Conformal theory: New light on dark matter, dark energy, and dark galactic halos

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Introduction

- The current ΛCDM model of cosmology invokes unobserved dark matter[1] to account for observed gravitational phenomena that cannot be explained by Einstein general relativity.

- Gravitational effects are observed as acceleration, more precisely as deflection of spacetime geodesics, such as particle trajectories or light rays. Examples are excessive rotational velocities for mass particles in galactic orbits, and galactic lensing of radiation.

- Theory that departs from standard particle physics and cosmology only by postulating universal conformal symmetry has been found to explain such phenomena, without requiring dark matter.

- Conformal gravity and the conformal Higgs model fit empirical data on galactic rotational velocities, on Hubble expansion including dark energy, and on dark galactic halos[2, 3, 4].

- Relativistic variational field theory[5, 2]:
  - Metric tensor $g_{\mu\nu}$, determinant g, such that $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$ for $x^\mu = \{t, x, y, z\}$.
  - Flat-space diagonal metric $\{1, -1, -1, -1\}$.
  - Riemannian scalar Lagrangian density $\mathcal{L}$.
  - Action integral $I = \int d^4x \sqrt{-g}\mathcal{L}$.
  - Coupled fields: stationary $I = \sum_a I_a$, given $\mathcal{L}_a$.
  - Metric functional derivative $X^{\mu\nu} = \frac{1}{\sqrt{-g}} \frac{\delta I}{\delta g_{\mu\nu}}$.
  - Energy-momentum tensor $\Theta_a^{\mu\nu} = -2X_a^{\mu\nu}$.
  - Einstein equations: $\sum_a X_a^{\mu\nu} = 0$ implies $X_g^{\mu\nu} = \frac{1}{2}\sum_{a\neq g} \Theta_a^{\mu\nu}$, trace $g_{\mu\nu}\sum_a X_a^{\mu\nu} = 0$.

- Local Weyl scaling (conformal) symmetry[6, 7]:
  - $g_{\mu\nu}(x) \to g_{\mu\nu}(x)\Omega^2(x)$, for fixed coordinates x^μ.
  - Scalar field: $\Phi(x) \to \Phi(x)\Omega^{-1}(x)$.
  - Postulate I_a invariant for any differentiable $\Omega(x)$, for all bare fields.
  - Massless fermion and gauge boson fields exhibit strict conformal symmetry.
  - Standard general relativity and the electroweak Higgs model are not conformal.
  - Trace $g_{\mu\nu}\Theta_a^{\mu\nu} = 0$ for conformal $\mathcal{L}_a$.

Surprising properties of conformal theory

- Universal conformal symmetry, postulated here for all elementary massless physical fields, retains standard fermion and gauge boson theory but modifies Einstein-Hilbert general relativity and the Higgs scalar field model, with no new physical fields.

- Conformal gravitational Lagrangian density $\mathcal{L}_g$ is the uniquely defined quadratic contraction of the conformal Weyl tensor[6, 7]. A theorem by Lanczos reduces it to $R^{\mu\nu}R_{\mu\nu} - \frac{1}{3}R^2$, for Ricci tensor $R^{\mu\nu}$ and scalar $R = g_{\mu\nu}R^{\mu\nu}$. The Weyl tensor vanishes identically in the uniform, isotropic geometry appropriate to Hubble expansion.

- $\mathcal{L}_\Phi$ for conformal Higgs scalar field Φ[8], which contains $(w^2 - \frac{1}{6}R - \lambda\Phi^\dagger\Phi)\Phi^\dagger\Phi$, is also uniquely defined. The term in R affects the gravitational field equation. Since the very early electroweak transition, the Higgs field must exist with nonzero amplitude throughout spacetime.

- Since uniform geometry eliminates $\mathcal{L}_g$, the most plausible assumption is that the conformal Higgs model is responsible for observed Hubble expansion.

- The effective gravitational constant derived from $\mathcal{L}_\Phi$ depends on parameters of the Higgs electroweak model. It has opposite sign and different magnitude from the Newton constant.

