SUPERTRANSLATIONS, ANGULAR MOMENTUM, AND COVARIANCE IN 4D ASYMPTOTICALLY FLAT SPACE

BASED ON

Reza Javadinezhad and MP
PRL 130 (2023) 1, 011401 arXiv: 2211.06538 [gr-qc]

earlier work with R. Javadinezhad and U. Kol

BUT MOSTLY ON:
Reza Javadinezhad and MP
PRL 132 (2024) 15, 151604 arXiv:2312.02458 [hep-th]
and
arXiv:24xx.xxxxx

- METRIC AND SYMMETRIES OF ASYMPTOTICALLY FLAT SPACE (BMS ALGEBRA)
- SUPERTRANSLATION AMBIGUITY OF ANGULAR MOMENTUM, BOOST AND THEIR FLUXES
- NEW DEFINITIONS OF ANGULAR MOMENTUM AND A SUPERTRANSLATION-INVARIANT FLUXES
- PHYSICAL INTERPRETATION
- LORENTZ-COVARIANCE AND ITS DISCONTENT
- A FULLY LORENTZ-COVARIANT DEFINITION OF LORENTZ CHARGES AND FLUXES
- LINK WITH OTHER WORKS AND A NOTE OF CAUTION
- TO DO LIST

Bondi charges: defined on spheres expanding at the speed of light

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To give a better expression to charges in general relativity and especially to angular momentum we need a bit of formalism..

The metric of an asymptotically flat spacetime near $\mathscr{I}^+$can be written in $(u = t - r, r, \theta^A)$ coordinates in the Bondi gauge as

$$ds^2 = -\,du^2 - 2dudr + r^2\Big(h_{AB} + \frac{C_{AB}}{r}\Big)d\theta^A d\theta^B + \frac{2m}{r}du^2 +$$
$$+\frac{1}{r}\Big(\frac{4}{3}(N_A + u\partial_A m) - \frac{1}{8}\partial_A(C_{BD}C^{BD})\Big)dud\theta^A + \ldots .$$

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This metric is invariant under:

1) Lorentz transformations, that act as conformal isometries of the celestial sphere $\theta^A \to \theta^A + V^A(\theta)$
2) Supertranslations. At leading order in r they are: $u \to u + f(\theta)$

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The supertranslation charges are

