d+1$ spacetime dimensions are characterized by couplings g which obey the renormalization group equation

$$u\frac{dg}{du} = \beta(g)$$

where u is the energy scale. The RG equation is *local* in energy scale, *i.e.* the RHS does not depend upon u.



J. McGreevy, arXiv0909.0518



Key idea: $\Rightarrow$ Implement r as an extra dimension, and map to a local theory in $d+2$ spacetime dimensions.

For a relativistic CFT in d spatial dimensions, the metric in the holographic space is uniquely fixed by demanding the following scale transformaion $(i = 1 \ldots d)$

$$x_i \to \zeta x_i \quad , \quad t \to \zeta t \quad , \quad ds \to ds$$

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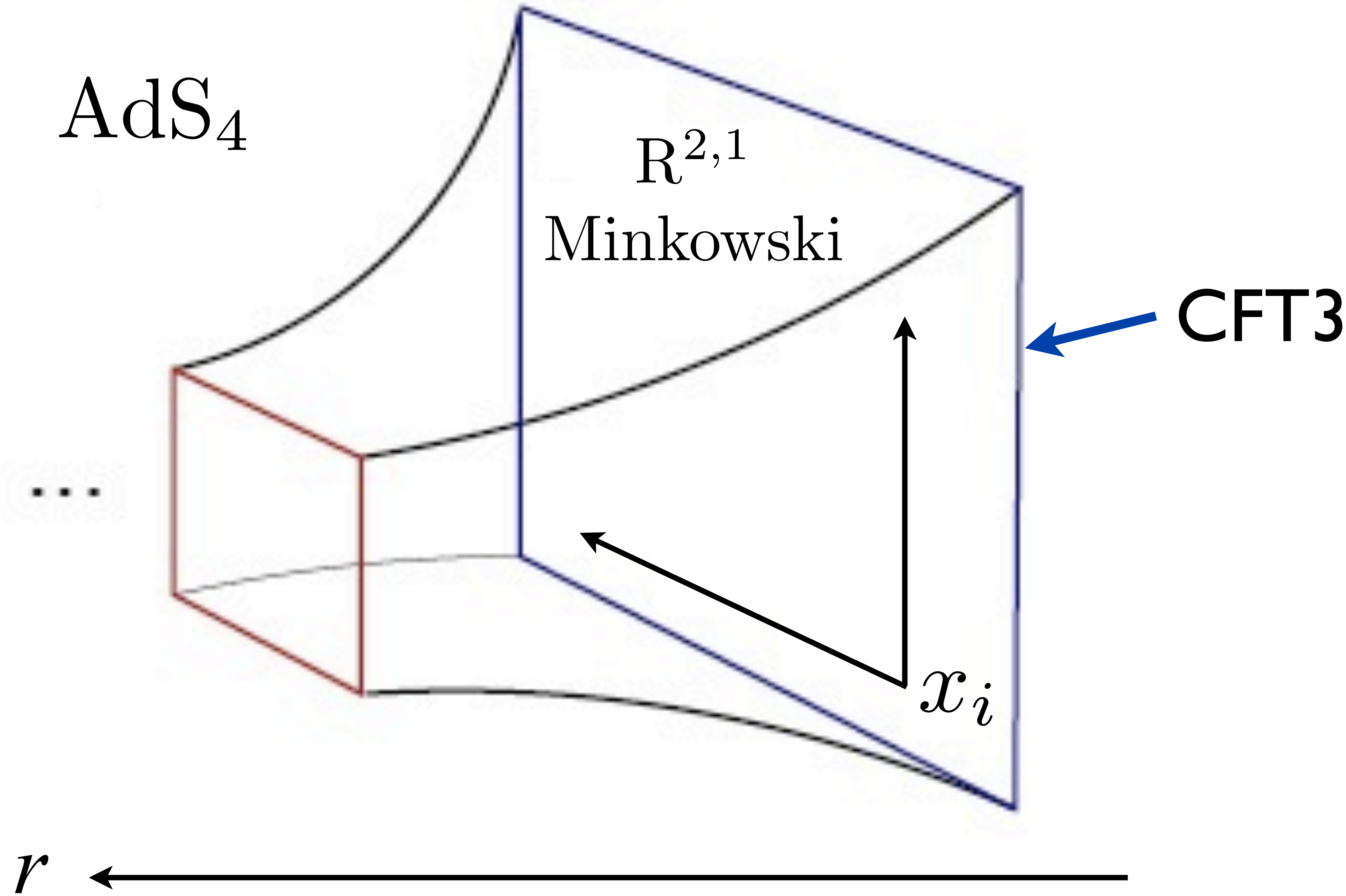
This gives the unique metric

$$ds^2 = \frac{1}{r^2}\left(-dt^2 + dr^2 + dx_i^2\right)$$

Reparametrization invariance in r has been used to the prefactor of dx_i^2 equal to $1/r^2$. This fixes $r \to \zeta r$ under the scale transformation. This is the metric of the space AdS_{d+2}.

AdS/CFT correspondence



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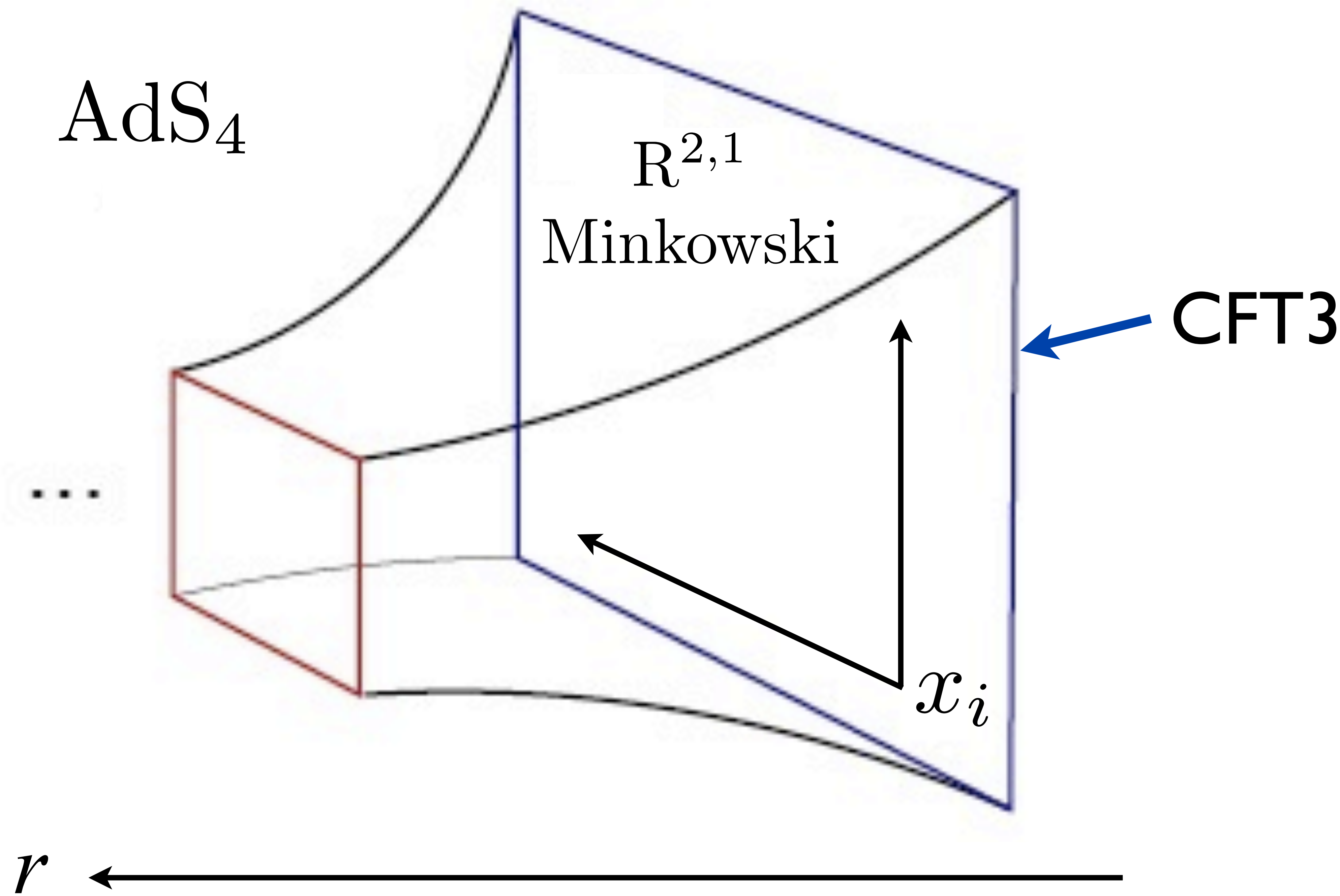


This emergent spacetime is a solution of Einstein gravity with a negative cosmological constant

$$\mathcal{S}_E = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) \right]$$

AdS/CFT correspondence

For every primary operator $O(\boldsymbol{x})$ in the CFT, there is a corresponding field $\phi(\boldsymbol{x}, r)$ in the bulk (gravitational) theory. For a scalar operator $O(\boldsymbol{x})$ of dimension Δ, the correlators of the boundary and bulk theories are related by

$$\langle O(\boldsymbol{x}_1) \ldots O(\boldsymbol{x}_n) \rangle_{\text{CFT}} = \\ Z^n \lim_{r \to 0} r_1^{-\Delta} \ldots r_n^{-\Delta} \langle \phi(\boldsymbol{x}_1, r_1) \ldots \phi(\boldsymbol{x}_n, r_n) \rangle_{\text{bulk}}$$

where the "wave function renormalization" factor $Z = (2\Delta - D)$.

