Does Quantum Gravity constrain Large Field Inflation?

Arthur Hebecker (Heidelberg)

(including work with P. Henkenjohann, S. Kraus, D. Lüst, P. Mangat, F. Rompineve, S. Theisen, A. Westphal, L. Witkowski, ...)

Outline

- Large-field inflation: Generalities
- Large-field inflation: Issues in quantum gravity / strings

  In particular: Weak Gravity Conjecture;
  Gravitational instantons

Slow-roll inflation and perturbations

Starobinsky '80; Guth '81
Mukhanov/Chibisov '81; Linde '82

- The simplest relevant action is

$$S = \int d^4x\sqrt{g}\left[\frac{1}{2}R[g_{\mu\nu}] + \frac{1}{2}(\partial\varphi)^2 - V(\varphi)\right] .$$

(We use $M_P \equiv 1$ here and below.)

- (Slow-roll) inflation requires

$$\epsilon = \frac{1}{2}\left(\frac{V'}{V}\right)^2 \ll 1 \quad \text{and} \quad |\eta| = \left|\frac{V''}{V}\right| \ll 1 \,.$$

- To gain some intuition, assume that

  $V \quad \sim \quad \varphi^n$ or $\ln(\varphi)$ (or some combination thereof).

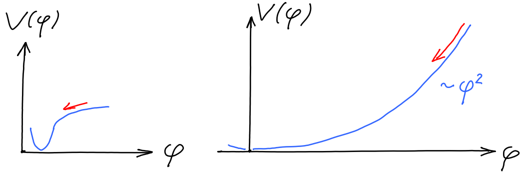
- This implies

$$\epsilon \sim \eta \sim 1/\varphi^2 \,,$$

  such that inflation is generic if $\varphi \gg 1$.

- As a result, one can roughly distinguish

Small- and Large-Field Models



- Small field: $V(\varphi)$ has some tuned very flat region.

- Large field: 'Generic' potentials.

  But: $\Delta\varphi \gg 1$ may lead to problems with quantum gravity.

Recently, the focus has been on large-field models
for two reasons....

1) Observations

- Recall the relation of tensor-to-scalar ratio and field-range:

$$r \equiv \frac{\Delta_T^2}{\Delta_R^2} = 16\epsilon \quad \Leftrightarrow \quad \Delta\varphi \simeq 20\sqrt{r}$$

Lyth '96

- The Planck/BICEP bounds are now somewhere near $r \simeq 0.1$.

- This will improve and we will see the discovery or demise of large-field models.

- If we manage (see below) to show that string theory forbids $\Delta\varphi > 1$, we can hope to rule out string theory!

...reasons for interest in large-field models...

2) Fundamental

- On the one hand, large-field models are more 'robust'
- On the other hand, there are generic arguments against large-field models in consistent quantum gravity theories

see e.g. Arkani-Hamed/Motl/Nicolis/Vafa '06 .... Conlon '12
....
Kaloper/Kleban/Lawrence/Sloth '15

- This goes hand in hand with persistent problems in constructing large-field models in string theory.

- However, triggered by BICEP and bulding on earlier proposals

  Kim, Nilles, Peloso '07
  McAllister, Silverstein, Westphal '08

  new promising classes of stringy large-field models have been constructed (e.g. F-term axion monodromy)

  Marchesano, Shiu, Uranga '14
  Blumenhagen, Plauschinn '14
  AH, Kraus, Witkowski '14

- At the same time, there are ongoing efforts to sharpen the 'no-go arguments' as well as to refute them

  Rudelius '14...'15
  Ibanez, Montero, Uranga, Valenzuela '15
  Brown, Cottrell, Shiu, Soler '15
  AH, Mangat, Rompineve, Witkowski '15
  ...

- I will try to explain some aspects of this debate....

