$N = 8$ Supergravity, and beyond

Or: what symmetry can teach us about quantum gravity and the unification of physics.

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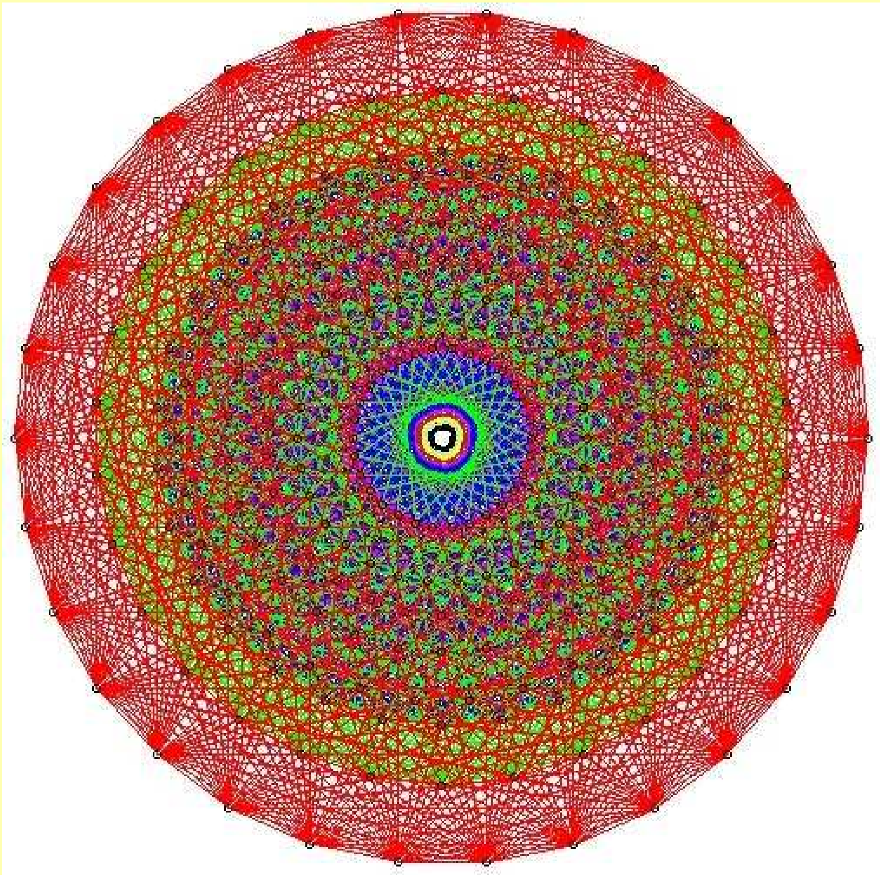
[with thanks to: Bernard de Wit, Thibault Damour, Marc Henneaux and Axel Kleinschmidt]

Main theme: Symmetry



Main theme: Symmetry

... arguably the most successful principle of physics!

- **Space-time symmetries**
  - **Rotations and translations in Newtonian physics**
  - **Special relativity and the Poincaré group**
  - **General relativity and general covariance**
- **Internal symmetries**
  - **Isospin $SU(2)$ symmetry:** $m_{\text{neutron}} = 1.00135 m_{\text{proton}}$
  - **Flavor symmetry $SU(3)$ and the strong interactions**
  - **Standard model and $SU(3)_c \times SU(2)_w \times U(1)_Y$**
- **The two fundamental theories of modern physics, General Relativity and the Standard Model of Particle Physics, are based on and largely determined by symmetry principles!**

Where we stand

Known laws of physics successfully describe observed phenomena over a huge range of distances from $10^{-18}\,m$ all the way to the visible horizon of our universe.