For a U(1) conserved current J_μ of the CFT, the corresponding bulk operator is a U(1) *gauge* field A_μ. With a Maxwell action for the gauge field

$$\mathcal{S}_M = \frac{1}{4g_M^2}\int d^{D+1}x\,\sqrt{g}F_{ab}F^{ab}$$

we have the bulk-boundary correspondence

$$\langle J_\mu(\boldsymbol{x}_1)\ldots J_\nu(\boldsymbol{x}_n)\rangle_{\text{CFT}} = (Zg_M^{-2})^n \lim_{r\to 0} r_1^{2-D}\ldots r_n^{2-D}\,\langle A_\mu(\boldsymbol{x}_1,r_1)\ldots A_\nu(\boldsymbol{x}_n,r_n)\rangle_{\text{bulk}}$$

with $Z = D-2$.

AdS/CFT correspondence

A similar analysis can be applied to the stress-energy tensor of the CFT, $T_{\mu\nu}$. Its conjugate field must be a spin-2 field which is invariant under gauge transformations: it is natural to identify this with the change in metric of the bulk theory. We write $\delta g_{\mu\nu} = (L^2/r^2)\chi_{\mu\nu}$, and then the bulk-boundary correspondence is now given by

$$\langle T_{\mu\nu}(\boldsymbol{x}_1) \ldots T_{\rho\sigma}(\boldsymbol{x}_n)\rangle_{\text{CFT}} =$$
$$\left(\frac{ZL^2}{\kappa^2}\right)^n \lim_{r\to 0} r_1^{-D} \ldots r_n^{-D} \langle \chi_{\mu\nu}(\boldsymbol{x}_1, r_1) \ldots \chi_{\rho\sigma}(\boldsymbol{x}_n, r_n)\rangle_{\text{bulk}} ,$$

with $Z = D$.

AdS/CFT correspondence

So the minimal bulk theory for a CFT with a conserved U(1) current is the *Einstein-Maxwell* theory with a cosmological constant

$$\mathcal{S} = \frac{1}{4g_M^2}\int d^4x\,\sqrt{g}F_{ab}F^{ab} + \int d^4x\sqrt{g}\left[-\frac{1}{2\kappa^2}\left(R+\frac{6}{L^2}\right)\right].$$

This action is characterized by two dimensionless parameters: g_M and L^2/κ^2, which are related to the conductivity $\sigma(\omega) = \mathcal{K}$ and the central charge of the CFT.

AdS/CFT correspondence

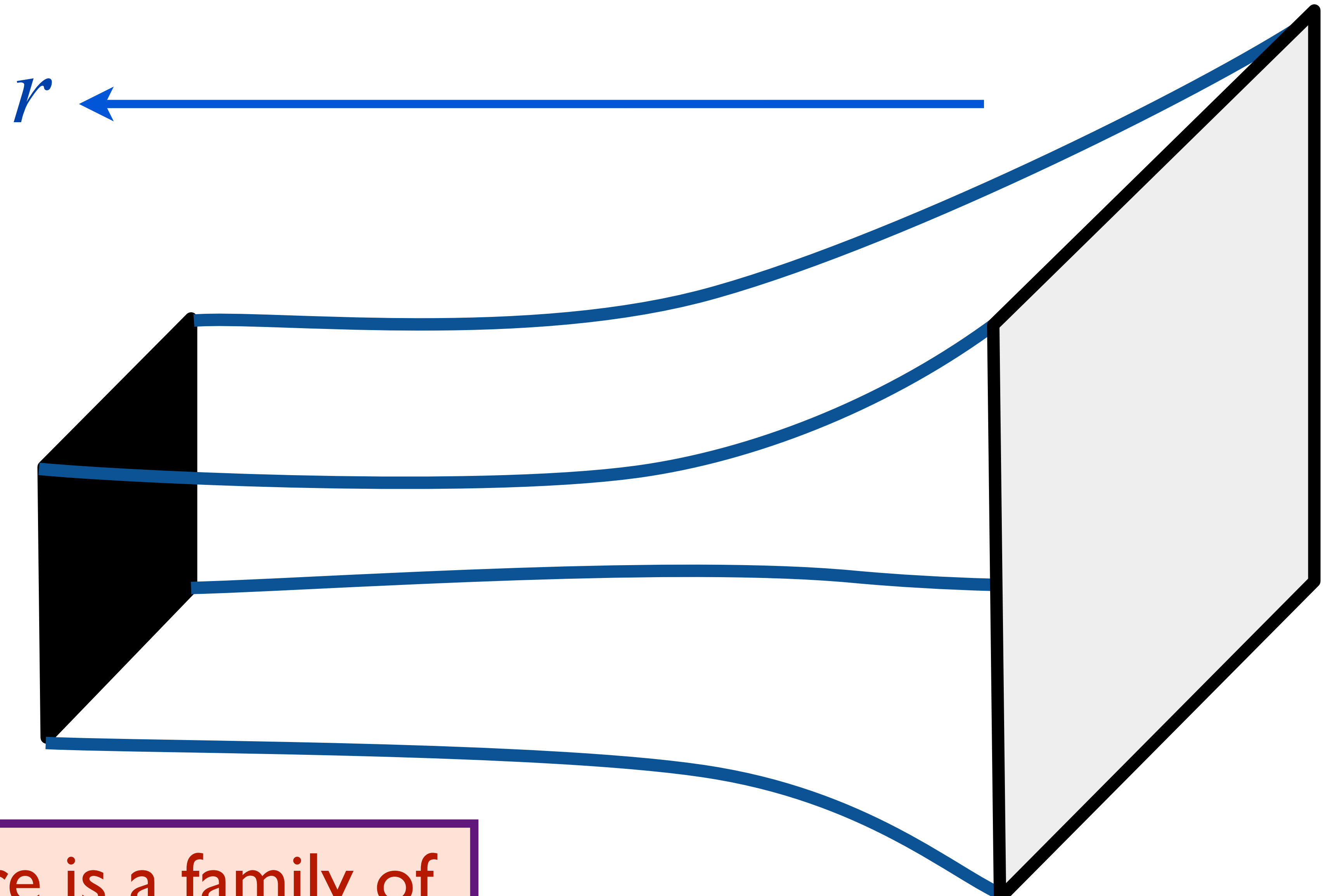
This minimal action also fixes multi-point correlators of the CFT: however these do not have the most general form allowed for a CFT. To fix these, we have to allow for higher-gradient terms in the bulk action. For the conductivity, it turns out that only a single 4 gradient term contributes

$$\mathcal{S}_{\rm bulk} = \frac{1}{g_M^2} \int d^4x \sqrt{g} \left[\frac{1}{4} F_{ab} F^{ab} + \gamma L^2 C_{abcd} F^{ab} F^{cd} \right] + \int d^4x \sqrt{g} \left[-\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) \right],$$

where C_{abcd} is the Weyl tensor. The parameter γ can be related to 3-point correlators of J_μ and $T_{\mu\nu}$. Both boundary and bulk methods show that $|\gamma| \leq 1/12$, and the bound is saturated by free fields.