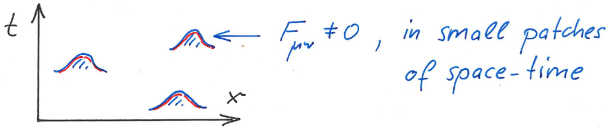
Natural (axionic) inflation in string theory

Freese/Frieman/Olinto '90

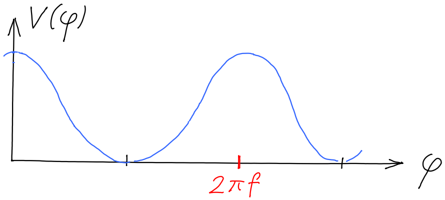
- In 4d effective theories of string compactifications, axion-like fields are abundant:

$$\mathcal{L} \supset -\frac{1}{2}(\partial\varphi)^2 - \frac{1}{32\pi^2}\left(\frac{\varphi}{f}\right)\mathrm{tr}(F\tilde{F})\,.$$

- The shift symmetry is generically broken by instantons:



$$\Rightarrow \quad V_{eff} \sim \cos(\varphi/f)\,, \qquad \varphi \equiv \varphi + 2\pi f\,.$$



- **Problem:** $f \ll 1$ in perturbatively controlled regimes.

- **Illustration:** 5d → 4d compactification with $\varphi \sim \int_{S^1} A_5$

  One finds $f \sim 1/R$, such that perturbative control restricts one to sub-planckian f.

- Based on many stringy examples,
  this appears to be a generic result (cf. Banks et al.)

- Three ideas about how to enlarge the axionic field range without losing calculational control:

  (a) KNP   Kim/Nilles/Peloso '04

  (b) N-flation   Dimopoulos/Kachru/McGreevy/Wacker '05

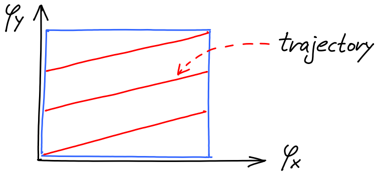
  (c) Axion-Monodromy   McAllister/Silverstein/Westphal '08

- The No-Go arguments alluded to earlier challenge these possibilities.

(a) KNP / Winding inflation

Kim/Nilles/Peloso '04; Berg/Pajer/Sjors '09; Ben-Dayan/Pedro/Westphal '14

- Consider a 'winding' trajectory on a 2d periodic field space:



- Clearly, such a trajectory can be much longer than the (naive) field range

- But: It is hard to realize the required potential in concrete string models

- Thus, even getting only an effective trans-planckian axion appears to be difficult. Is there a fundamental reason?

No-go argument I: Weak gravity conjecture

Arkani-Hamed/Motl/Nicolis/Vafa '06

- Some recent papers developing this in (more or less close) relation to large-field inflation:

  Cheung/Remmen; de la Fuente/Saraswat/Sundrum . . . '14
  Rudelius; Ibanez/Montero/Uranga/Valenzuela; Brown/Cottrell/Shiu/Soler;
  Bachlechner/Long/McAllister; AH/Mangat/Rompineve/Witkowski;
  Junghans; Heidenreich/Reece/Rudelius; Kooner/Parameswaran/Zavala;
  Harlow; AH/Rompineve/Westphal; . . . '15
  Ooguri/Vafa, Conlon/Krippendorf . . . '16
  Dolan/Draper/Kozaczuk/Patel; AH/Henkenjohann/Witkowski/Soler . . . '17

Weak gravity conjecture

- Roughly speaking: 'Gravity is always the weakest force.'
- More concretely (mild form):

  For any U(1) gauge theory there exists a charged particle with

  $$\boxed{q/m > 1\,.}$$

- Strong form:

  The above relation holds for the lightest charged particle.

Weak gravity conjecture (continued)

- The historical supporting argument:

  In the absence of sufficiently light, charged particles, extremal BHs are stable. Such remnants are believed to cause inconsistencies.

  see e.g. Susskind '95

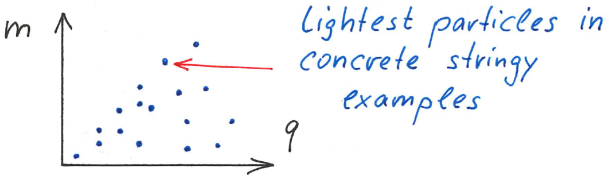
  The boundary of stability of extremal black holes is precisely $q/m = 1$ for the decay products.