$$Q[f] = \frac{1}{4\pi G}\int d^2\theta\sqrt{h}f(\theta)m(\theta, u)$$

and $Q(\theta) = P^0 + P_m Y_{1\,m}(\theta)$ is the energy-momentum 4-vector

The Bondi Lorentz charges at retarded time u are

$$J_Y(u) = \frac{1}{8\pi G}\int d^2\theta\sqrt{h}\,Y^A N_A(\theta,u)$$

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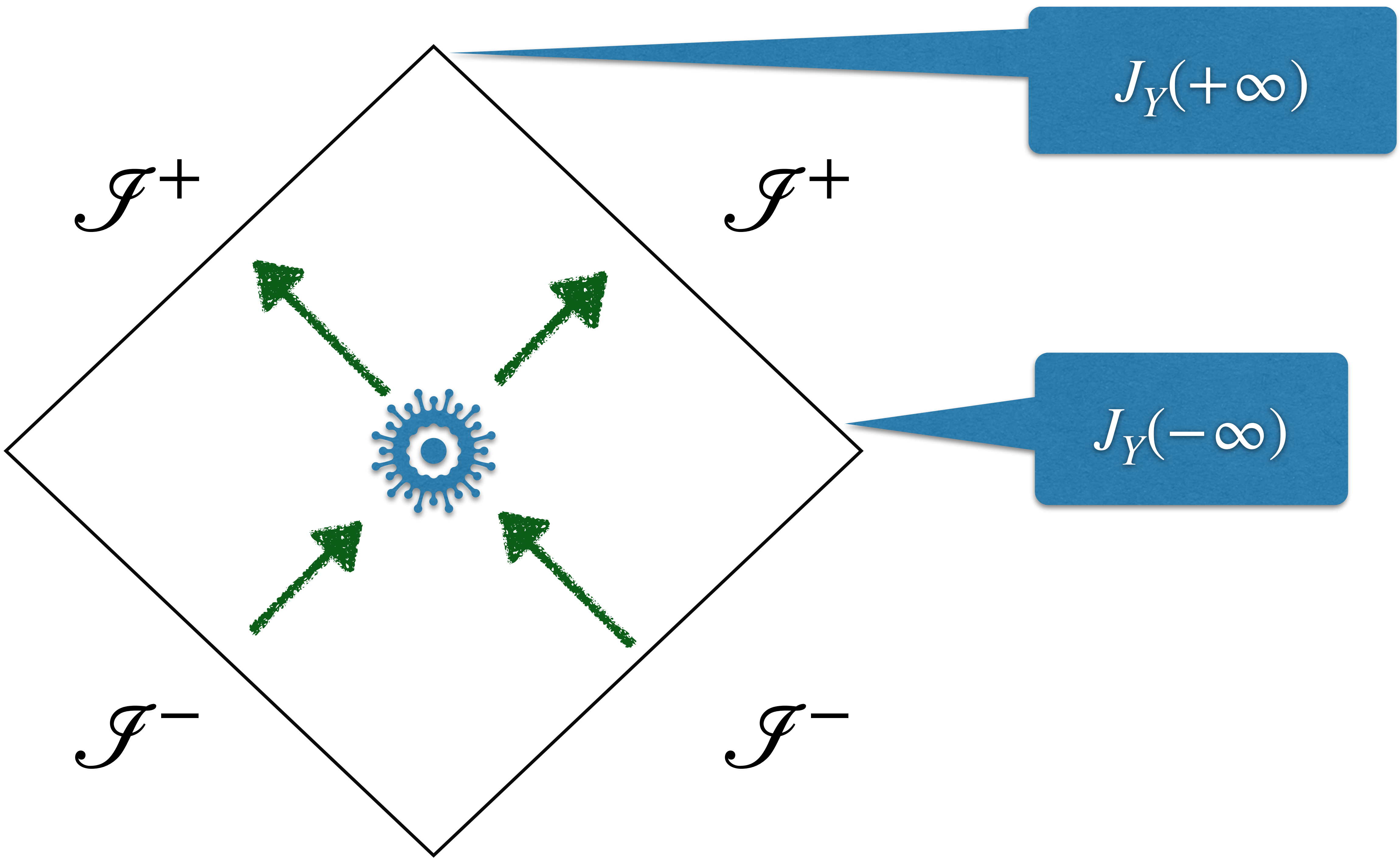
The total (conserved) angular momentum is $J_Y(-\infty)$ and the angular momentum flux is $\Delta J \equiv J(+\infty) - J(-\infty)$

The flux of a conserved charge (e.g. angular momentum) is defined as the difference of charges defined at null infinity

=

charge radiated away by massless particles



The flux of angular momentum can be changed by supertranslations, i.e. by adding an infinite-wavelength gravitational wave!