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)
D. Chowdhury, S. Raju, S. Sachdev, A. Singh, and P. Strack, arXiv:1210.5247

AdS$_4$-Schwarzschild black-brane



There is a family of solutions of Einstein gravity which describe non-zero temperatures

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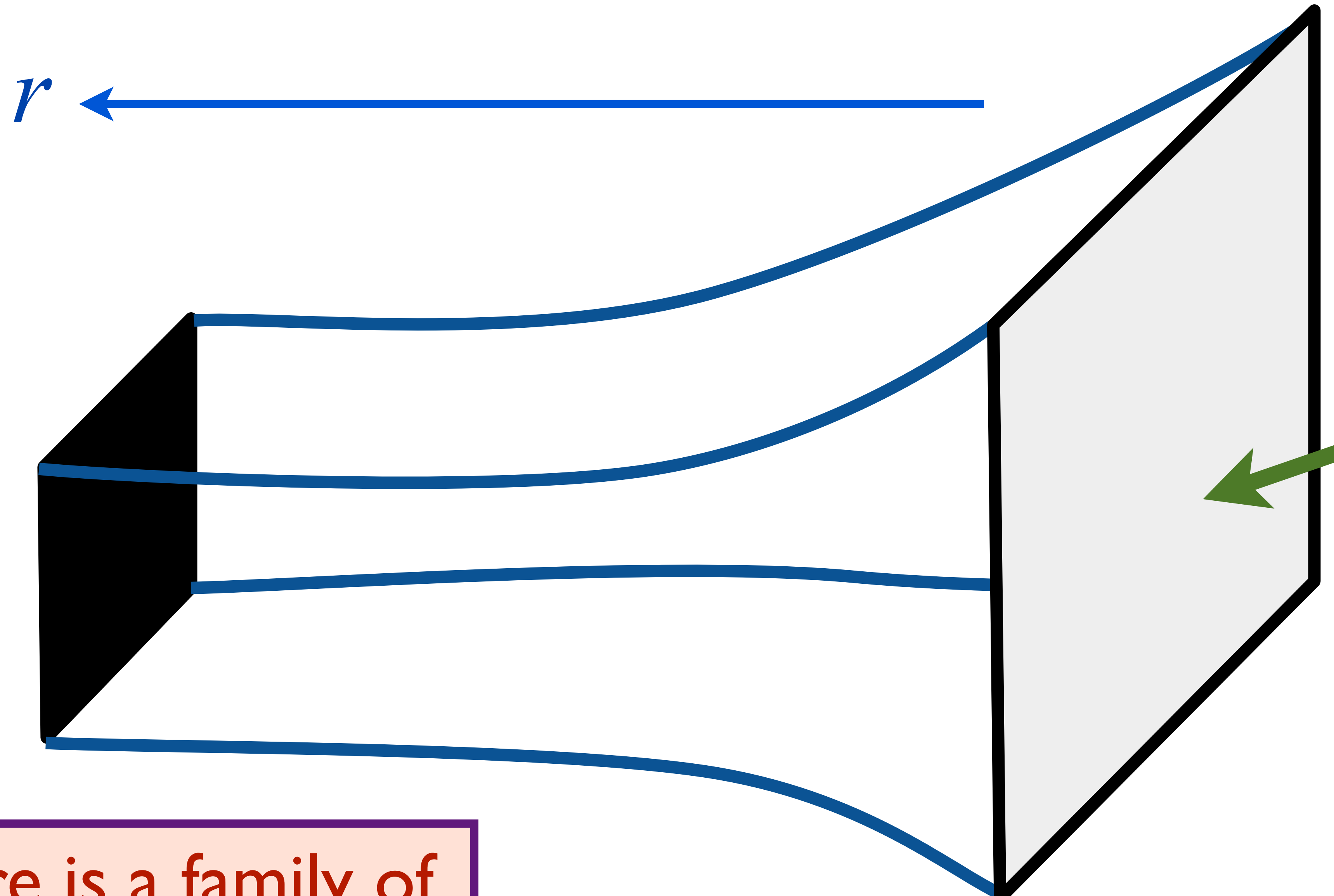

A 2+1 dimensional system at its quantum critical point: $k_B T = \frac{3\hbar}{4\pi R}$.

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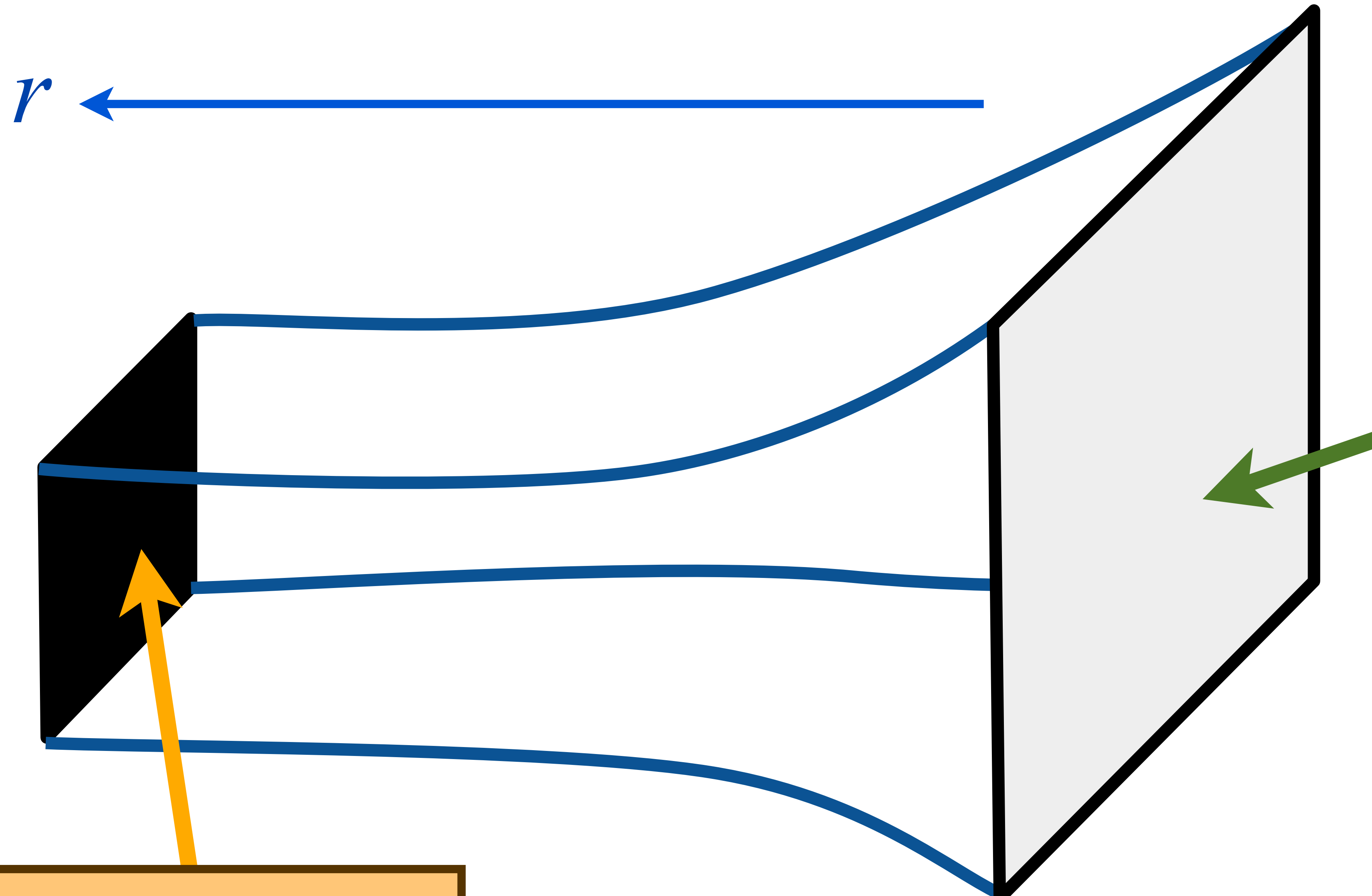
$$ds^2 = \left(\frac{L}{r}\right)^2 \left[\frac{dr^2}{f(r)} - f(r)dt^2 + dx^2 + dy^2\right]$$
with $f(r) = 1 - (r/R)^3$

AdS$_4$-Schwarzschild black-brane



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Black-brane at temperature of 2+1 dimensional quantum critical system

