# Muon $g-2$: much ado about nothing?

## A lattice QCD calculation of the leading hadronic contribution


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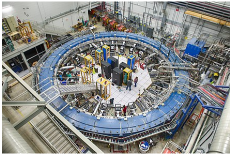
Budapest-Marseille-Wuppertal collaboration [BMWc]

Borsanyi, Fodor, Guenther, Hoelbling, Katz, LL, Lippert, Miura, Szabo, Parato, Stokes, Toth, Torok, Varnhorst

Nature 593 (2021) 51, online 7 April 2021 $\rightarrow$ BMWc '20
PRL 121 (2018) 022002 (Editors' Selection) $\rightarrow$ BMWc '17
& Aoyama et al., Phys. Rept. 887 (2020) 1-166 $\rightarrow$ WP '20

# Much ado about nothing?

## NO !!!



(Wikimedia)

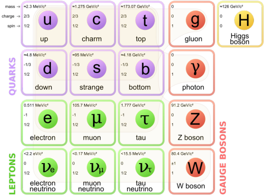
Beautiful experiments BNL (1999-2006), FNAL ($\geq$ 2007) and soon JPARC ($\geq$ 2025)

$\rightarrow$ measurement of muon magnetism with breathtaking precision:

[PhysRev D73 '06, PhysRevLett 126 '21]

$$\left[a_\mu^{\rm exp} = \frac{g_\mu^{\rm exp} - 2}{2}\right] \times 10^{10} = 11659206.1 \pm 4.1 \; [0.35\text{ppm}]$$

$\Leftrightarrow$ bathroom scale sensitive to weight of single small eyelash




(Wikimedia)

May hold clues to answer important questions left open by SM:

- Why three families of matter particles?
- Why do they have such different masses?
- How do neutrinos get their mass?
- Are electromagnetic, weak and strong forces three facets of a more fondamental force?
- Is Higgs mechanism all there is to electroweak symmetry breaking?
- What is dark matter?
- ...

# Much ado about nothing?

Difference between measurement and prediction of SM from [WP '20]

$$a_\mu^{\rm exp} - a_\mu^{\rm SM} = (25.1 \pm 5.9) \times 10^{-10}\ [4.2\sigma]$$

- $4.2\sigma \Rightarrow \sim 1/40,000$ chance that the two numbers actually agree ...
- ... assuming that uncertainties in SM prediction, which have large *systematic* component (known unknown), can be interpreted statistically
- Too large to ignore, but too small to claim new fundamental physics (usually $5\sigma$)
- With planned uncertainty of final FNAL measurement ca. 2025, could have $\sim 6\sigma$ w/out even improving SM prediction, marking discovery of new elementary particles or forces
- Must follow this opportunity to the end

Caution:

- asked to say a few words about new calculation by BMWc

→ very small part of huge international effort around muon $g-2$

→ cannot do justice to extraordinary work of many FNAL, JPARC and theory colleagues

→ cannot do justice to long history that contributed to development of SM over last $\sim$ 100 yrs

# Hadronic or strong force contribution

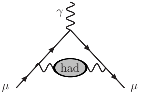
- All three forces (electromagnetic, strong & weak) and all particles of SM needed to make precise enough prediction for muon $g-2$

- In particular

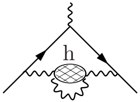
$$a_\mu^{\text{had}} = 0.6\% \times a_\mu \qquad\qquad \text{err}(a_\mu^{\text{had}}) = 100\% \times \text{err}(a_\mu)$$

$\Rightarrow$ focus on leading-order (LO) hadronic vacuum polarization (HVP) that gives most of hadronic contribution and uncertainty

$$a_\mu^{\text{LO-HVP}} = 0.6\% \times a_\mu \qquad\qquad \text{err}(a_\mu^{\text{LO-HVP}}) = 93\% \times \text{err}(a_\mu)$$




+




$$\to a_\mu^{\text{LO-HVP}} = O\left(\left(\frac{\alpha}{\pi}\right)^2 \left(\frac{m_\mu}{M_\rho}\right)^2\right) = O(1000. \times 10^{-10})$$

# Hadronic or strong force contribution

Challenging to compute because strong interactions highly nonlinear for $E \sim m_\mu c^2$