- **General Relativity: gravity from space-time curvature (general covariance and equivalence principle).**
- **Standard Model of Particle Physics: combines quantum mechanics and special relativity to describe**

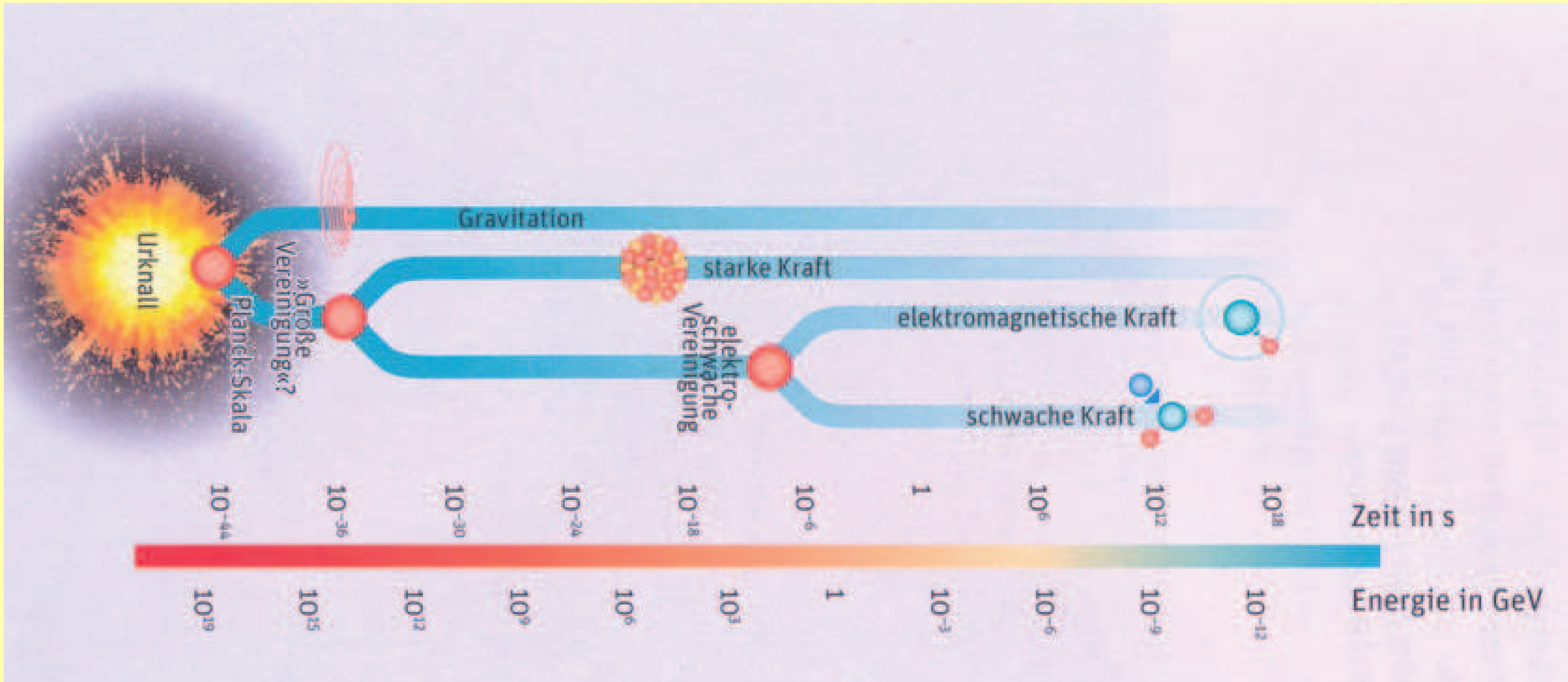
  Matter = three generations of 16 spin-$\frac{1}{2}$ fermions

  Forces = electromagnetic, weak and strong

  via $G = SU(3)_c \times SU(2)_w \times U(1)_Y$ gauge symmetry.

... but both theories are incomplete and possibly even inconsistent: singularities and infinities!

Symmetry and Unification



Like a ferromagnet: symmetry is broken more and more with decreasing temperature as universe expands.