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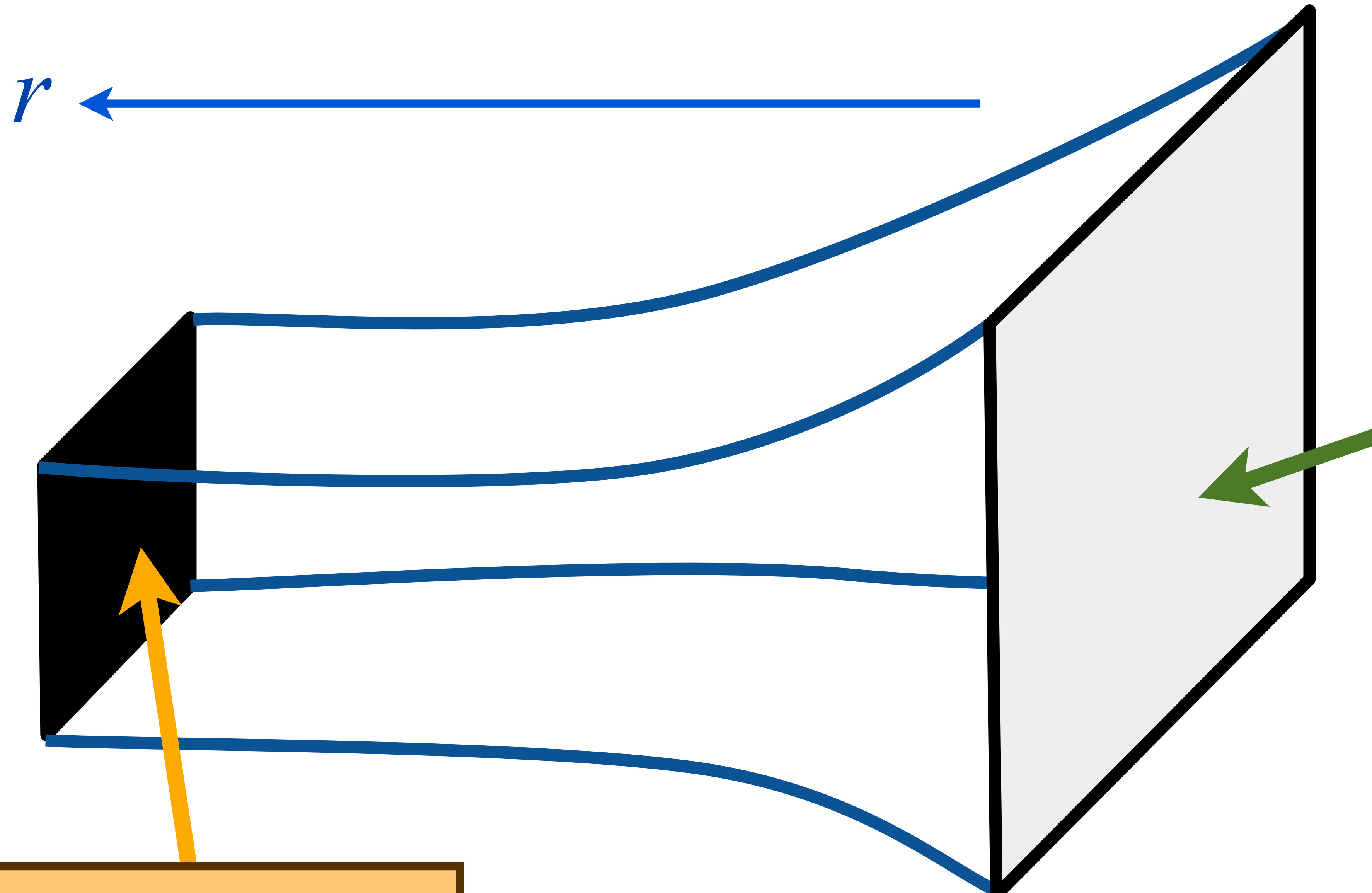

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Black-brane at temperature of 2+1 dimensional quantum critical system

Beckenstein-Hawking entropy of black brane = entropy of CFT3

AdS/CFT correspondence at non-zero temperatures

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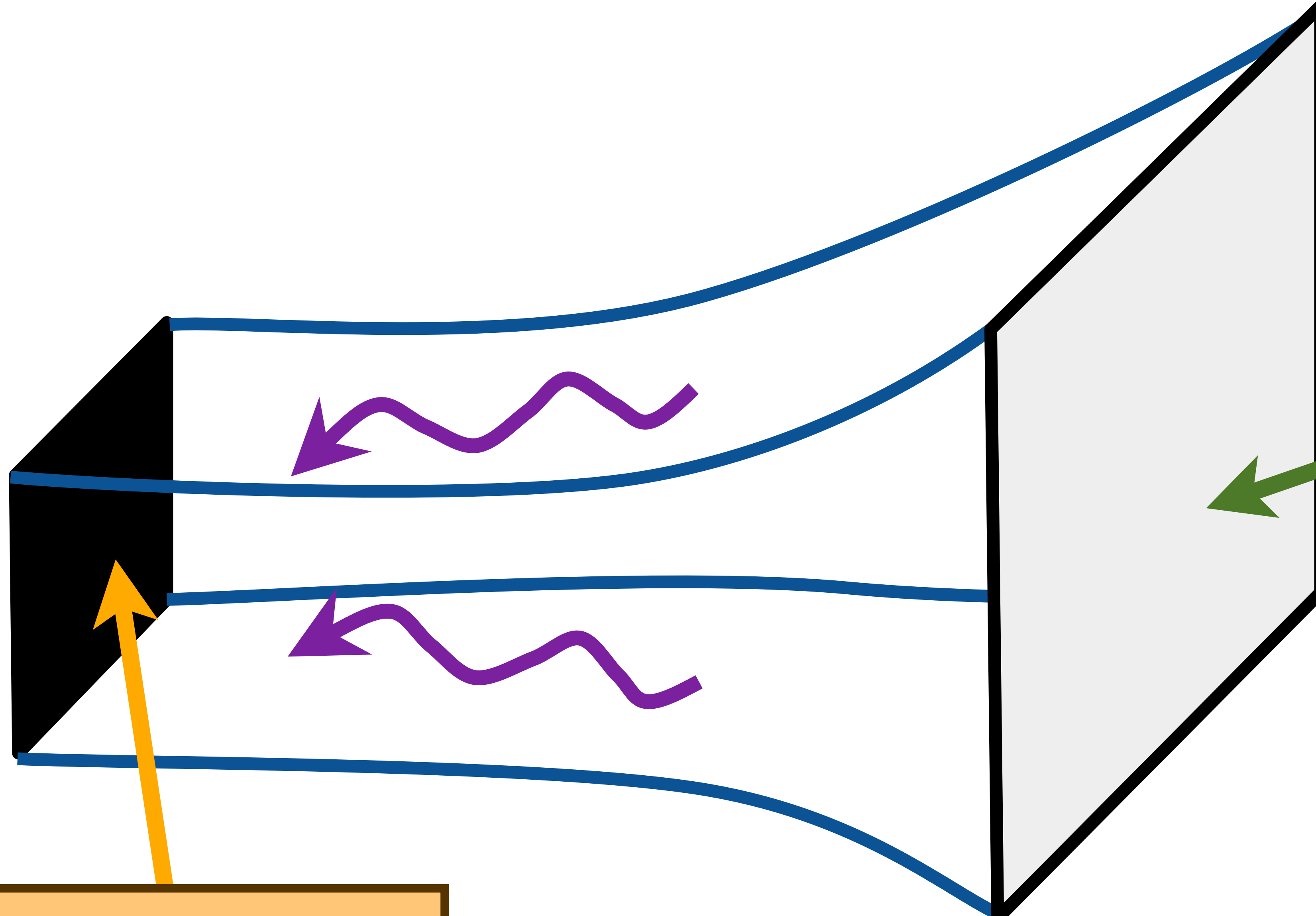

A 2+1 dimensional system at its quantum critical point:

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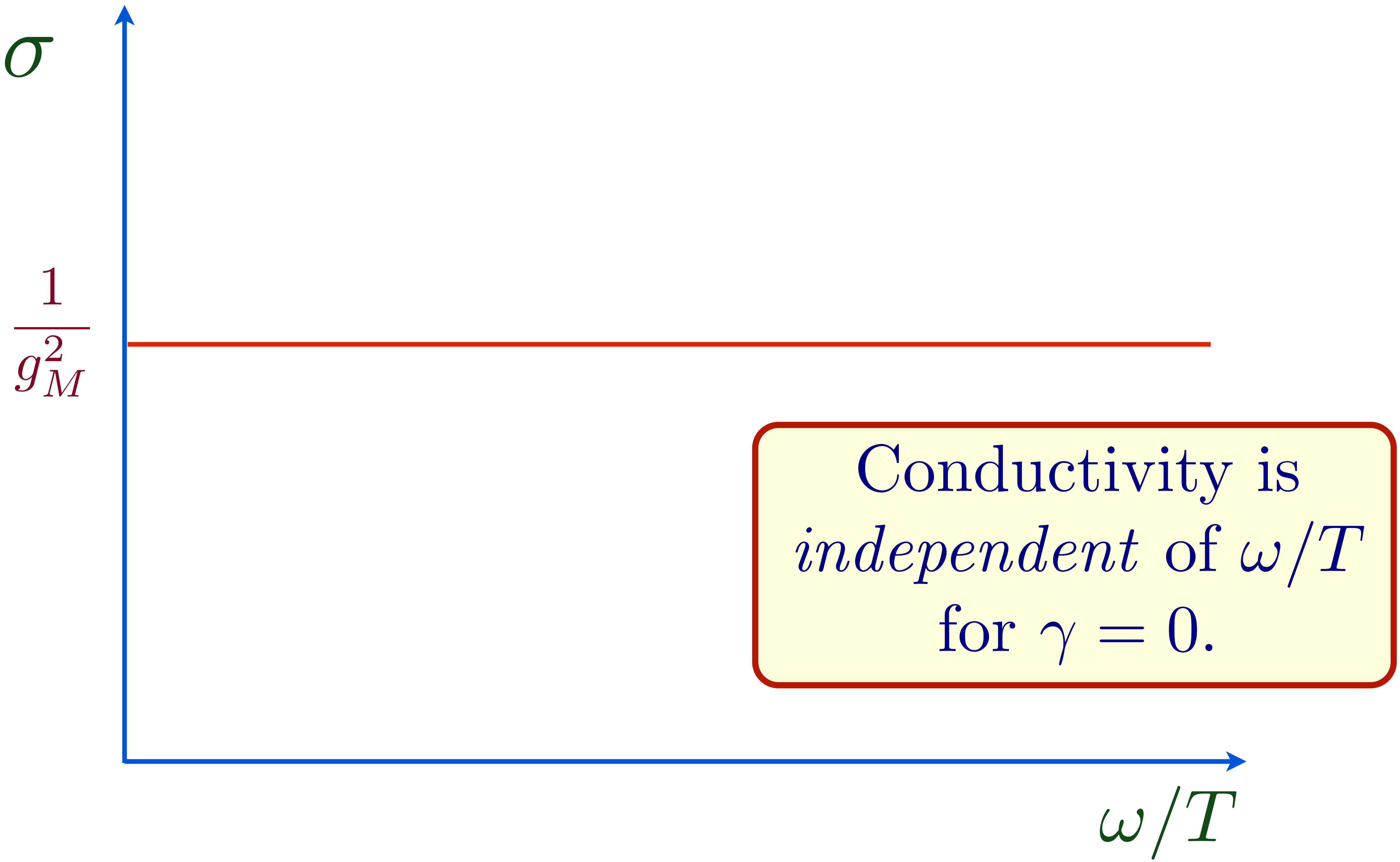
Black-brane at temperature of 2+1 dimensional quantum critical system