- Uniform mass-energy density ρ_m drives centrifugal cosmic acceleration since the earliest cosmos, possibly the mechanism for the early acceleration characterized as a Big Bang.

- Parameter w^2 becomes dark energy in the gravitational field equation of the conformal Higgs model (modified Friedmann equation). Net acceleration due to ρ_m and w^2 decreases asymptotically to zero in the remote future.

- Dependence of Hubble acceleration on ρ_m and cosmic curvature can be shown to determine centripetal acceleration attributed to dark halos, observed in galactic rotation velocities.

The conformal Higgs model and Hubble expansion

- The mathematics of general relativity can be intimidating. This is vastly simplified in two very special geometric situations, both used here to quantify observed cosmological phenomena.

- In a uniform, isotropic universe, averaged over all local structure such as galaxies, the geometry reduces to a single active coordinate, the time as measured by any so-called comoving observer, at any selected spacetime event.

- This implies the Robertson-Walker (RW) metric, $ds^2_{RW} = dt^2 - a^2(t)(\frac{dr^2}{1-kr^2} + r^2 d\omega^2)$, originally introduced by Friedmann and LeMaitre to describe cosmic evolution. Here k is a curvature constant.

- The conformal Higgs model in RW geometry implies a modified time-dependent Friedmann cosmic evolution equation[8].

- Given cosmic average mass-energy density $\rho_m = \Theta_m^{00}$, the modified Friedmann equation for scale factor $a(t)$ is $\frac{\dot{a}^2}{a^2} + \frac{k}{a^2} - \frac{\ddot{a}}{a} = \frac{2}{3}(\bar{\kappa}\rho_m + \bar{\Lambda})$. Vanishing trace removes the usual second Friedmann equation.

- Gravitational constant $\bar{\kappa} < 0$ and dark energy $\bar{\Lambda} > 0$ are defined by $\mathcal{L}_\Phi$ for the Higgs scalar field[8].

- This modified Friedmann equation, which derives dark energy from the Higgs model, has been applied to fit detailed observed data on the Hubble expansion[8].

- Dividing the Friedmann equation by $\frac{\dot{a}^2}{a^2}$ determines dimensionless sum rule $\Omega_m + \Omega_k + \Omega_\Lambda + \Omega_q = 1$. This adds acceleration weight $\Omega_q = -q = \frac{\ddot{a}a}{\dot{a}^2}$ to the sum rule of standard theory, usually presented as a pie-chart of the cosmic energy budget.

- Mannheim[9] fitted supernovae data as accurately as ΛCDM theory by setting $\Omega_\Lambda = 0.37$ and $\Omega_m = 0$ in the standard sum rule, eliminating dark matter.

- However, for $\Omega_m = 0$, standard sum rule $\Omega_k + \Omega_\Lambda = 1$ would imply current curvature weight $\Omega_k(t_0) = 0.63$, much larger than its consensus empirical value.

- The conformal Higgs model removes this problem of excessive curvature. Solving the modified Friedmann equation with $\Omega_m = \Omega_k = 0$[8], a fit to Type Ia supernovae data for redshifts $z \leq 1$ finds $\Omega_\Lambda(t_0) = 0.732$, in agreement with consensus empirical value $\Omega_\Lambda(t_0) = 0.726 \pm 0.015$.

- The computed acceleration weight is $\Omega_q(t_0) = 0.268$. Only one independent parameter is required for $z \leq 1$. In the current epoch, dark energy and acceleration terms are of comparable magnitude, the curvature term is small, and other terms are negligible.

- A numerical solution from $t = 0$ to current $t = t_0$[8] is determined by four fixed parameters, fitted to two dimensionless ratios that characterize CMB acoustic peak structure[10] as well as to redshifts for $z \leq 1$.

Dark energy

- Higgs parameter w^2, which breaks conformal symmetry, must be a dynamical consequence of the theory. w^2 becomes dark energy in the modified Friedmann cosmic evolution equation[8].

- Coupled scalar and gauge boson fields produce gauge boson mass through the Higgs mechanism. Conformal symmetry introduces an interaction with gravitation in the scalar field Lagrangian density, which extends the Higgs model to include the metric tensor.