Disclaimer

- There is at this time no evidence from experiment or observation for any particular approach to quantum gravity and a fully unified theory, in spite of numerous *ansätze* and proposals.
- No particular proposal at this time can claim to offer a compelling explanation for the observed low energy physics (Standard Model) or recent cosmological observations (inflation, dark energy).

In fact...

- The 'tension' between *fundamental* theory and observation has become even more pronounced with recent experimental data (LHC, PLANCK,...).

So where do we go from here?

Idea: symmetry enhancement as a guiding principle!

- **Grand Unification:**

$$SU(3)_c \times SU(2)_w \times U(1)_Y \subset SU(5) \subset SO(10) \subset E_6 \subset \ldots ?$$

$\rightarrow$ **quark lepton unification, proton decay, ...**

- **'Fusion' of space-time and internal symmetries?**
- **Duality symmetries, e.g. electromagnetic duality**

$$\mathbf{E} + i\mathbf{B} \to e^{i\alpha}\big(\mathbf{E} + i\mathbf{B}\big) \quad , \qquad q + ig \to e^{i\alpha}(q + ig)$$

- **Quantum symmetry and quantum space-time?**

QUESTION: is it possible to pin down the 'right' theory simply by imposing a symmetry principle?

Supersymmetry

Or: the search for exceptionality in physical theories!

- **A new kind of symmetry relating Bosons $\leftrightarrow$ Fermions.**

  Or: Forces (vector bosons) $\leftrightarrow$ Matter (quarks & leptons)?

- **Probably one of the most important developments in mathematical physics over the last 40 years.**

- **Supersymmetry is generally believed to be essential for constructing a perturbatively consistent (finite) theory of quantum gravity.**

- **Discovery of supersymmetry at LHC would open many new avenues and revolutionize particle physics.**

  (But indications so far from ATLAS and CMS are not encouraging for *low energy supersymmetry* ...)

Supersymmetry in a Nutshell

Simple example: ***supersymmetric quantum mechanics*** **with** ***supercharge*** $Q, Q^\dagger$ **and** ***Hamiltonian*** H**:**

$$H = \frac{1}{2}\{Q\,,\,Q^\dagger\} \;\Rightarrow [Q, H] = [Q^\dagger, H] = 0$$

Hence for any eigenstate $|E\rangle$ **(with** $E > 0$**) we have**

$$H|E\rangle = E|E\rangle \quad \Rightarrow \quad H(Q|E\rangle) = E(Q|E\rangle)$$

$\Rightarrow$ **degeneracy of bosonic/fermionic energy levels.**

[$\rightarrow$ **equal masses for bosons and fermions in supersymmetric quantum field theory.]**

New mathematical concept: Superalgebra contains commutators and anticommutators ($\equiv$ **graded Lie algebra with** $\mathbb{Z}_2$ **grading** $(-1)^F$ **and fermion number** F**).**

Supersymmetric (Quantum) Field Theory

For (semi-)realistic field theories need to 'marry' supersymmetry with other symmetries (Poincaré and internal symmetries): $P_\mu, M_{\mu\nu}, \ldots$.

Supercharges Q^i_α **and** $\bar{Q}_{\dot\alpha j}$ **are now** ***space-time spinors*****.** **The key relation of the relativistic superalgebra is**

$$\{Q^i_\alpha\,,\,\bar{Q}_{\dot\beta j}\} = 2\delta^i_j \sigma^\mu_{\alpha\dot\beta} P_\mu$$

N**-extended supersymmetry (for** $i,j = 1,\ldots,N$**) merges** ***spacetime and internal symmetries*** **when** $N \geq 2$**.**

In QFT, supersymmetry entails (partial) cancellations of UV infinities in Feynman diagrams, e.g. only ***logarithmic*** **instead of quadratic divergences** $\rightarrow$ **solve (or alleviate) hierarchy problem with MSSM or NMSSM?**

Representations (Supermultiplets)

For massless multiplets use Wigner method by starting from states of highest helicity $|h\rangle$:

$$|h\rangle\,,\;|h-\tfrac{1}{2};i\rangle \equiv Q^i|h\rangle\,,\;|h-1;[ij]\rangle \equiv Q^iQ^j|h\rangle\,,\cdots$$

Theories become more and more restricted with increasing N, and *unique* for maximal supersymmetry.