Friction of quantum criticality = waves falling into black brane

D. T. Son

AdS4 theory of electrical transport in a strongly interacting CFT3 for *T > 0*



C. P. Herzog, P. K. Kovtun, S. Sachdev, and D. T. Son, *Phys. Rev.* D **75**, 085020 (2007).

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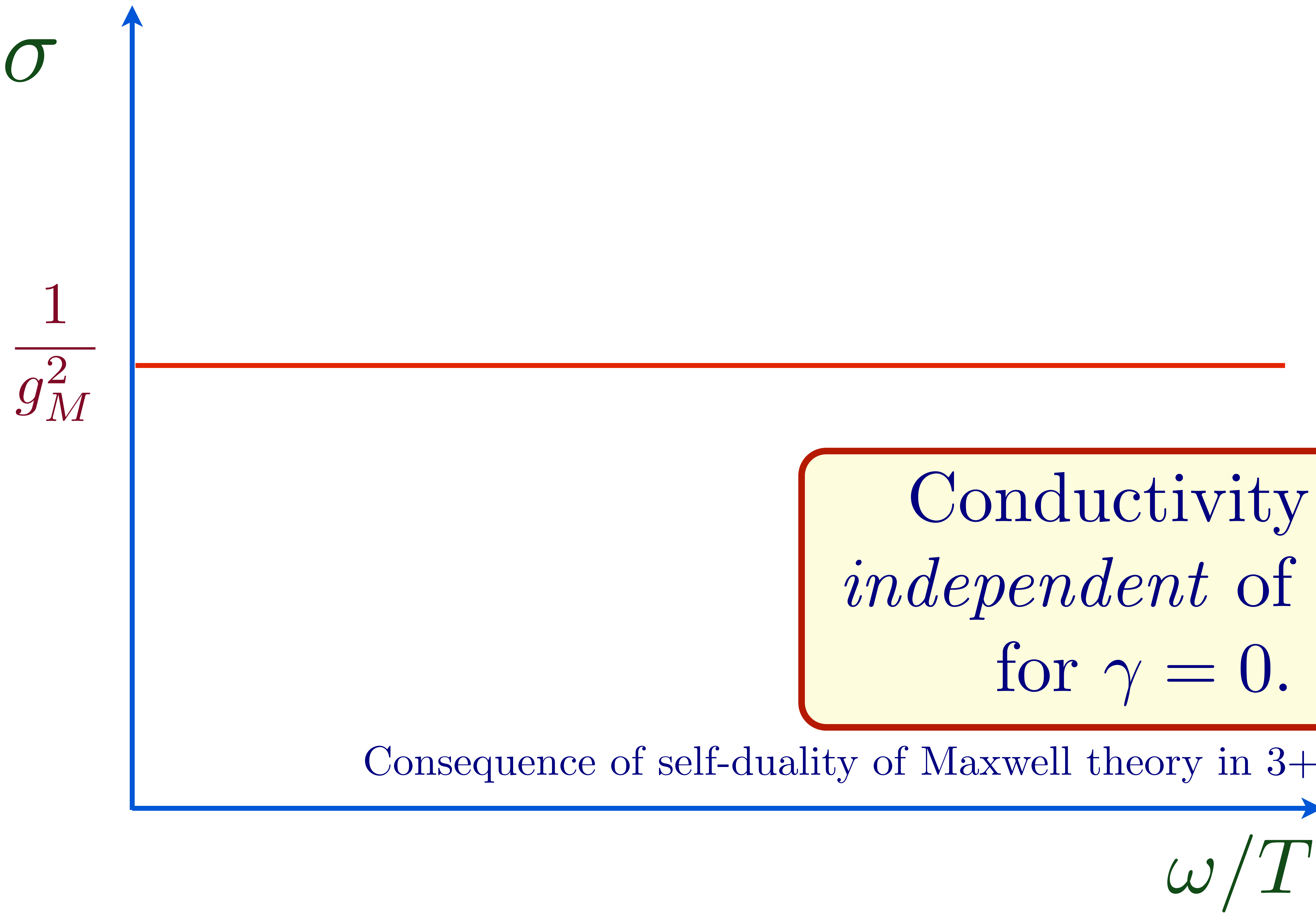
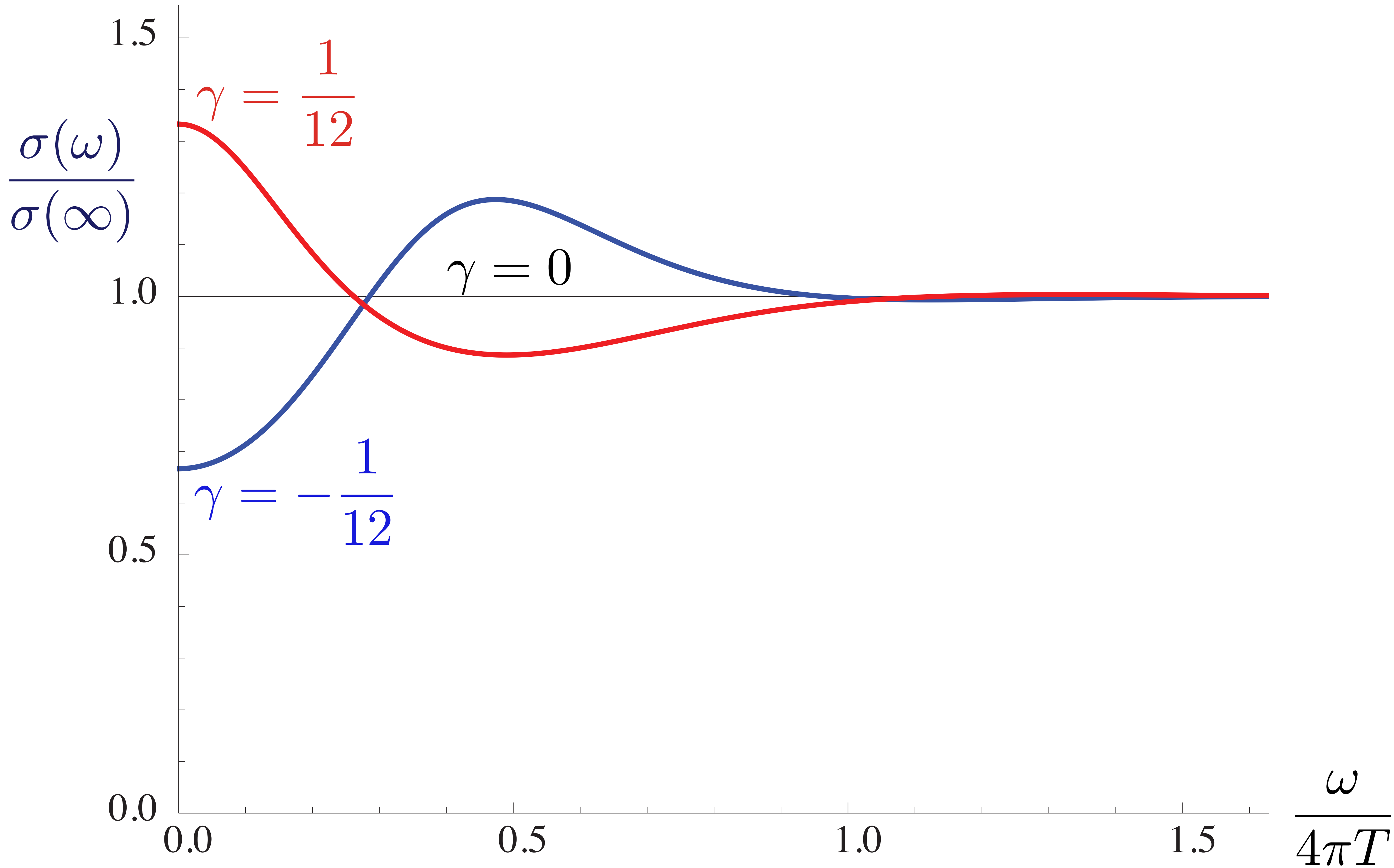


Conductivity is *independent* of ω/T for $\gamma = 0$.