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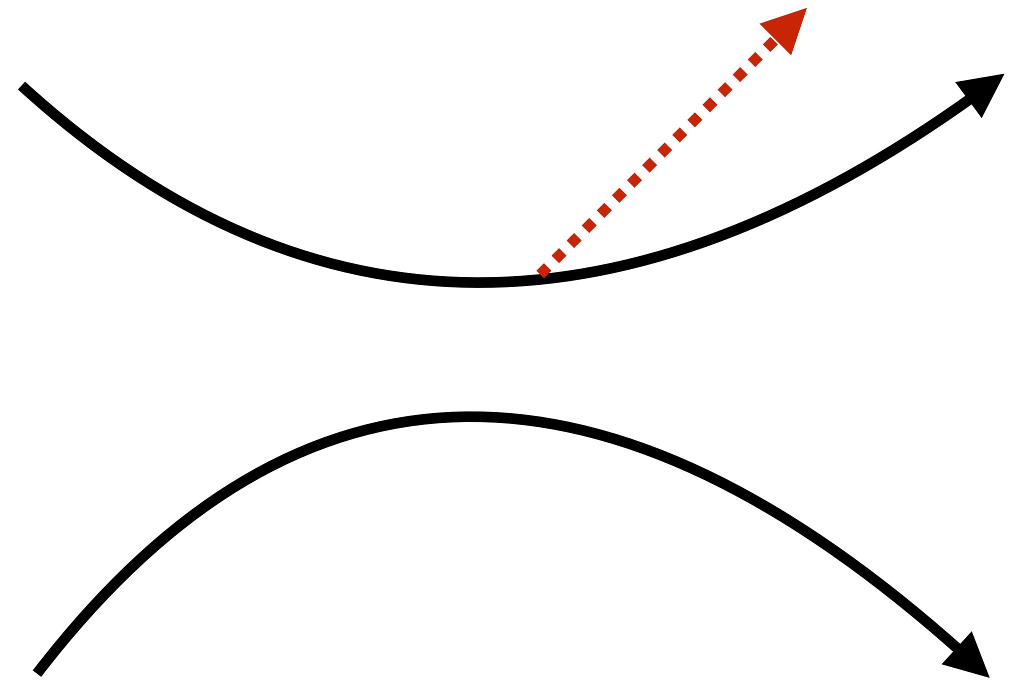
Not crazy ($p \times x$ can be large even when the momentum is small)

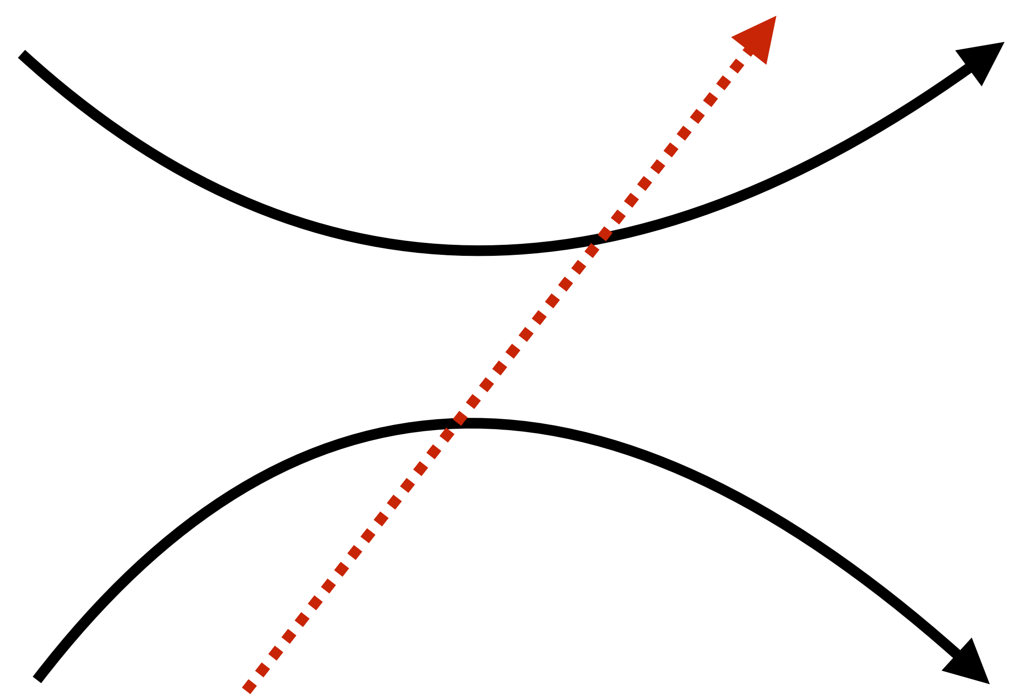
yet not useful: we want to be able to tell wheat from chaff separating the radiation due to a scattering process (e.g. black hole mergers) from unobservable backgrounds

But… it may be difficult to tell one from the other!