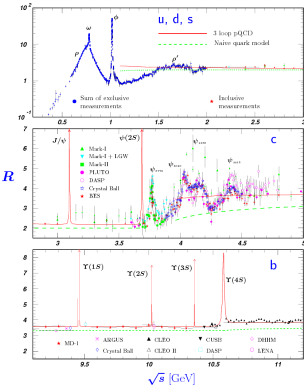
$\Rightarrow$ techniques used for electromagnetic and weak interactions do not work

- Reference approach combines general properties of SM and experimental data for $e^+e^- \to$ hadrons [DHMZ '19, KNT '19, CHHKS '19]

$$\to a_\mu^{\text{LO-HVP}} = (693.1 \pm 4.0) \times 10^{-10} \; [0.6\%]$$

- We and other groups solve the equations of fundamental theory using supercomputers to provide independent crosscheck of this most uncertain contribution [Blum '02, ..., BMWc '17, RBC/UKQCD '18, ETM '19, PACS '19, FHM '19, Mainz '19, LM '20, BMWc '20]

  $\to$ lattice quantum chromodynamics (LQCD) w/ initial subpercent target




(PDG compilation)

# What is lattice QCD (LQCD)?

To describe ordinary matter, QCD requires $\geq$ 104 numbers at every point of spacetime

$\rightarrow$ $\infty$ number of numbers in our continuous spacetime

$\rightarrow$ must temporarily "simplify" the theory to be able to calculate

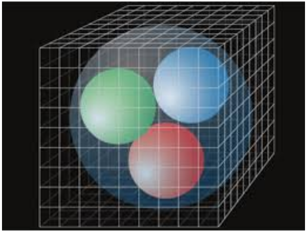
Lattice QCD (Ken Wilson '74): construct a version of QCD on spacetime that is a finite cubic lattice of points such that it reduces to real world QCD when mesh of lattice is infinitely fine and volume sufficiently large

$\rightarrow$ number of numbers to describe a state of the system becomes finite

$\rightarrow$ solve the problem with a computer

$\rightarrow$ repeat calculation for larger and finer lattices

$\rightarrow$ get predictions of real world QCD



(KEK)

# Lattice QCD: huge challenge

- Theory, algorithms and effects included have continuously improved since early 80s
- Need $\sim$ 1 billion numbers to describe accurately state of system
- Still an *uncountably* $\infty$ number of possible states !
- To quantize system: must average over all states, weighing each one with a quantum probability !!
- And must repeat calculations with more or less fine and large lattices, etc. !!!

- Most states have negligible probability

  $\rightarrow$ extremely effective algorithms to find most probable states
- Based on repeated random sampling methods of Monte Carlo type
- For very simple cases, a few hundred states suffice

Only very recently have theoretical tools and supercomputers improved enough to make subpercent calculations

# Our “accelerators”

Such computations require some of the world’s most powerful supercomputers

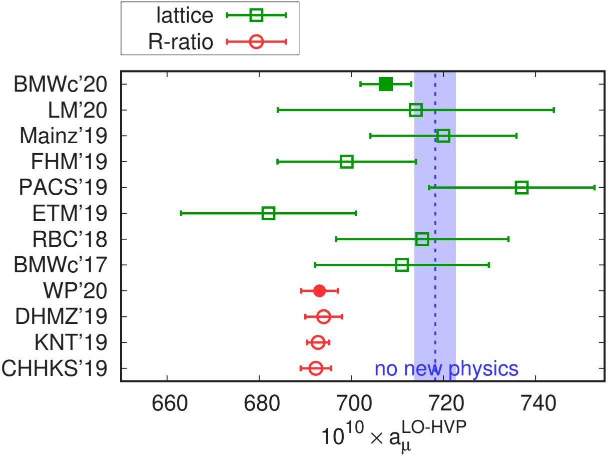




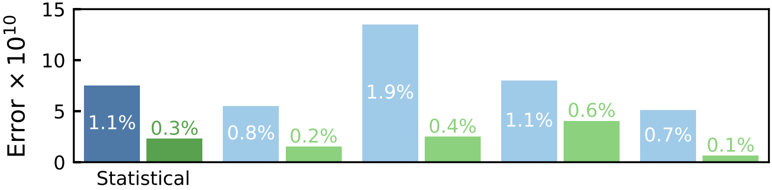

- 1 year on supercomputer $\sim$ 100 000 years on laptop
- In Germany, those of the Forschungszentrum Jülich, the Leibniz Supercomputing Centre (Munich), and the High Performance Computing Center (Stuttgart); in France, Turing and Jean Zay at the Institute for Development and Resources in Intensive Scientific Computing (IDRIS) of the CNRS, and Joliot-Curie at the Very Large Computing Centre (TGCC) of the CEA, by way of the French Large-scale Computing Infrastructure (GENCI).