Weak gravity conjecture (continued)

- Another (possibly stronger?) supporting argument:

  Quantum gravity forbids global symmetries. We should not be able to take the limit of small gauge couplings.

  The WGC quantifies this on the basis of stringy examples.



Generalizations of the weak gravity conjecture

- The basic lagrangian underlying the above is

$$S \sim \int (F_2)^2 + m \int_{1-dim.} d\ell + q \int_{1-dim.} A_1 \,.$$

- This generalizes to charged strings, domain walls etc. Crucially, the degree of the corresponding form-field (gauge-field) changes:

$$S \sim \int (F_{p+1})^2 + m \int_{p-dim.} dV + q \int_{p-dim.} A_p$$

with

$$F_{p+1} = dA_p \,.$$

Generalizations to instantons

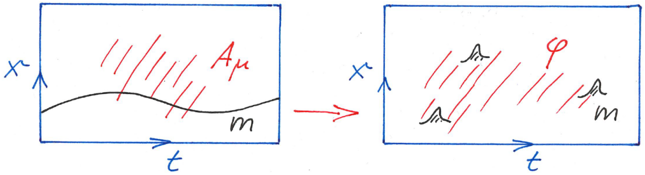
- One can also lower the dimension of the charged object, making it a point a in space-time:

$$S \sim \int (d\varphi)^2 \; + m + q\,\varphi(x_{inst.})\,.$$

This should be compared with

$$\text{cf.} \quad S \sim \int (d\varphi)^2 + \int \mathrm{tr}(F^2) + \int \left(\frac{\varphi}{f}\right) \mathrm{tr}(F\tilde{F})\,,$$

where $\int \mathrm{tr}(F^2) \sim S_{inst.} \sim m\,.$



WGC for instantons and inflation

- The consequences for inflation are easy to derive.
- First, recall that the instantons induce a potential

$$V(\varphi) \;\sim\; e^{-m}\cos(\varphi/f)\,.$$

- Since, for instantons, $q \equiv 1/f$, we have

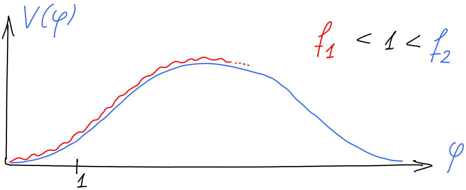
$$q/m > 1 \qquad \Rightarrow \qquad \boxed{\;m\,f < 1\;}\,.$$

- Theoretical control (dilute instanton gas) requires $m > 1$.
- This implies $f < 1$ and hence
large-field 'natural' inflation is in trouble.

A Loophole

Rudelius '15

- Suppose that only the mild form of the WGC holds.
- In this case, we can have one 'sub-planckian' instanton maintaining the WGC, together with a lighter 'super-planckian' instanton realizing inflation:



For other arguments and loopholes see e.g.
de la Fuente, Saraswat, Sundrum '14
Bachlechner, Long, McAllister '15.

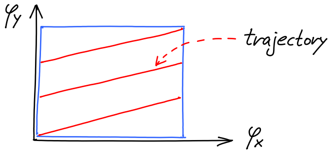
String theory appears to realize this loophole...

AH/Mangat/Rompineve/Witkowski '15

- The fields φ_x and φ_y are two 'string theory axions', both with $f < 1$ (obeying the WGC).
- They are also moduli. Hence, fluxes (e.g. $\langle F_3 \rangle \neq 0$ on the compact space) can be used to stabilize them.
- A judicious flux choice allows for stabilizing just one linear combination, forcing the remaining light field on the winding trajectory:

$$V \supset (\varphi_x - N\varphi_y)^2 + e^{-M}\cos(\varphi_x/f) + e^{-m}\cos(\varphi_y/F)$$

with $N \gg 1$.