“wheat”



vs “chaff”



Several choices of angular momentum exist in the literature.
We will choose one proposed by Chen, Wang, Wang and Yau

$$J_Y^{CWWY}(\pm\infty) = J_Y^{\pm} - j_Y(m^{\pm}, C^{\pm})$$

$$j_Y(A,B) = \frac{1}{4\pi G}\int d^2\theta\sqrt{h}A(\delta_Y^{-1/2}B) = -\frac{1}{4\pi G}\int d^2\theta\sqrt{h}(\delta_Y^{3/2}A)B$$

$$D^A D^B C_{AB}(\pm\infty,\theta) \equiv D^2(D^2+2)C^{\pm}$$

$$\delta_Y^w X \equiv w D_C Y^C X + Y^C D_C X$$

invariant because $\delta_f C^{\pm} = f, \quad \delta_f m^{\pm} = 0$

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Other definitions dress the angular momentum by using only the boundary graviton C^- as additional degree of freedom. Some do not capture $O(G^2)$ contributions to the flux needed to explain radiative back-reaction effects in gravitational scattering.

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THESE ARE THE FRAMES WHERE THE BONDI CHARGES J_Y COINCIDE WITH CANONICAL ADM CHARGES

(Veneziano and Vilkovisky following Ashtekar et al.)

This may explain why they coincide with scattering amplitudes computations and other perturbative computations that implicitly or explicitly work in the "round-metric" canonical frame.

THE CWWY FLUX AND COVARIANCE

The flux ΔJ_Y^{CWWY} is covariant if m^+, m^-, C^+, C^- transform covariantly i.e.

$$\delta_Y^{3/2} m^\pm = \frac{3}{2} D_A Y^A m^\pm + Y^A D_A m^\pm$$

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They are the transformations of SL(2,R) conformal fields of weight

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These transformations follow from the asymptotic form of Lorentz transformations on the metric

Generated by J_Y^- not by $J_Y^- - j_Y(m^-, C^-)$

THE FIRST PROBLEM WITH LORENTZ BOOSTS

Expand the definition of $C(\theta, u)$ in spherical harmonics

$$l(1-l^2)(l+2)C_{lm}(u) = 4m_{lm}(u) - 4m_{lm}^{-} + \int_{-\infty}^{u} dv T_{uu}^{lm}(v) + C_{lm}^{-}, \quad l > 1$$

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Lorentz transformations mix l>1 with l=0,1 so the condition

$C^{\pm}_{lm} = 0 \quad l = 1,0$

is incompatible with Lorentz invariance

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supertranslation invariance + covariance = translation invariance

invariant flux is ALSO TRANSLATION INVARIANT

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JACOBI:

$$[[F^{\mu\nu}, J^{\rho\sigma}], S^a] + [[S^a, F^{\mu\nu}], J^{\rho\sigma}] + [[J^{\rho\sigma}, S^a], F^{\mu\nu}] = 0$$

COVARIANCE

$$[F^{\mu\nu}, J^{\rho\sigma}] = -\eta^{\mu\rho}F^{\nu\sigma} - \eta^{\nu\sigma}F^{\mu\rho} + \eta^{\nu\rho}F^{\mu\sigma} + \eta^{\mu\sigma}F^{\nu\rho}$$