Consequence of self-duality of Maxwell theory in 3+1 dimensions

C. P. Herzog, P. K. Kovtun, S. Sachdev, and D. T. Son, *Phys. Rev.* D **75**, 085020 (2007).

AdS_4 theory of quantum criticality



R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

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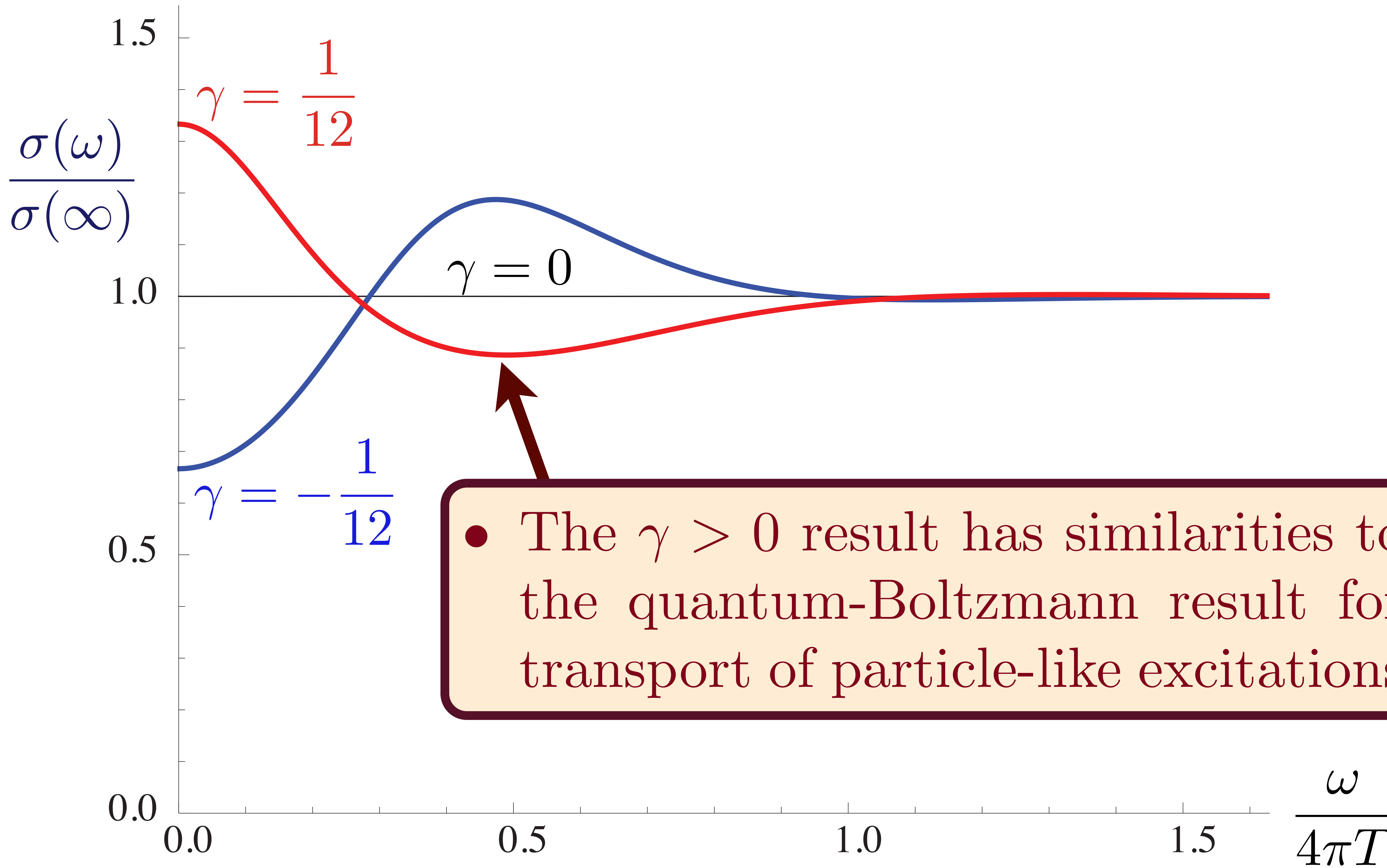
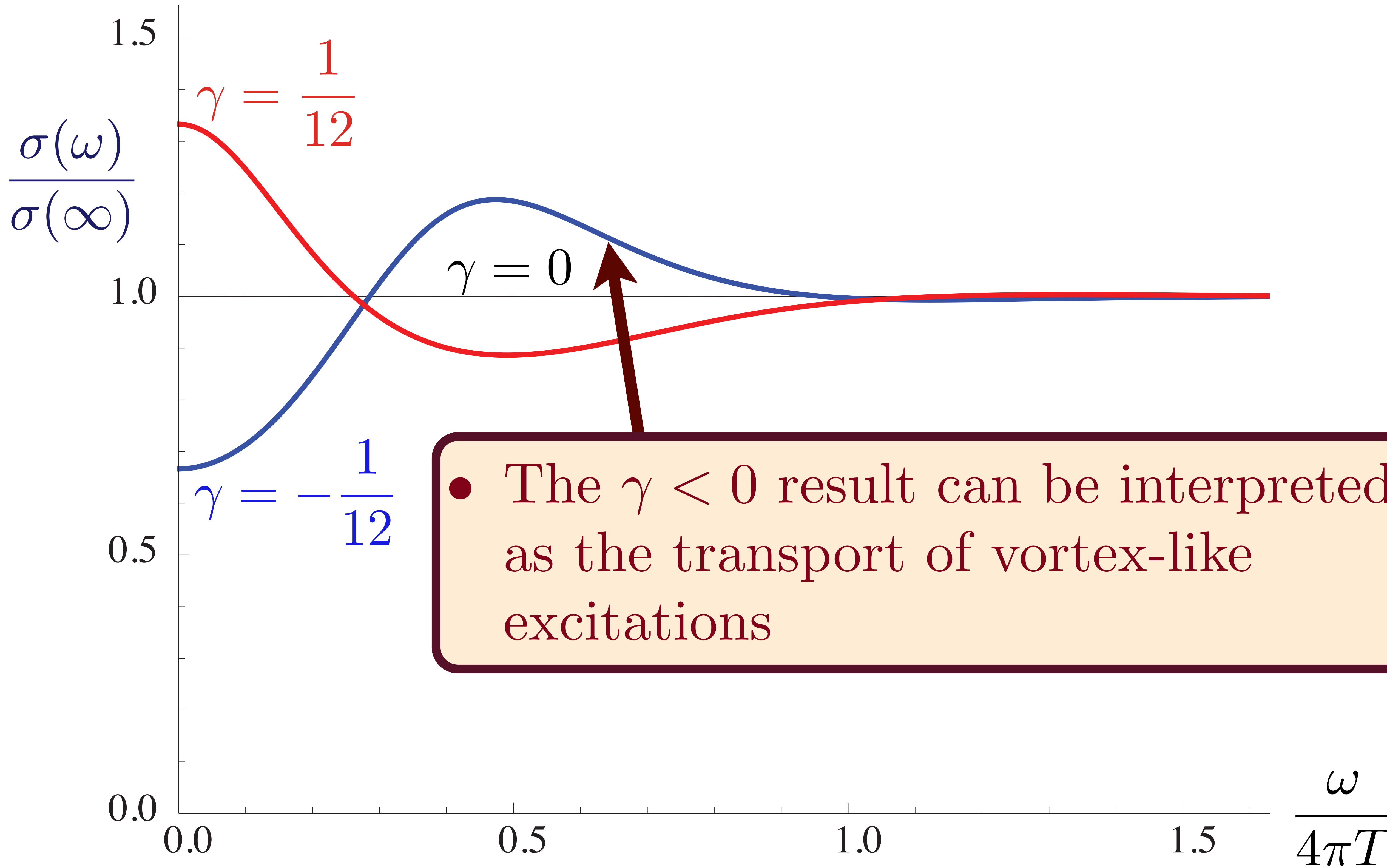


- The $\gamma > 0$ result has similarities to the quantum-Boltzmann result for transport of particle-like excitations