- This preserves the Higgs mechanism for gauge boson masses and the trace condition for the coupled field equations[4]. The resulting cosmological time dependence of Ricci scalar R determines a nonvanishing neutral gauge field source current density[11].

- The Higgs scalar field is dressed by an induced neutral gauge boson field. Parameter w^2 is determined as self-interaction of the scalar field due to the induced accompanying gauge field.

- The very small implied scale parameter offers an explanation, unique to conformal theory, of the huge disparity in magnitude between parameters relevant to cosmological and elementary-particle phenomena.

- If derivatives of Φ can be neglected, the scalar field equation has an exact solution given by $\Phi^\dagger\Phi = \phi_0^2 = (w^2 - \frac{1}{6}R)/2\lambda$. The phase is arbitrary, so ϕ_0 can be a real constant. Its experimental value is $\phi_0 = 180GeV$.

- From the scalar field equation, $\phi_0^2 = -\zeta/2\lambda$, where $\zeta = \frac{1}{6}R - w^2$. Computed from the integrated modified Friedmann equation, $\zeta(t_0) = 1.224 \times 10^{-66}eV^2$[11]. Given $\phi_0 = 180GeV$, the empirical value of dimensionless Higgs parameter $\lambda = -\frac{1}{2}\zeta/\phi_0^2$ is -0.189×10^{-88}.

- Using parameters fitted by integrating the modified Friedmann equation from $t = 0$ to t_0, dark energy weight $\Omega_\Lambda(t_0) = w^2 = 0.717$, in Hubble units. Hence $w = 0.847\hbar H_0 = 1.273 \times 10^{-33} eV$, where H_0 is the Hubble constant.

- Solving the coupled field equations for $g_{\mu\nu}, \Phi$, and induced neutral gauge field Z_μ, using computed time derivative $\frac{\dot{\phi}_0}{\phi_0}(t_0)$, gives $w \simeq 2.651\hbar H_0 = 3.984 \times 10^{-33} eV$[11]. A more accurate calculation should include charged fields $W^\pm_\mu$ and the presently unknown time dependence of Higgs parameter λ.

- This approximate calculation agrees in magnitude with the value implied by dark energy Hubble weight $\Omega_\Lambda(t_0)$. This level of agreement justifies the conclusion that conformal theory explains both the existence and magnitude of dark energy.

- For $\lambda < 0$ the conformal Higgs scalar field does not have a stable fluctuation, required to define a massive Higgs particle. The recent observation of a particle or resonance at 125 GeV is consistent with such a Higgs boson, but may prove to be an entirely new entity when more definitive secondary properties are established.

- Because the conformal Higgs field retains the finite field amplitude essential to gauge boson and fermion mass, while accounting for empirically established dark energy, an alternative explanation of the recent 125GeV resonance might avoid a severe conflict with observed cosmology.

Conformal gravity and galactic rotational velocities

- In static, spherically symmetric geometry, the only relevant coordinate is the radius from an assumed central point, for example the mass center of the sun or of our galaxy. The tensorial equations of relativity reduce to ordinary differential equations in the radial coordinate.

- This is the basic idea behind the exterior Schwarzschild (ES) metric, written in conformal theory as
  $ds^2_{ES} = B(r)dt^2 - \frac{dr^2}{B(r)} - r^2 d\omega^2$,
  where $d\omega^2 = d\theta^2 + \sin^2\theta d\phi^2$.

- Mannheim and Kazanas[12, 7] derived an exact ES solution for conformal gravity outside a spherical source density, valid for rotation velocities in the outer reaches of galaxies. The gravitational potential is $B(r) = 1 - 2\beta/r + \gamma r - \kappa r^2$. A circular orbit with velocity v is stable if
  $v^2 = \frac{1}{2} r dB/dr = \beta/r + \gamma r/2 - \kappa r^2$.

- Parameter $\beta = GM$ is proportional to total galactic baryonic mass M if dark matter is omitted. Defining N^* as total visible plus gaseous mass in solar units, and neglecting κ, Mannheim determined two universal parameters such that $\gamma = \gamma^* N^* + \gamma_0$ fits rotational data for eleven typical galaxies, not invoking dark matter[13, 2].