$\rightarrow$ **distinguish two possibilities (with N supercharges):**

- **Global (= rigid) supersymmetry:** $s \leq 1 \;\leftrightarrow N \leq 4$

  $N = 4$ **multiplet:** $\quad 1{\times}[1] \,\oplus\, 4{\times}\left[\tfrac{1}{2}\right] \,\oplus\, 6{\times}[0]$

- **Local supersymmetry (supergravity):** $s \leq 2 \leftrightarrow N \leq 8$

$N = 8$ **multiplet:** $\quad 1{\times}[2] \oplus 8{\times}\left[\tfrac{3}{2}\right] \oplus 28{\times}[1] \oplus 56{\times}\left[\tfrac{1}{2}\right] \oplus 70{\times}[0]$

$N = 8$ Supergravity (I)

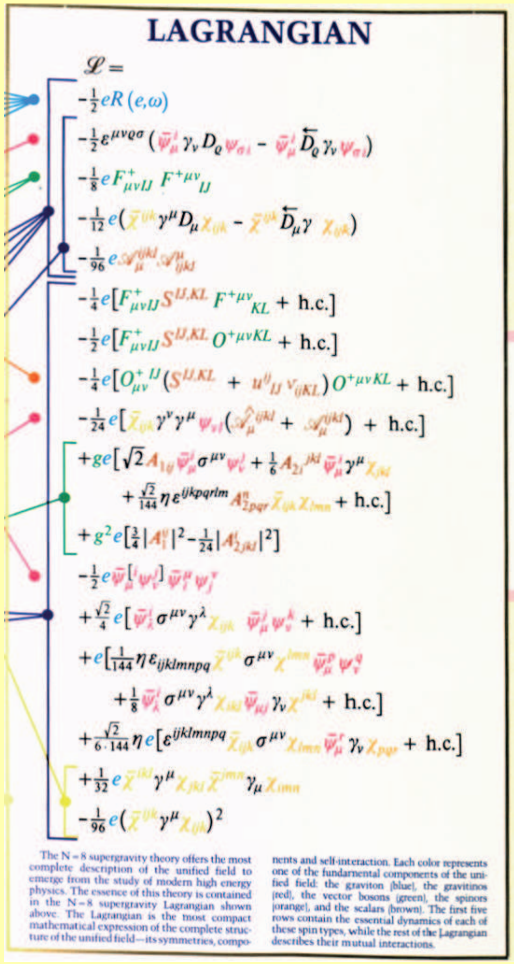
Unique theory (modulo 'gauging'), *most symmetric* known field theoretic extension of Einstein's theory!