# Three years of progress

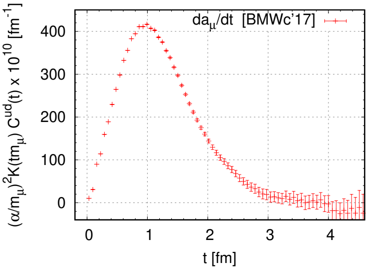
- Recently published subpercent LQCD calculation of HVP contribution to muon $g-2$ [BMWc '20]
- First lattice calculation w/ errors comparable to data-driven approach
- $(3 \div 4)\times$ improved precision over our previous [BMWc '17] and comparable calculations
- Many improvements needed to reach such precision



# Key improvements: statistical noise reduction

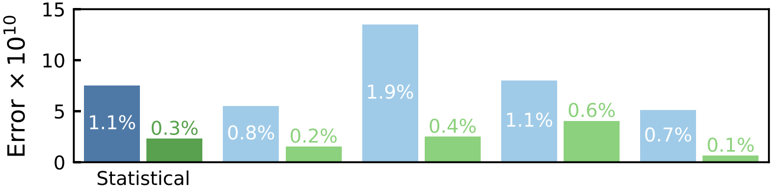



Statistical noise of up and down quark contributions increases exponentially w/ spacetime size of HVP "bubble"




($144 \times 96^3$, $a \sim 0.064$ fm, $M_\pi \sim 135$ MeV)

# Key improvements: statistical noise reduction

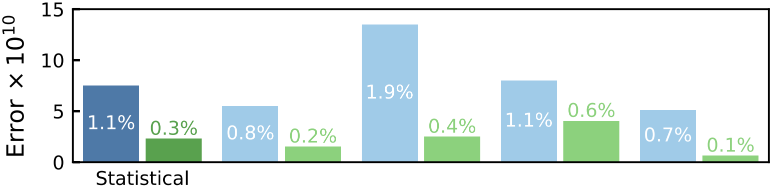



Statistical noise of up and down quark contributions increases exponentially w/ spacetime size of HVP "bubble"

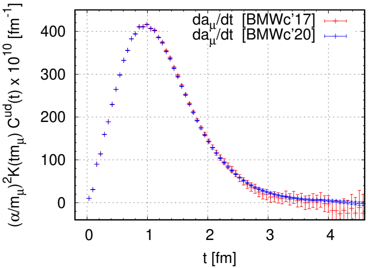
Solve w/:

- Algorithmic improvements (EigCG, solver truncation [Bali et al '09], all mode averaging [Blum et al '13]) to generate more statistics: $> 25,000$ gauge configurations & tens of millions of measurements
- Exact treatment of long-distance modes to reduce long-distance noise (low mode averaging [Neff et al '01, Giusti et al '04, ...])
- Rigorous upper/lower bounds on long-distance contribution [Lehner '16, BMWc '17]

# Key improvements: statistical noise reduction

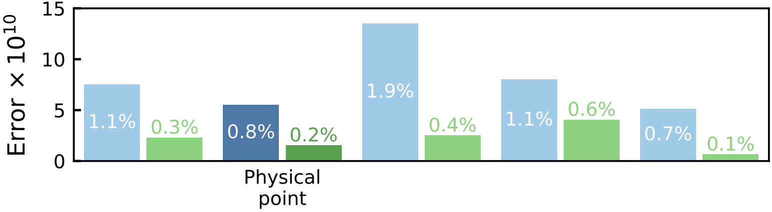



Statistical noise of up and down quark contributions increases exponentially w/ spacetime size of HVP "bubble"




($144 \times 96^3$, $a \sim 0.064$ fm, $M_\pi \sim 135$ MeV)