- This fit of conformal gravity to rotational data has recently been extended, including parameter κ, to 138 galaxies whose orbital velocities are known outside the optical disk[14, 15, 16], using only three universal parameters in addition to Newtonian G.

- The fit of mass-independent γ_0 to observed data implies a significant effect of the cosmic background, external to a baryonic galactic core[13]. This implies an isotropic gravitational field with a cosmological source, hence a spherical gravitational halo.

- For spiral galaxies, v^2 is nearly constant over a large range of r where Keplerian acceleration $a \simeq a_0$, an empirical universal constant. Empirical Tully-Fisher relation $v^4 = a_0 GM$ for galactic mass M is valid in particular for largely gaseous galaxies, whose baryonic mass is well-defined[17]. It does not follow readily from the ΛCDM model of a dark matter galactic halo.

- Well inside halo radius r_H, conformal velocity function $v^2(r) = GM/r + \gamma r/2$ has a broad local minimum at $r_x^2 = 2GM/\gamma$. This confirms the observed flat $v^2(r)$ curve for spiral galaxies.

- Evaluated at r_x, $\gamma r_x/2 = GM/r_x$, such that $v^4(r_x) = 4(\gamma r_x/2)(GM/r_x) = 2\gamma GM$. If $\gamma^* N^* \ll \gamma_0$ and dark matter is omitted, this is an exact baryonic Tully-Fisher relation.

Dark halos without dark matter

- Extended dark halos are detected by gravitational effects including excessive rotation velocities and gravitational lensing. Understanding these phenomena requires use of both ES and RW metrics.

- Starting from the primordial cosmos, postulated to have uniform, isotropic mass-energy density ρ_m, any concentration of matter must extract this matter from a surrounding depleted background. Primordial ρ_m is condensed into galactic density ρ_g. Total galactic mass M is missing from a surrounding depleted uniform, isotropic background.

- Gravitational effects of a depleted background could be attributed to a halo of dark matter surrounding a galaxy. This is the current consensus model of galactic halos. Conformal theory, which eliminates the need for dark matter to explain excessive rotation velocities, provides an alternative interpretation of halo phenomena[18, 4].

- Modeled by a totally depleted sphere of radius r_H, $4\pi\rho_m r_H^3/3 = M$. The integral of $\rho_g - \rho_m$ must vanish. A very large halo radius r_H is implied. If $\rho_g/\rho_m \simeq 10^5$ the ratio of radii is $10^{\frac{5}{3}} = 46.4$, so that a galaxy of radius 10kpc would be embedded in a depleted halo of radius 464kpc.

- Hubble expansion is characterized by centrifugal acceleration[8]. In conformal theory[18], removal of mass-energy density ρ_m from a halo reduces this background acceleration. The net observed effect is centripetal acceleration in outer galactic orbits within a dark halo, parametrized by Mannheim's parameter γ_0[2].

- Gravitational field equation $X_g^{\mu\nu} + X_\Phi^{\mu\nu} = \frac{1}{2}\Theta_m^{\mu\nu}$ has an exact solution for $r_g \leq r \leq r_H$. ES metric parameters proportional to galactic mass M, valid for $X_g^{\mu\nu} = 0, r \geq r_g$, are determined by matching an assumed spherically averaged internal solution at r_g to the explicit external solution. $X_\Phi^{\mu\nu} = 0$ is solved exactly for $r \leq r_H$ in the RW metric by the modified Friedmann equation, omitting ρ_m.

- The value of acceleration parameter γ_0 in the ES metric must be adjusted to fit the observable acceleration inferred in the RW metric from the Friedmann equation with and without a mass term.

- The procedure is to compute acceleration parameter Ω_q for mass-energy densities $\rho(halo) = 0$ and $\rho(cosmos) = \rho_m$. The physically relevant γ_0 is determined from the difference $\Delta\Omega_q = \Omega_q(halo) - \Omega_q(cosmos)$.