Concrete realization at (partially) large complex stucture

- Let $z_1, \cdots, z_n, u, v$ be complex structure moduli of a type-IIB orientifold, let $\mathrm{Im}(u) \gg \mathrm{Im}(v) \gg 1$.

$$K = -\log\left(\mathcal{A}(z, \bar{z}, u - \bar{u}, v - \bar{v}) + \mathcal{B}(z, \bar{z}, v - \bar{v})e^{2\pi i v} + \mathrm{c.c.}\right)$$

$$W = w(z) + f(z)(u - Nv) + g(z)e^{2\pi i v}$$

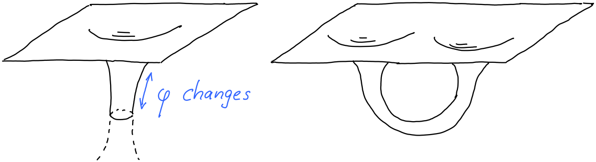
- Without exponential terms, it is clear that W leaves one of the originally shift-symmetric directions $\mathrm{Re}(u)$ and $\mathrm{Re}(v)$ flat
- If $N \gg 1$, this direction is closely aligned with $\mathrm{Re}(u)$
- The exponential terms induce a long-range cosine potential for this light field φ:

$$e^{2\pi i v} \quad \to \quad \cos(2\pi\varphi/N)$$

No-go argument II: (Gravitational) instantons

- In Euclidean Einstein gravity, supplemented with an axionic scalar φ , instantonic solutions exist:

Giddings/Strominger '88
. . .



- The 'throat' is supported by the kinetic energy of $\varphi = \varphi(r)$, with r the radial coordinate of the throat/instanton.

- The relevance for inflation arises through the induced instanton-potential for the originally shift-symmetric field φ.

Montero/Uranga/Valenzuela '15

Gravitational instantons (continued)

- The underlying lagrangian is simply

$$\mathcal{L} \sim \mathcal{R} + f^2|d\varphi|^2\,, \quad \text{now with} \quad \varphi \equiv \varphi + 2\pi\,.$$

- This can be dualized $(dB_2 \equiv f^2 * d\varphi)$ to give

$$\mathcal{L} \sim \mathcal{R} + \frac{1}{f^2}|dB_2|^2\,.$$

- The 'throat' exists due the compensation of these two terms. Reinstating M_P, allowing n units of flux (of $H_3 = dB_2$) on the transverse S^3, and calling the typical radius R, we have

$$M_P^2\,R^{-2} \sim \frac{n^2}{f^2}R^{-6} \quad \Rightarrow \quad M_P\,R^2 \sim \frac{n}{f}\,.$$

Gravitational instantons (continued)

- Returning to units with $M_P = 1$, their instanton action is

$$S \sim n/f \qquad \text{(with } n \text{ the instanton number).}$$

- Their maximal curvature scale is $\sqrt{f/n}$, which should not exceed the UV cutoff:

$$f/n < \Lambda^2$$

- This fixes the lowest n that we can trust and hence the minimal size of the instanton correction to the potential $V(\varphi)$:

$$\boxed{\delta V \;\sim\; e^{-S} \;\sim\; e^{-n/f} \;\sim\; e^{-1/\Lambda^2}}$$

Gravitational instantons (continued)

- For gravitational instantons not to prevent inflation, the relative correction must remain small:

$$\frac{\delta V}{V} \sim \frac{e^{-1/\Lambda^2}}{H^2} \ll 1$$

- For a Planck-scale cutoff, $\Lambda \sim 1$, this is never possible
- However, the UV cutoff can in principle be as low as H
- Then, if also $H \ll 1$, everything might be fine....

$$\frac{\delta V}{V} \sim \frac{e^{-1/H^2}}{H^2}$$

AH, Mangat, Rompineve, Witkowski '15

Can one at least obtain reasonably model-independent bounds in high-cutoff models ?

AH/Mangat/Theisen/Witkowski '16

- Look at the case where we expect the strongest bound: A string model with $g_s = 1$ on T^6 at self-dual radius.