# Key improvements: tuning of QCD parameters

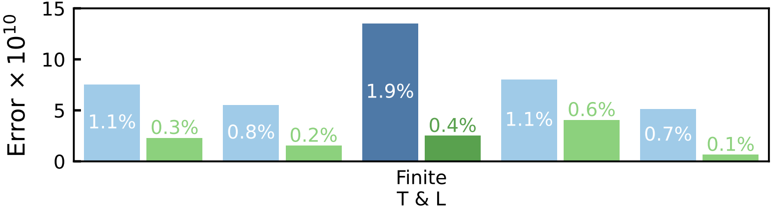



Must tune parameters of QCD very precisely: $m_u$, $m_d$, $m_s$, $m_c$ & overall mass scale

Solve w/:

- Permil determination of overall QCD scale
- Set w/ $\Omega^-$ baryon mass computed w/ 0.2% uncertainty
- Use Wilson flow scale [Lüscher '10, BMWc '12] to separate out electromagnetic corrections

# Key improvements: remove finite spacetime distortions

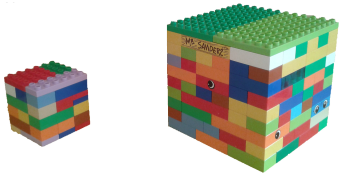



Even on "large" lattices ($L \gtrsim 6\,\mathrm{fm}$, $T \gtrsim 9\,\mathrm{fm}$), early pen-and-paper estimate [Aubin et al '16] suggested that exponentially suppressed finite-volume distortions are still $O(2\%)$

Solve by:

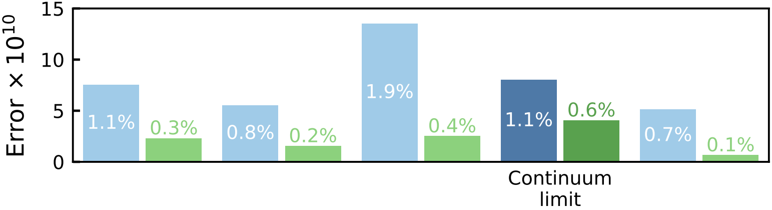
- Finding a way to perform dedicated supercomputer simulations to calculate effect between above and much larger $L = T = 11\,\mathrm{fm}$ volume directly in QCD, i.e. "big" − "ref"
- Computing remnant $\sim 0.1\%$ effect in "big" volume w/ simplified models of QCD that correctly predict "big" − "ref"

"ref" →

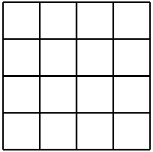


← "big"

# Key improvements: controlled continuum limit




Our world corresponds to spacetime w/ lattice spacing $a \to 0$



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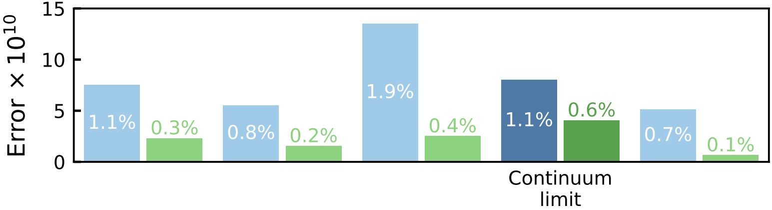


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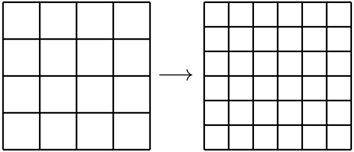

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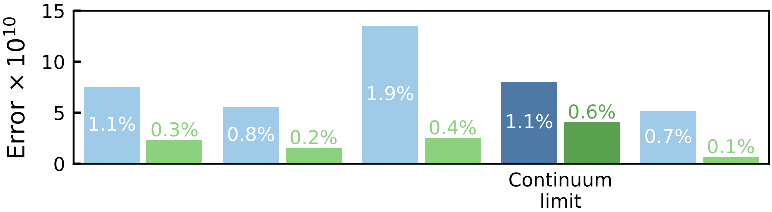


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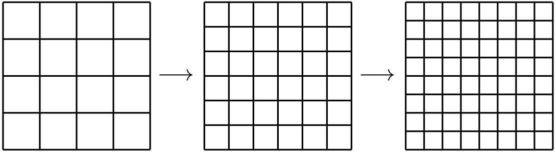