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

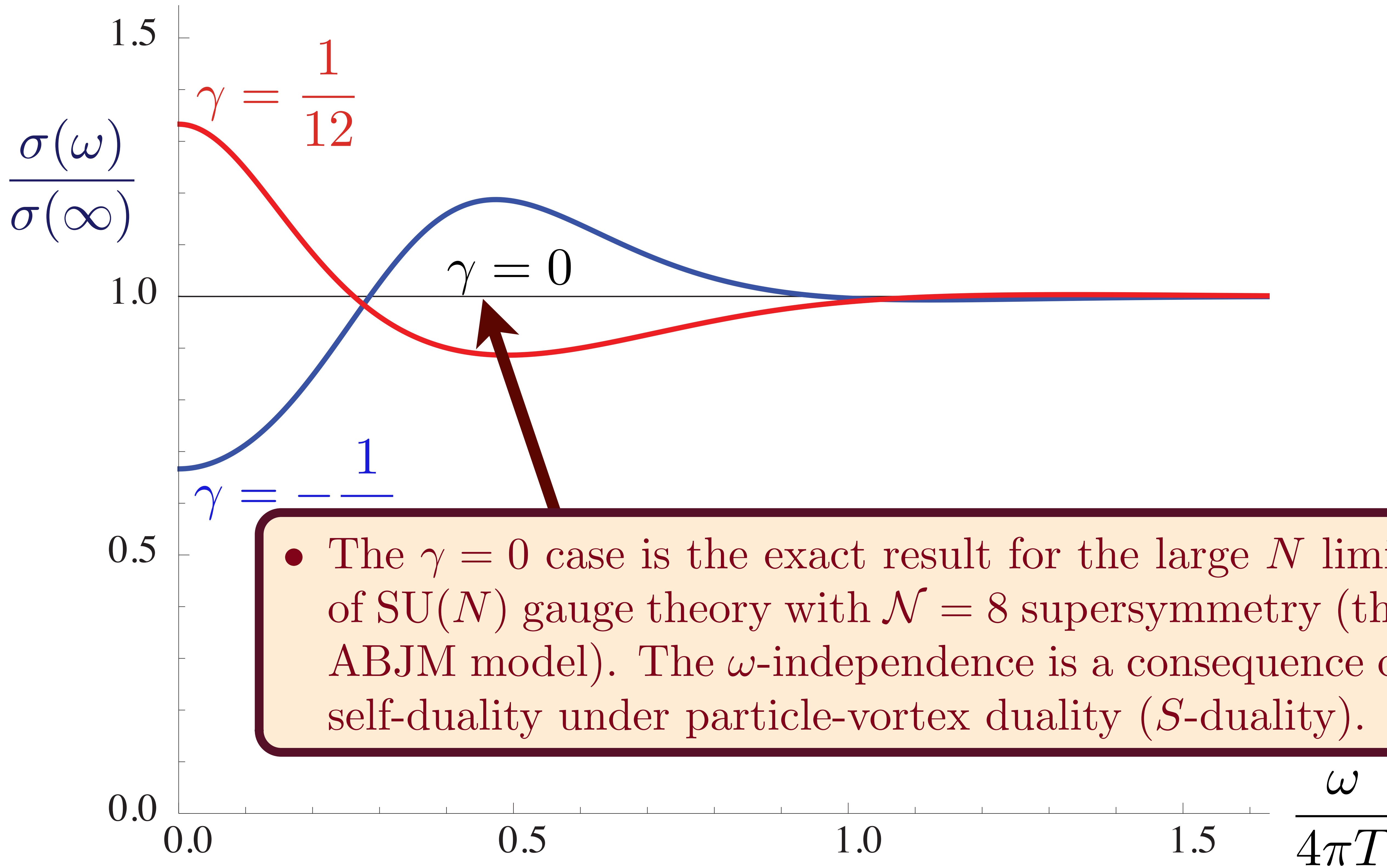
AdS$_4$ theory of quantum criticality



- The $\gamma < 0$ result can be interpreted as the transport of vortex-like excitations

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

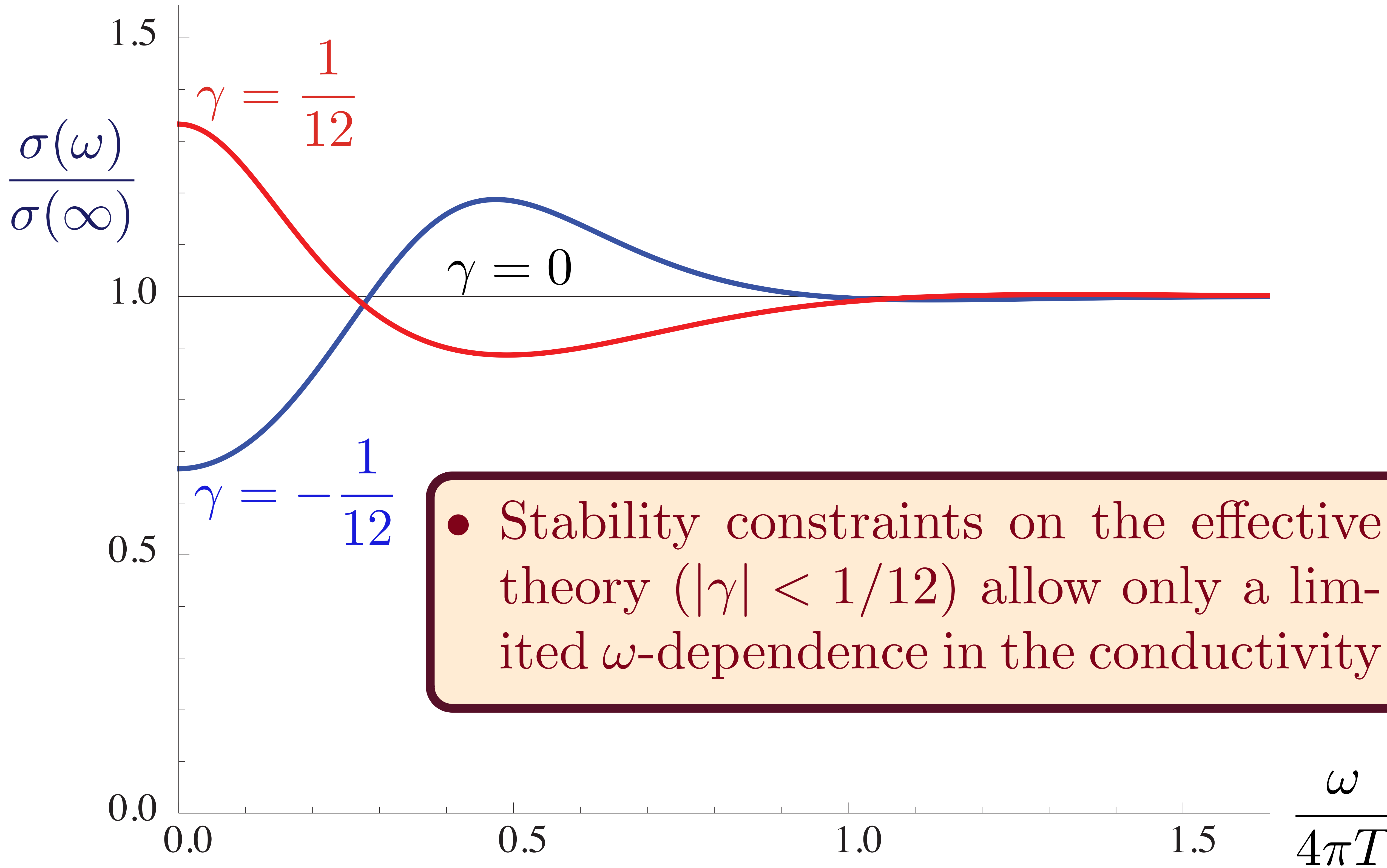
AdS$_4$ theory of quantum criticality



- The $\gamma = 0$ case is the exact result for the large N limit of SU(N) gauge theory with $\mathcal{N} = 8$ supersymmetry (the ABJM model). The ω-independence is a consequence of self-duality under particle-vortex duality (S-duality).