- Need to decide when to trust a wormhole / extremal instanton

  (i.e., what is the smallest allowed S^3-radius r_c)

  The following two choices appear 'natural':

$$2\pi^2 r_c^3 = \mathcal{V}_{self-dual}^{1/2} \qquad \text{and} \qquad 2\pi r_c = \mathcal{V}_{self-dual}^{1/6}$$

- One finds:

First case: $r_c M_p \simeq 1.3$ Second case: $r_c M_P \simeq 0.56$

- The crucial numerical effect comes from the π's in the instanton action:

$$S_{inst.} = 3\pi^3 \, (r_c M_p)^2$$

- The correction to the potential is suppressed as

First case: $e^{-S} \simeq 10^{-68}$ First case: $e^{-S} \lesssim 10^{-13}$.

- Thus, one needs to look into the quantum-gravity regime of gravitational instantons.

- For recent work on bounds from gravitational instantons in the small-f regime, see

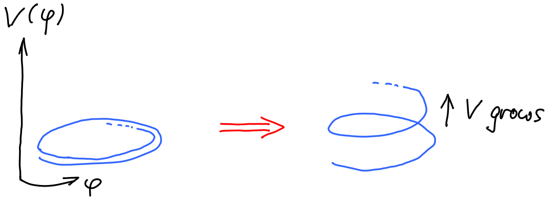
Alonso/Urbano '17

(c) Monodromy inflation

Silverstein/Westphal/McAllister '08

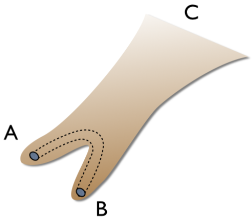
Very general but simple-minded definition:

- Start with a single, shift-symmetric, periodic inflaton φ
- Break the periodicity **weakly** by the scalar potential



The 'classical' model ...

$$S_{\mathrm{NS5}} \sim \int \sqrt{-\det(g_{\mu\nu} + F_{\mu\nu} + C_{\mu\nu})}$$



Bifid throat with shared 2-cycle
(figure from Retolaza et al. '15)

... has issues with the explicit geometry and quantitative control.

For recent progress see e.g. McAllister/Silverstein/Westphal/Wrase '14
...
Retolaza/Uranga/Westphal '15

F-term axion monodromy

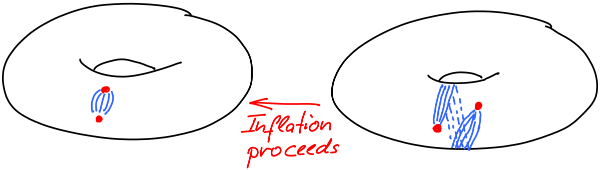
- More recently, classes of monodromy models with 4d supergravity description and stabilized compact space have emerged.

  Marchesano/Shiu/Uranga '14
  Blumenhagen/Plauschinn '14
  AH/Kraus/Witkowski '14

- One option is that inflation corresponds to brane-motion

  Dvali/Tye '98....Dasgupta et al. '02....Lüst et al. '11

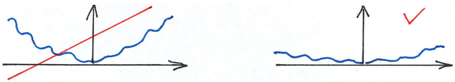
- The monodromy arises from a flux sourced by the brane



Challenges in axion monodromy

- It remains controversial whether one can (e.g by tuning) make the monodromy as small as necessary for moduli stabilization

cf. recent work by Blumenhagen, Valenzuela, Palti, Marchesano,... (and by our group)



- The WGC applies only indirectly (in its domain-wall version), but the constraints are not strong enough for inflation

Brown/Cottrell/Shiu/Soler, Ibanez/Montero/Uranga/Valenzuela, AH/Rompineve/Westphal '15

- It has been attempted to use the Swampland Conjecture to argue against axion monodromy inflation

Baume/Palti, Klaewer/Palti '15 ... '16

The Landscape/Swampland paradigm and the Size of Moduli Spaces

- The idea is to ask which subset of effective theories can be UV-completed in Strings (or Quantum Gravity in general)

Vafa '05, Ooguri/Vafa '06

- The WGC may be viewed as one particular instance of this

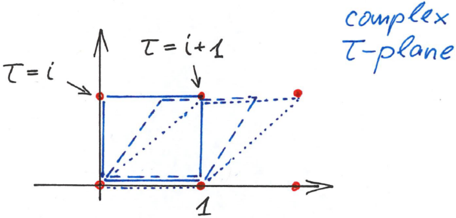
- One suggested criterium is that, "as we move a distance L in moduli space, the cutoff must come down as $\exp(-L)$."

