

# Classical and quantum matrix product states: exploring the structure of non-equilibrium states

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The Higgs Centre for Theoretical Physics, JCMB, University of Edinburgh

## Abstracts:

### Overview of DMRG and related methods - S. White

### Matrix ansatz for exclusion processes - B. Derrida

This talk will explain the basic idea of the matrix ansatz to describe the steady state of exclusion processes and review a number of exact results obtained via this approach:

- the phase diagram of the TASEP
- the equal time density fluctuations
- the matrix ansatz on the infinite line
- the calculation of large deviation functions.

### Applications of matrix product states to quantum transport - J. Moore

Matrix-product-state (MPS) methods that exploit translation invariance and minimize entanglement growth enable rather accurate studies of several interesting dynamical problems in one spatial dimension. We explain a few examples where such methods either allow testing of a theory in detail or suggested a new theory where none existed before, and discuss some general features of the MPS description of gapless systems. Results include some exact statements about far-from-equilibrium thermal transport in the XXZ model and scaling after a quench in a one-dimensional system with a boundary Majorana fermion.

### Nested Matrix Ansatz for the Exclusion Process - K. Mallick

In this talk, we shall explain how the stationary measure of multi-species exclusion processes on a ring can be expressed by a Matrix Product Representation. The algebras involved are constructed recursively by performing tensor products of the fundamental quadratic algebra that appeared in the solution of the archetypal ASEP problem (Derrida, Evans, Hakim and Pasquier, 1993).

In the second part of the talk, we show that similar structures play a role in the calculation of the current fluctuations in the single-species exclusion process with open boundaries.

### Stochastic Matrix Product States - F. Verstraete

The success of DMRG and matrix product states for simulating quantum spin systems stems from the existence of area laws and rapidly decaying Schmidt coefficients for ground states. We will explore the stochastic analogue of such area laws, and the complications that arise by the fact that stochastic matrix product states are normalized in 1-norm as opposed to 2-norm.

## **Matrix Product State and Matrix Product Operator in Symmetry Protected Topological Phases - X. Chen**

Symmetry protected topological (SPT) phases have unusual robust gapless edge states around an otherwise rather featureless gapped symmetric bulk. Their theoretical proposal and subsequent experimental discovery generated a lot of excitement in condensed matter physics. In this talk, I will discuss the important role the matrix product formalism played in the systematic study of SPT phases in general interacting spin / boson systems. In particular, the matrix product state led to the complete classification of SPT phases in one dimensional spin / boson models and the matrix product operator was essential to the establishment of nontrivial SPT order in a systematic construction of 2D SPT models. Their contribution highlights the crucial dependence of SPT order on the nontrivial entanglement pattern in many-body systems and the matrix product formalism provides an efficient characterization of such global entanglement patterns.

## **Matrix product solutions of boundary driven quantum chains - T. Prosen**

I will review recent progress on constructing non-equilibrium steady state density operators of boundary driven locally interacting quantum chains, where driving is implemented via Markovian dissipation channels attached to the chain's ends. I will elaborate on the connection between the matrix product form of nonequilibrium states and the integrability structures of the bulk Hamiltonian. However, there is a remarkable distinction with respect to the conventional quantum inverse scattering method, namely addressing nonequilibrium steady state density operators requires non-unitary irreducible representations of Yang-Baxter algebra which are typically of infinite dimensionality. Such constructions result in non-Hermitian, and often also non-diagonalisable families of commuting transfer operators which in turn result in novel conservation laws of the integrable bulk Hamiltonians.

## **Steady states of dissipative chains using matrix product operators - M. Banuls**

Tensor network states have proven successful in describing ground states of quantum many body systems. The paradigmatic example is that of Matrix Product States (MPS), which underlie the celebrated DMRG method for the study of one dimensional systems. Using these methods it is also possible to simulate dynamics. MPS can be also extended to describe density operators. In the case of open systems, this extension has been combined with the evolution algorithms to find the stationary state under some Markovian dynamics. However, growing entanglement in the evolved state might represent a drawback for such strategy. We have devised a new variational algorithm to find a MPS description for the steady states of dissipative one dimensional systems. We have applied it to different dissipative spin chains and shown that the strategy allows for an efficient exploration of the the nature of the steady state over a broad range of parameters.

## Higgs Centre Colloquium

### Entanglement Matters: from Bell states to Quantum Tensor Networks - F. Verstraete

The theory of quantum entanglement is revolutionizing the field of quantum many body physics. One of the most relevant developments in this direction has been the introduction of quantum tensor networks, which provide a natural framework for describing the entanglement structure in ground states of strongly correlated systems. I will give an overview of this new research direction, and discuss applications in the field of low dimensional quantum many body systems, in systems exhibiting topological quantum order, and in simulating gauge field theories.

### Exact matrix product solution for the boundary-driven Lindblad XXZ chain- G.Schutz

We demonstrate that the exact nonequilibrium steady state of the one-dimensional Heisenberg XXZ spin chain driven by boundary Lindblad operators can be constructed explicitly with a matrix product ansatz for the nonequilibrium density matrix where the matrices satisfy a quadratic algebra. This algebra turns out to be related to the quantum algebra  $U_q[\mathfrak{sl}(2)]$ . For the isotropic Heisenberg chain, coupled at the ends to boundary reservoirs polarized in different directions, we calculate the exact magnetization profiles and magnetization currents in the nonequilibrium steady state of a chain with  $N$  sites.

The magnetization profiles are harmonic functions with a frequency proportional to the twisting angle  $\theta$ . The currents of the magnetization components lying in the twisting plane and in the orthogonal direction behave qualitatively differently: In-plane steady-state currents scale as  $1/N^2$  for fixed and sufficiently large boundary coupling, and vanish as the coupling increases, while the transversal current increases with the coupling and saturates to  $2\theta/N$ .

### Integrability and Matrix ansatz - E. Ragoucy

We present the Matrix ansatz from the integrability point of view. This can be done on very general grounds, allowing to formulate explicitly the matrix ansatz (namely the DEHP algebra) for different models. We illustrate the technics on a reaction-diffusion model.