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SUPERTRANSLATION INVARIANCE

$$[S^a, F^{\mu\nu}] = 0$$

On the other hand:

$$[J^{\rho\sigma} S^a] = supertranslations + TRANSLATIONS$$

so Jacobi implies $[F^{\mu\nu}, P^\rho] = 0$

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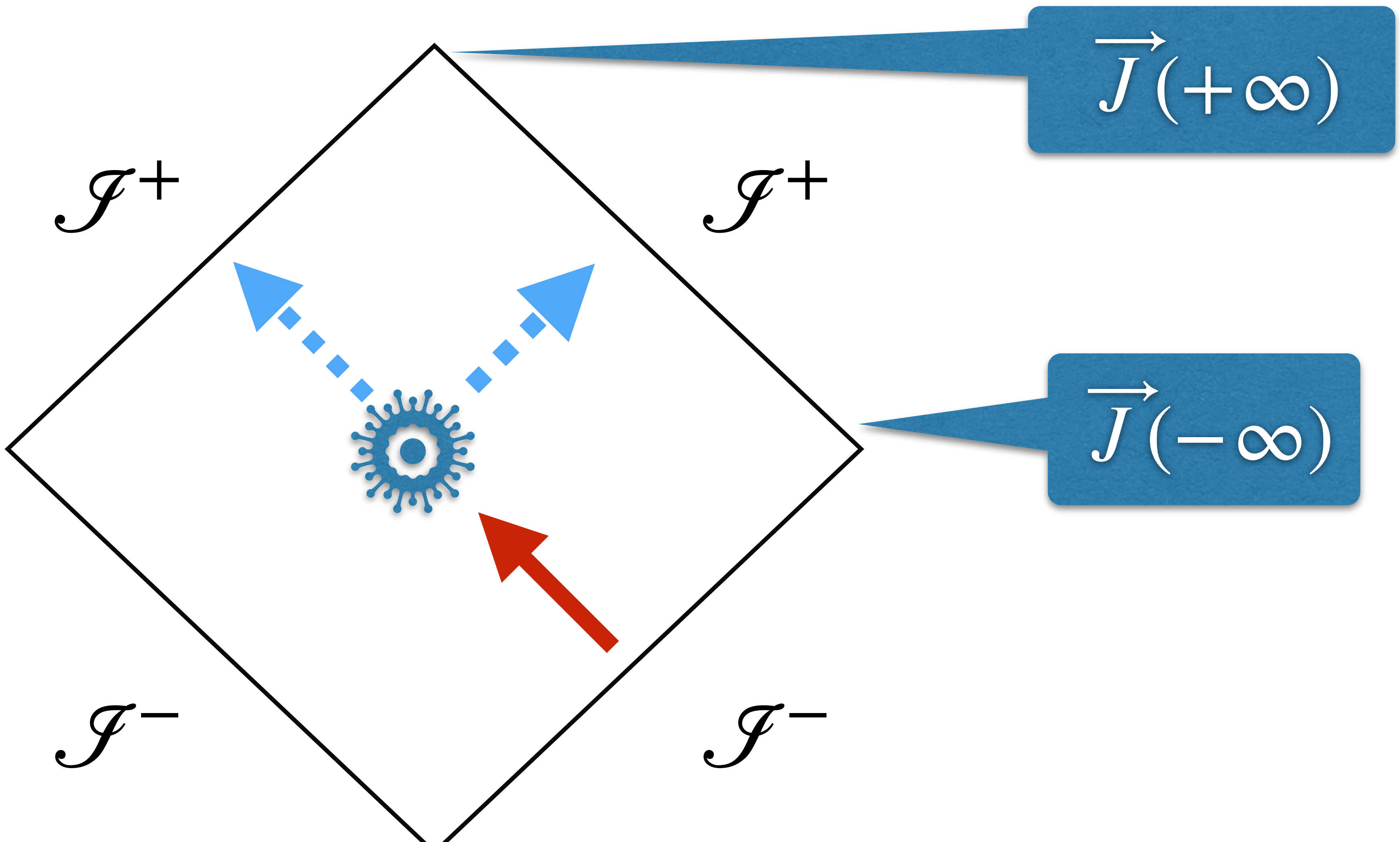