In the present context, see especially Palti et al.

- I will try to explain this using the toy model of a torus moduli space and make some new observations

AH/Henkenjohann/Witkowski 1708.06761

- Recall that a torus can be viewed as a lattice in $\mathbb{C}$ and its shape is parametrized by $\tau \in \mathbb{C}$.



- There are many identifications (e.g. $\tau = i$ and $\tau = i + 1$ correspond to the same torus)

- Moreover, the metric in the τ-plane (both in math in the 4d EFT with a complex modulus field τ) reads

$$ds^2 = \frac{d\tau\, d\overline{\tau}}{4\,(\mathrm{Im}\tau)^2} \qquad \text{`Hyperbolic plane'}$$

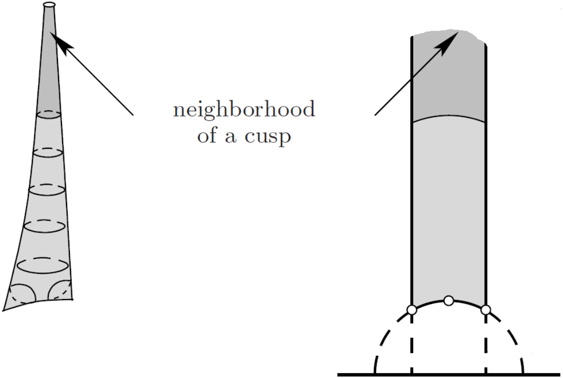
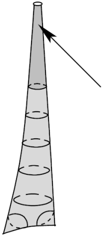


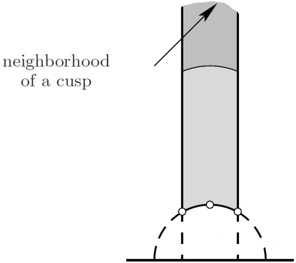
Fig. from A. Zorich, 'Flat surfaces'

- The fundamental domain is an infinitely long, vertical strip with $i \times \infty$ corresponding to a very thin torus.

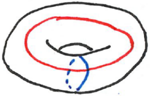
- The modulus space has an infinite extension, but the cutoff comes down exponentially fast if one goes there (due to light winding strings).

- The 'axionic' horizontal direction is at most $\mathcal{O}(1)$ in size ($f \lesssim M_p$)



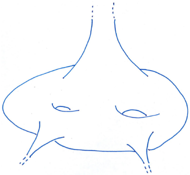


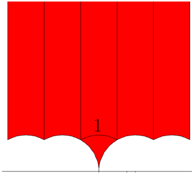
- Now, if the torus carries flux (think of rubber bands marking the cycles), the picture changes.



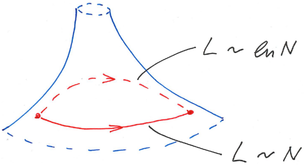
- Some of the identifications are lost and the fundamental domain increases ($\rightarrow$ fund. domain of congruence subgroups of $SL(2,\mathbb{Z})$).

- The cusp or ‘throat’ becomes much wider (super-planckian f),





...but the geodesic distances remain short ($\sim \ln(1/\text{cutoff})$)



- We formulate this in a ‘moduli space size conjecture’ which tries to unify axionic WGC and Swampland Conjecture
- The implications for inflation require further work....

Summary/Conclusions

- Quantum gravity (Instantons / Weak gravity conjecture) may be constraining large-field inflation at a very fundamental level

- Concrete problems with large-field inflation in string theory reflect these fundamental ‘issues’

- Progress is being made both in understanding the generic constraints as well as in constructing counterexamples (i.e. models)

In primordial gravity waves / large-field inflation, fundamental quantum gravity problems may meet reality!