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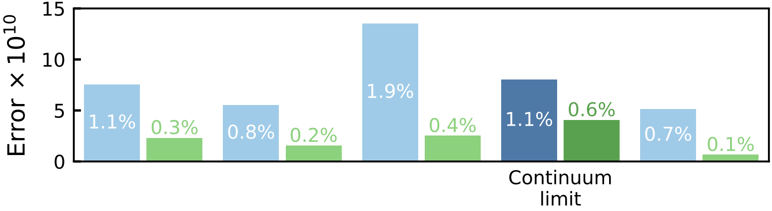


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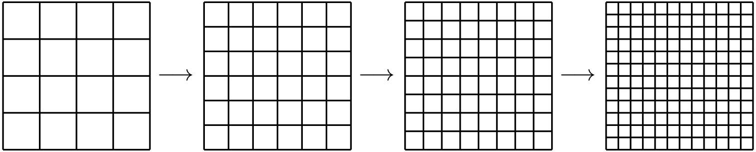

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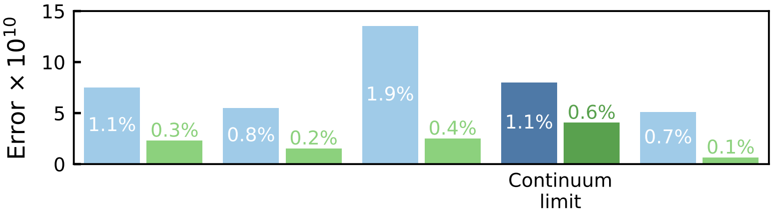


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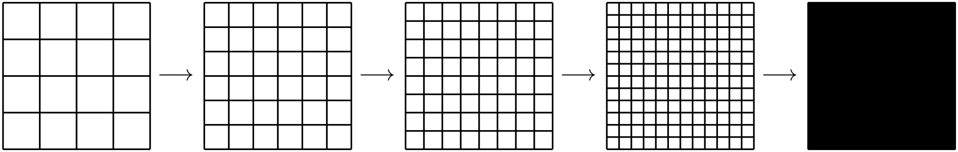

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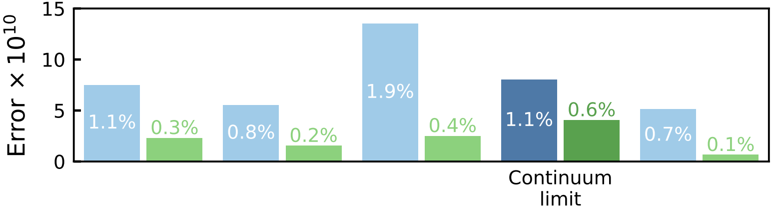
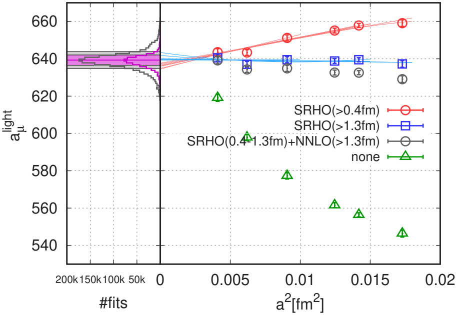


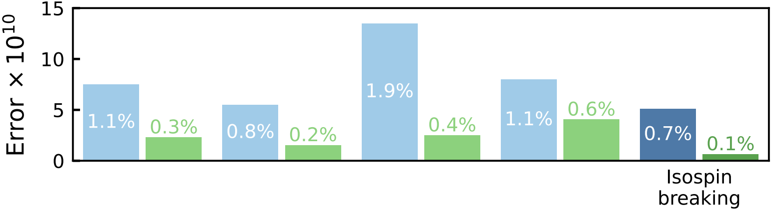

Our world corresponds to spacetime w/ lattice spacing $a \to 0$

Control $a \to 0$ extrapolation of results by:

- Performing all calculations on lattices w/ 6 values of $a$ in range $0.134\ \mathrm{fm} \to 0.064\ \mathrm{fm}$
- Reducing statistical error at smallest $a$ from 1.9% to 0.3% !
- Improving approach to continuum limit w/ simplified models for QCD [Sakurai '60, Bijnens et al '99, Jegerlehner et al '11, Chakraborty et al '17, BMWc '20] shown to reproduce distortions observed at $a>0$
- Extrapolate results to $a=0$ using theory as guide