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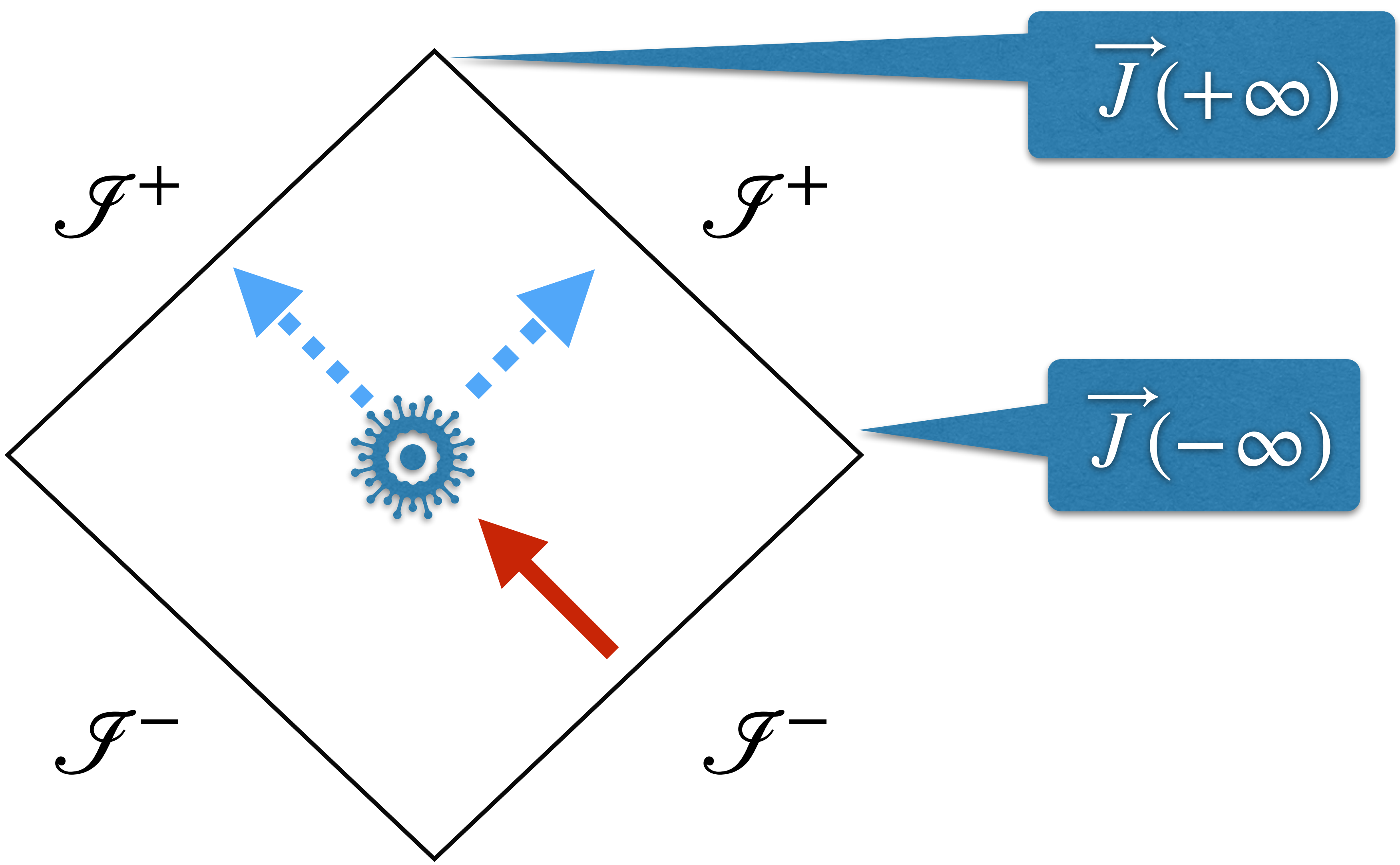
Translate frame by $\overrightarrow{a}$: $\overrightarrow{J}(-\infty) = \overrightarrow{a} \times \overrightarrow{P}, \quad \overrightarrow{J}(+\infty) = 0$:

Angular momentum flux does not vanish!