- This analysis relates $\Delta\Omega_q = \Omega_q(halo) - \Omega_q(cosmos)$ to $\Delta\rho = \rho_g - \rho_m$, which reduces to $-\rho_m$ in the halo. From the dimensionless sum rule, $\Omega_q = 1 - \Omega_k - \Omega_\Lambda - \Omega_m$. If curvature and dark energy are independent of ρ_m, $\Delta\Omega_q = -\Delta\Omega_m = \Omega_m(\rho_m) < 0$. This centripetal acceleration is consistent with parameter $\gamma_0 > 0$.

- In the ES metric, radial acceleration due to a galactic halo is centripetal $\frac{\ddot{r}}{r} = -\frac{1}{2}\gamma_0 c^2$. At t_0, $\gamma_0 = -2h^2\Delta\Omega_q = -2\Omega_m(\rho_m, t_0)$ in Hubble units.

- To the extent that dark energy Ω_Λ and cosmic background Ω_k are independent of ρ_m, they cancel out of $\Delta\Omega_q$. Because fixed coordinate r is multiplied by $a(t)$, observed radial acceleration due to a halo is $\frac{\ddot{r}}{r} = \Delta\frac{\ddot{a}}{a} = \Delta\frac{\dot{a}^2}{a^2}\Omega_q = \frac{\dot{a}^2}{a^2}\Omega_m(\rho_m)$, if Ω_k is independent of mass density.

- Given empirical $\gamma_0 = 3.06 \times 10^{-28} m^{-1}$ and Hubble length $\frac{c}{H_0} = 1.313 \times 10^{26} m$ in MKS units, dimensionless $\Omega_m(\rho_m; t_0) = -\frac{1}{2}\gamma_0\frac{c}{H_0} = -2.01 \times 10^{-2}$. This has the correct magnitude for fitted parameters $\Omega_k(t_0) + \Omega_m(t_0) = -1.25 \times 10^{-2}$.

- What, if any, should be the gravitational effect of a depleted background density? An analogy, in well-known physics, is vacancy scattering of electrons in conductors. In a complex material with a regular periodic lattice independent electron waves are by no means trivial functions, but they propagate without contributing to scattering or resistivity unless there is some lattice irregularity, such as a vacancy.

- Impurity scattering depends on the difference between impurity and host atomic T-matrices. Similarly, a photon or isolated mass particle follows a geodesic in the cosmic background unless there is some disturbance of the uniform density ρ_m. Observed acceleration is deflection of a geodesic.

- Equivalence of galactic and displaced halo mass resolves the paradox for ΛCDM that despite any interaction other than gravity, the amount of dark matter inferred for a galactic halo is strongly correlated with the galactic luminosity or baryonic mass[19].

Implications for cosmology and particle physics

- Observed gravitational phenomena can be understood without dark matter, at least for an isolated galaxy.

- A new basic rule for mass-depleted halos is that total depleted volume must be conserved in a galactic collision. Hence halos cannot overlap.

- The Newtonian virial theorem is not valid.

- Interactions between galaxies must be reconsidered with revised theory.

- Conformal theory retains the Higgs mechanism for particle masses, but does not imply the existence of a Higgs particle.

- Dark energy is an unanticipated consequence of the Higgs scalar field model for gauge boson masses.

- Dark energy is self-interaction due an induced neutral gauge field, producing Higgs parameter w^2.

- Higgs parameter λ could be produced by an induced diboson field, W_+W_- interacting with Z_0Z_0, at approximately 125GeV[20].

- The recently observed 125GeV resonance may be such a completely new particle or field.

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Appendix: semiclassical coupled field equations

- The Higgs model derives gauge boson mass from coupling via gauge covariant derivatives to a postulated SU(2) doublet scalar field Φ. SU(2) symmetry is broken by a solution of the scalar field equation such that $\Phi^\dagger\Phi = \phi_0^2$, a spacetime constant. Only the charge-neutral component of doublet field Φ is nonzero.

- Generation of gauge field masses follows from a semiclassical theory of coupled scalar and gauge fields, extended here to include the gravitational metric tensor field in RW geometry. This is further simplified by considering only neutral vector field Z_μ. Numerical results follow from solving nonlinear coupled field equations.