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

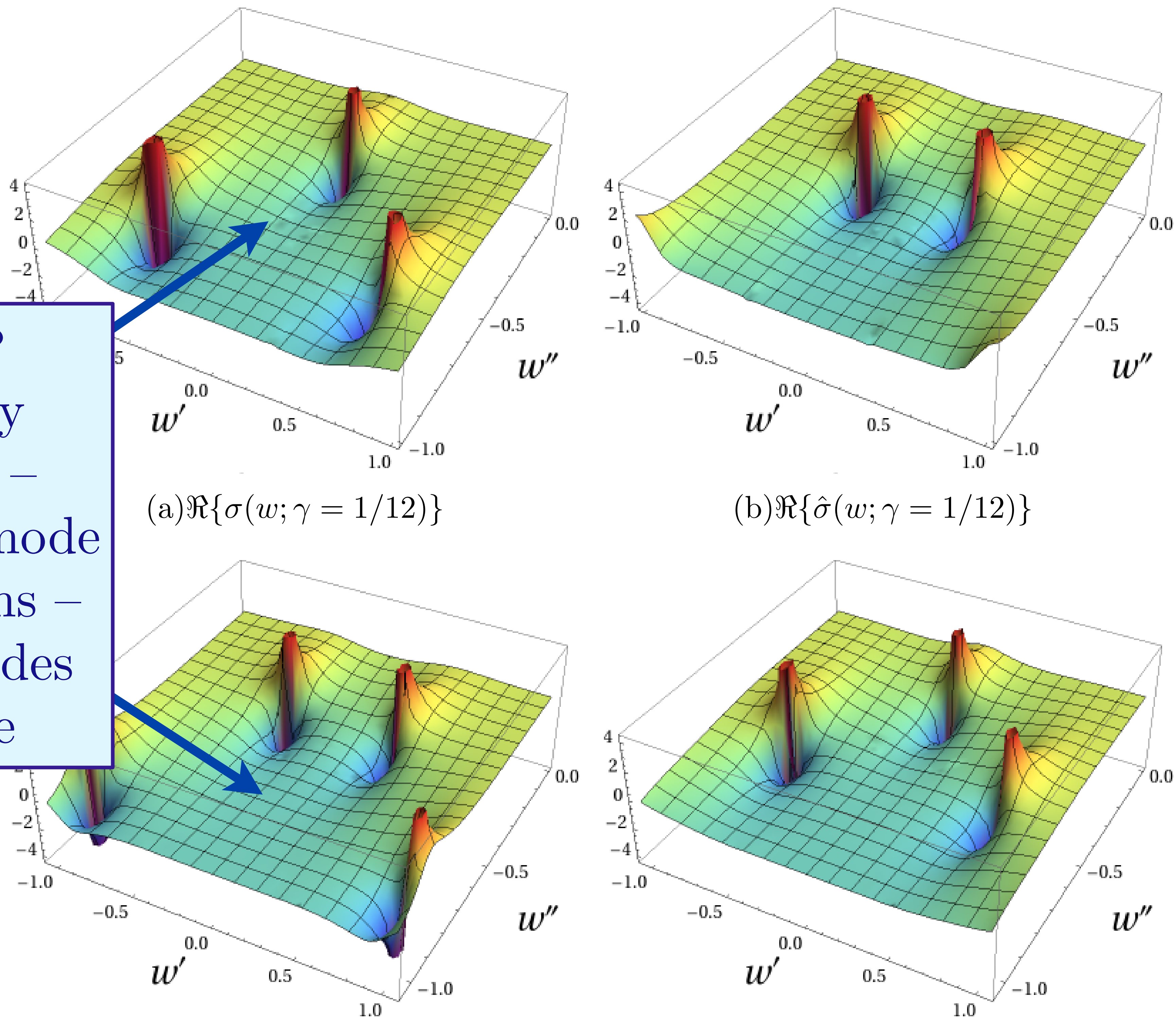
AdS$_4$ theory of quantum criticality



- Stability constraints on the effective theory ($|\gamma| < 1/12$) allow only a limited ω-dependence in the conductivity

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

AdS$_4$ theory of quantum criticality



(a)$\Re\{\sigma(w;\gamma=1/12)\}$ (b)$\Re\{\hat{\sigma}(w;\gamma=1/12)\}$

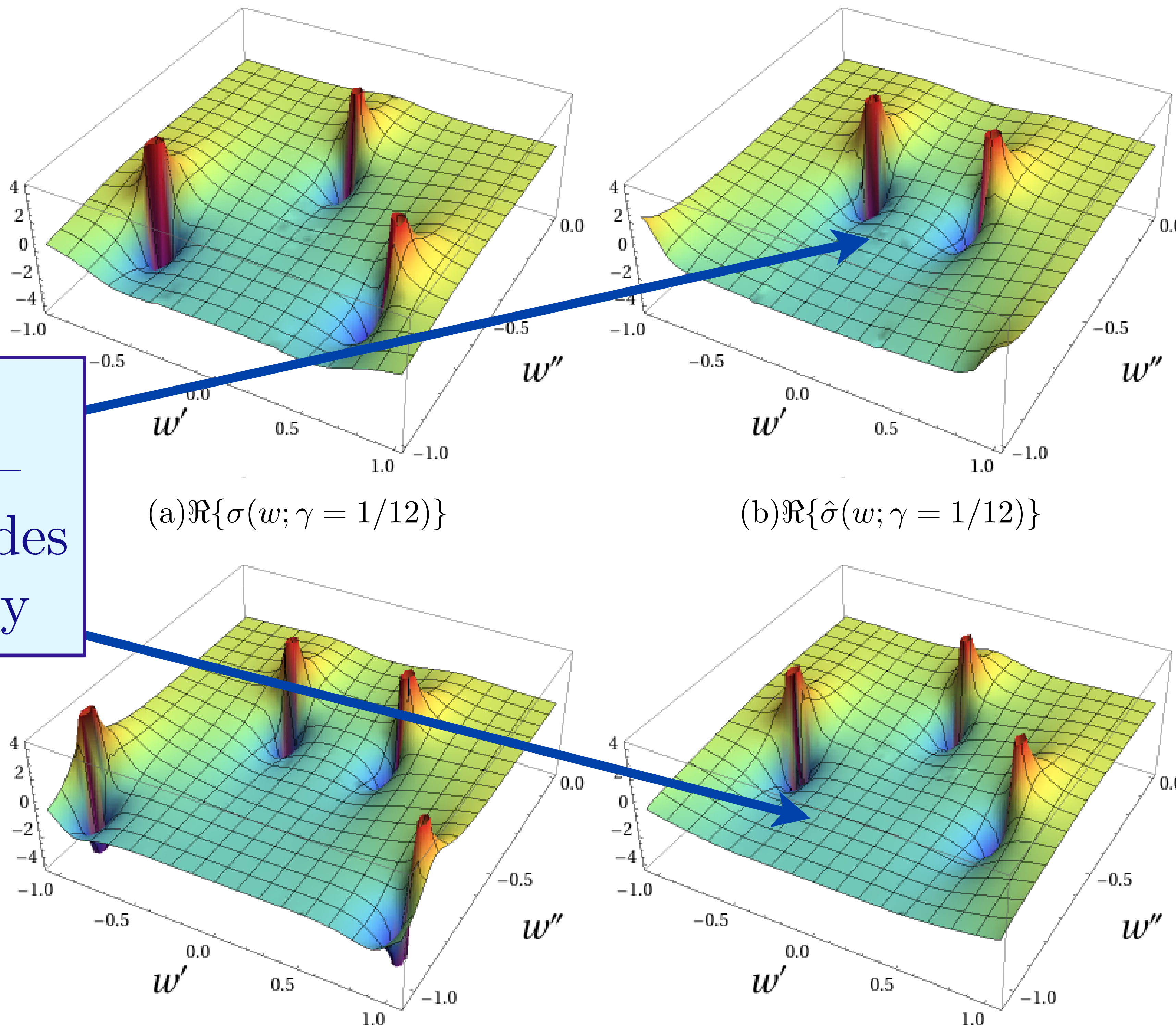
(c)$\Re\{\sigma(w;\gamma=-1/12)\}$ (d)$\Re\{\hat{\sigma}(w;\gamma=-1/12)\}$

Poles in LHP of conductivity at $\omega \sim k_B T/\hbar$ – analog of Higgs mode in two dimensions – quasinormal modes of black brane

W. Witzack-Krempa and S. Sachdev, *Physical Review D* **86**, 235115 (2012)

AdS$_4$ theory of quantum criticality

Poles in LHP of resistivity — quasinormal modes of S-dual theory



(a) $\Re\{\sigma(w;\gamma = 1/12)\}$ (b) $\Re\{\hat{\sigma}(w;\gamma = 1/12)\}$

(c) $\Re\{\sigma(w;\gamma = -1/12)\}$ (d) $\Re\{\hat{\sigma}(w;\gamma = -1/12)\}$

W. Witzack-Krempa and S. Sachdev, *Physical Review D* **86**, 235115 (2012)

AdS$_4$ theory of quantum criticality

The holographic solutions for the conductivity satisfy two sum rules, valid for all CFT3s. (W. Witzack-Krempa and S. Sachdev, Phys. Rev. B **86**, 235115 (2012))

$$\int_0^\infty d\omega \mathrm{Re}\left[\sigma(\omega) - \sigma(\infty)\right] = 0$$

$$\int_0^\infty d\omega \mathrm{Re}\left[\frac{1}{\sigma(\omega)} - \frac{1}{\sigma(\infty)}\right] = 0$$

The second rule follows from the existence of a EM-dual CFT3.

Boltzmann theory chooses a "particle" basis: this satisfies only *one* sum rule but not the other.

Holographic theory satisfies both sum rules.

Traditional CMT

- Identify quasiparticles and their dispersions

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- Compute scattering matrix elements of quasiparticles (or of collective modes)

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- Start with strongly interacting CFT without particle- or wave-like excitations

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- Solve Einsten-Maxwell equations. Dynamics of quasi-normal modes of black branes.

"Complex entangled" states of quantum matter, *not* adiabatically connected to independent particle states

Gapped quantum matter

Z_2 Spin liquids, quantum Hall states

Conformal quantum matter

Graphene, ultracold atoms, antiferromagnets

Compressible quantum matter

Strange metals, Bose metals

S. Sachdev, 100th anniversary Solvay conference (2011), arXiv:1203.4565

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Important insights from holography

S. Sachdev, 100th anniversary Solvay conference (2011), arXiv:1203.4565

Conclusions

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Conformal quantum matter

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- Good prospects for experimental tests of frequency-dependent, non-linear, and non-equilibrium transport