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We fix the origin of the the initial center of mass rest frame by requiring

$$J^{-}_{\bar{Y}} - j_{\bar{Y}}[m^{-}, C^{-}] = 0$$

for all boosts, which can be written as

$$\bar{Y}^A = \text{boost} = D^A\psi, \quad D^2\psi = -2\psi$$

$$J_{\bar{Y}} - j_{\bar{Y}}[m^-, C^-] = 0$$

defines the l=1 components of $C^- = C(-\infty)$. After some straightforward calculations:

$$\frac{3m_0^-}{4\pi G} C^-_{1m} = J_{\bar{Y}^A} - j_{\bar{Y}^A}[m^-, C^-|_{l>1}], \quad \bar{Y}^A = D^A \mathrm{Y}_{1-m}$$

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In tensor notation we have just set $J_{i0}^{-} = 0$
in a generic frame we impose the manifestly covariant condition

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In the initial rest frame $\overrightarrow{P}^{-} = 0$ so we recover $J_{i0}^{-} = 0$

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The condition $J^-_{\bar{Y}} - j_{\bar{Y}}[m^-, C^-] = 0$ is implicitly used in most papers on gravitational radiation because the frame of choice is a CMRF where the origin of coordinates coincides with the COM

To completely define the a covariant and (super)translation invariant flux we have to define also $C^+|_{l\leq 1}$

To make contact with most of literature we choose

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When the final momentum $m^+_{l=1}$ is nonzero

$$j_{\bar{Y}}[m^+, C^+_{00}] = \frac{C^+_{00}}{4\pi G}\int d^2\Theta\sqrt{h}\psi m^+_{l=1}, \quad \bar{Y}_A = D_A\psi$$

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In tensor notations: write $J^+_{i0} = J^T_{i0} + cP^+_i$ and we can fix C^+_{00} by setting $c = 0$

After $C^{\pm}|_{l\leq 1}$ is fixed, the covariant flux is

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It was recently argued that J_Y^- in the $h_{AB} + O(1/r^2)$ frame is the ADM angular momentum (using results by Ashtekar et al.)
So $\Delta J_Y^{CWWY} = J_Y^+ - J_Y^-$ is the change in the total ADM angular momentum. Riva Vernizzi and Wong verified in some cases that it coincides with the formulas given by Bini, Damour and Manohar et al.

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This identification is valid only for the ADM Lorentz generators defined with special asymptotic boundary conditions that forbid supertranslations (there is no single angular momentum otherwise)

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generate the Poincaré algebra. How is this consistent with all that we just found?

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Because $\mathfrak{J}_Y \equiv J^-_Y - j_Y(m^-, C^-|_{l>1})$ does not generate the Lorentz transformations induced on the asymptotic metric by the coordinate change $u = K(\bar{\mathbf{x}})\bar{u}, \quad \mathbf{x} = g(\bar{\mathbf{x}}), \quad r = K(\bar{\mathbf{x}})$

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This is obvious because supertranslation invariance implies

$$\hat{\delta}_Y C^-|_{l>1} \equiv [\mathfrak{J}_Y, C^-|_{l>1}] = 0, \quad \hat{\delta}_Y m|_{l>1} \equiv [\mathfrak{J}_Y, m^-|_{l>1}] = 0$$

SUMMARY

- The formula may be interpreted as the difference of ADM angular momentum before and after a burst of radiation through $\mathscr{I}$

- Why the formula works at all? What is special about the "canonical frame" or in fact the standard ADM angular momentum?

- Can we make sense of the other formulas -e.g. by identifying ΔJ_Y^{BMS} with the amount of radiated angular momentum that can be detected or extracted by appropriate asymptotic observers?

- Link with ADM quantities computed at i^0 when asymptotic falloff of the metric is relaxed to allow for the full BMS algebra?

- IR safe fluxes? So far we removed the effect of radiation with $\lambda = \infty$. How to remove the effect of finite-wavelength radiation?