# Key improvements: QED and $m_u \neq m_d$ corrections



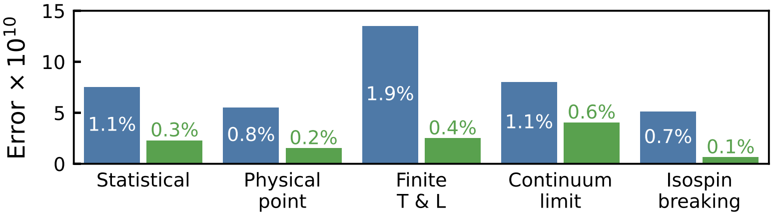

For subpercent accuracy, must include small effects from electromagnetism and due to fact that masses of $u$ and $d$ quarks are not quite equal

- Effects are proportional to powers of $\alpha = \frac{e^2}{4\pi} \sim 0.01$ and $\frac{m_d - m_u}{(M_p/3)} \sim 0.01$

$\Rightarrow$ for SM calculation at permil accuracy sufficient to take into account contributions proportional to only first power of $\alpha$ or $\frac{m_d - m_u}{(M_p/3)}$

- We include *all* such contributions for *all* calculated quantities needed in calculation

# Robust determination of uncertainties




Thorough and robust determination of statistical and systematic uncertainties

- Stat. err.: resampling methods
- Syst. err.: extended frequentist approach [BMWc '08, '14]
  - Hundreds of thousands of different analyses of correlation functions
  - Weighted by AIC weight
  - Use median of distribution for central values & $16 \div 84\%$ confidence interval to get total error

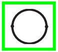
(Nature paper has 95 pp. Supplementary information detailing methods)

# Summary of contributions to $a_\mu^{\text{LO-HVP}}$

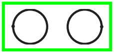
## Isospin symmetric

connected light
633.7(2.1)(4.2)

connected strange
53.393(89)(68)

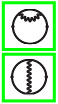


connected charm
14.6(0)(1)

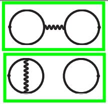


disconnected
-13.36(1.18)(1.36)

## QED isospin-breaking: valence



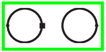
connected -1.23(40)(31)



disconnected -0.55(15)(10)

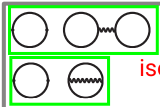
## Strong isospin-breaking

connected
6.60(63)(53)

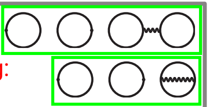


disconnected
-4.67(54)(69)

## QED isospin-breaking: sea



connected 0.37(21)(24)



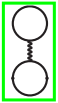
disconnected -0.040(33)(21)

## Etc.

bottom; higher order; perturbative

0.11(4)

## QED isospin-breaking: mixed



connected -0.0093(86)(95)

disconnected 0.011(24)(14)

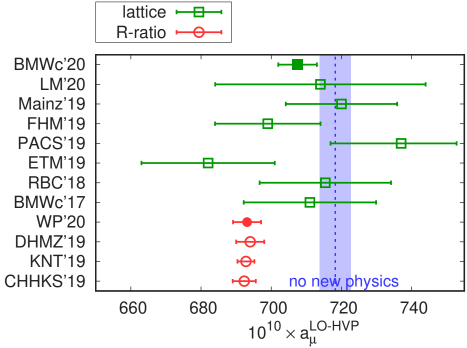
## Finite-size effects

isospin-symmetric
18.7(2.5)

isospin-breaking
0.0(0.1)