- Gauge invariance replaces bare derivative ∂_μ by gauge covariant derivative

$$D_\mu = \partial_\mu - \frac{i}{2} g_z Z_\mu \quad (1)$$

This retains $\mathcal{L}_Z$ and augments conformal

$$\mathcal{L}^0_\Phi = (\partial_\mu \Phi)^\dagger \partial^\mu \Phi - \frac{1}{6} R \Phi^\dagger \Phi \quad (2)$$

by coupling term

$$\Delta\mathcal{L} = (D_\mu \Phi)^\dagger D^\mu \Phi - (\partial_\mu \Phi)^\dagger \partial^\mu \Phi =$$
$$\frac{i}{2} g_z Z^*_\mu \Phi^\dagger \partial^\mu \Phi - \frac{i}{2} g_z Z^\mu (\partial_\mu \Phi)^\dagger \Phi + \frac{1}{4} g_z^2 \Phi^\dagger Z^*_\mu Z^\mu \Phi. \quad (3)$$

- Given mass parameter m_Z and source current density J_Z^μ, the parametrized part of $\mathcal{L}$ for a complex vector field is

$$\mathcal{L}_Z + \frac{1}{4}Z^*_{\mu\nu}Z^{\mu\nu} = \frac{1}{2}m_Z^2 Z^*_\mu Z^\mu - \frac{1}{2}(Z^*_\mu J^\mu_Z + Z^\mu J^*_{Z\mu}). \quad (4)$$

The parametrized field equation for Z^μ is

$$\partial_\nu Z^{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta \Delta I}{\delta Z^*_\mu} = m_Z^2 Z^\mu - J^\mu_Z. \quad (5)$$

$\Delta\mathcal{L}$ from D_μ, Eq.(3), determines parameters for field Z^μ:

$$\frac{2}{\sqrt{-g}}\frac{\delta \Delta I}{\delta Z^*_\mu} = \frac{1}{2}g_z^2 \Phi^\dagger \Phi Z^\mu + i g_z \Phi^\dagger \partial^\mu \Phi \quad (6)$$

implies not only Higgs mass formula $m_Z^2 = \frac{1}{2}g_z^2\Phi^\dagger\Phi$, but also field source density $J^\mu_Z = -i g_z \Phi^\dagger \partial^\mu \Phi$.

- $\Delta\mathcal{L}_\Phi = (w^2 - \lambda\Phi^\dagger\Phi)\Phi^\dagger\Phi$ in the parametrized Higgs model. The parametrized scalar field equation is

$$\partial_\mu\partial^\mu\Phi + \frac{1}{6}R\Phi = \frac{1}{\sqrt{-g}}\frac{\delta\Delta I}{\delta\Phi^\dagger} = (w^2 - 2\lambda\Phi^\dagger\Phi)\Phi. \quad (7)$$

  Using $\Delta\mathcal{L}$ derived from D_μ, Eq.(3),

$$\frac{1}{\sqrt{-g}}\frac{\delta\Delta I}{\delta\Phi^\dagger} = \frac{1}{4}g_z^2 Z_\mu^* Z^\mu\Phi + \frac{i}{2}g_z(Z_\mu^* + Z_\mu)\partial^\mu\Phi. \quad (8)$$

  Comparison implies $w^2 = \frac{1}{4}g_z^2 Z_\mu^* Z^\mu$.

- Neglecting derivatives of induced gauge field Z^μ, Eq.(5) reduces to $Z^\mu = J_Z^\mu/m_Z^2$. Time variation determines pure imaginary $J_Z^0 = -ig_z\phi_0^*\partial^0\phi_0 = -ig_z\frac{\dot\phi_0}{\phi_0}\phi_0^*\phi_0$, defining real parameter $\frac{\dot\phi_0}{\phi_0}$. Implied pure imaginary Z^0 does not affect λ. The scalar field equation implies $w^2 = \frac{1}{4}g_z^2|Z|^2 = (\frac{\dot\phi_0}{\phi_0})^2$.