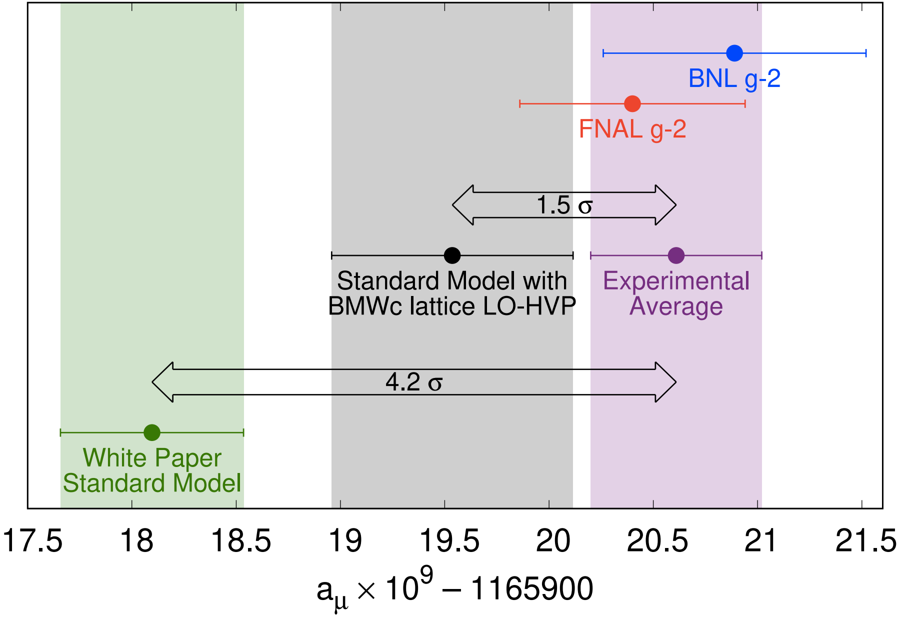
$$10^{10}\times a_\mu^{\text{LO-HVP}} = 707.5(2.3)_{\text{stat}}(5.0)_{\text{sys}}[5.5]_{\text{tot}}$$

# Comparison




- Consistent with other lattice results
- Total uncertainty is divided by $\sim 3 \div 4$ ...
- ... and comparable to R-ratio and experiment
- $2.1\sigma$ larger than R-ratio average value [WP '20]
- Consistent w/ experiment @ $1.5\sigma$ ("no new physics" scenario) !

# Fermilab plot, April 7 2021, BMWc version



# Take home messages: current situation

- Much ado about nothing? NO!
- Muon $g - 2$ experiment has significant potential to reveal presence of yet unknown particles or forces in nature ...
- ...but too early in process to know for sure
- Reference, data-driven SM prediction suggests that such a scenario is very likely
- ... but new lattice QCD calculation suggests that SM may may still be OK at current level of precision
- Reference approach is based on very basic principles and data from many, mostly independent experimental measurements of another process that would have to be collectively off ...
- ...or that process itself would have to be affected by new, unknown particles or forces
- Lattice QCD calculation is state-of-the-art and very thorough ...
- ... but has to be confirmed by equally precise lattice QCD calculations by other groups
- If confirmed, differences w/ data-driven approach must be understood and resolved

# Take home messages: what's next

- A third, independent approach, based on the measurement $\mu e \to \mu e$ is currently under study (MUonE experiment) that should help clarify situation

- Result on muon $g-2$ presented by FNAL is obtained from only 6% of data which they plan to accumulate

  $\Rightarrow$ error on average of BNL and FNAL measurement will be reduced by further factor of $2 \div 3$ by ca. 2025

  $\Rightarrow$ that alone can signficantly change current picture

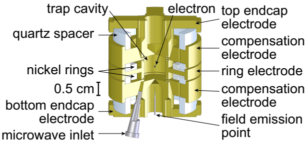
- To fully leverage precision of those measurements, critical to reduce theory error, but also proportion of systematic uncertainties in that error …

- … because the latter make the significance of any observed deviation between SM prediction and measurement difficult to determine

- Work in all of these directions is underway

# BACKUP

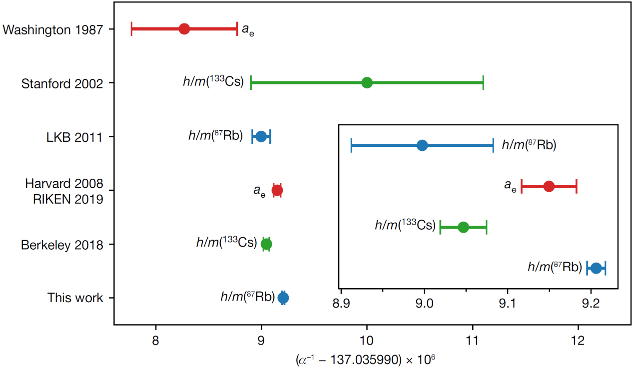
# Measurement of $a_e$ and $\alpha$




$$a_e^{exp} = 1\,159\,652\,180.73(28) \times 10^{-12} \text{ [0.24 ppb]}$$

(Hanneke et al '08)

With 5-loop QED $\Rightarrow \sigma_\alpha/\alpha = 2.4 \times 10^{-10}$ vs $0.81 \times 10^{-10}$ from Rb




(Morel et al '20)