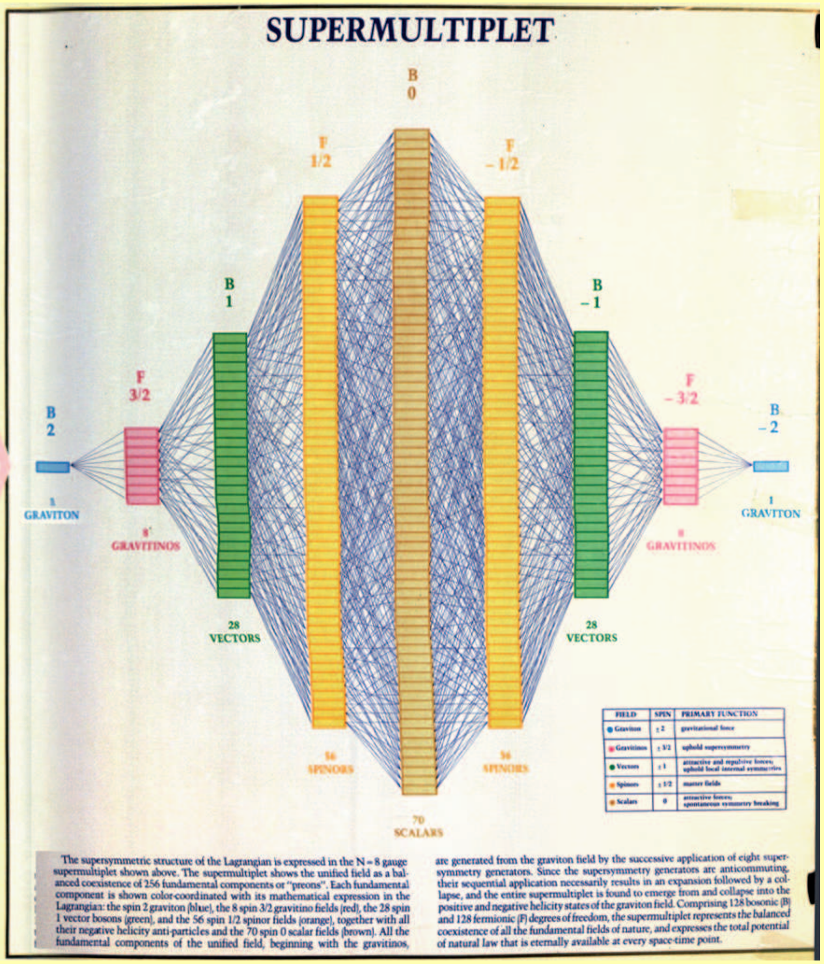
$$1\times[2] \oplus 8\times\left[\tfrac{3}{2}\right] \oplus 28\times[1] \oplus 56\times\left[\tfrac{1}{2}\right] \oplus 70\times[0]$$

- ***General covariance and local Lorentz symmetry***
- $N = 8$ ***local supersymmetry***
- $SU(8)$ ***R-symmetry (local or rigid)***
- ***Linearly or non-linearly realized duality symmetry*** $E_{7(7)}$

The unexpected 'hidden' $E_{7(7)}$ symmetry is a *duality symmetry* combining 28 'electric' vector fields and their 28 'magnetic' duals into a single 56 of $E_{7(7)}$.

$N = 8$ Supergravity (II)





$N = 8$ Supergravity (III)

In the late 1970s this theory was thought to be a promising candidate for a unified theory of quantum gravity and matter interactions. However,

- Existence of supersymmetric counterterms suggested the appearance of non-renormalizable infinities from three loops onwards;
- Its properties [absence of chiral fermions, huge negative cosmological constant for gauged theory] seem to be in obvious conflict with observations.

And: in both regards superstring theory seemed to do much better!

BUT: remarkable recent progress on UV finiteness...

Finiteness: to be or not to be?

Einstein gravity is *perturbatively non-renormalizable*

$$\Gamma^{(2)}_{div} = \frac{1}{\varepsilon}\frac{209}{2880}\frac{1}{(16\pi^2)^2}\int dV\, C_{\mu\nu\rho\sigma}C^{\rho\sigma\lambda\tau}C_{\lambda\tau}{}^{\mu\nu}$$

where $C_{\mu\nu\rho\sigma}$ = Weyl tensor. [Goroff, Sagnotti(1986); van de Ven(1992)]

NB: calculation of the coefficient $\frac{209}{2880}$ requires consideration of $\mathcal{O}(100\,000)$ Feynman diagrams!

It can be shown that this 2-loop counterterm does not allow for a supersymmetric extension. At three loops, a possible supersymmetric invariant can be built by squaring the Bel-Robinson tensor.

Hence, supergravity can be expected to be UV infinite from three loops onwards!

However, for $N = 8$ supergravity computation of 3-loop counterterm coefficient would require consideration of $\mathcal{O}(10^{xxx})$ Feynman diagrams $\rightarrow$ hopeless? ...

NO! Completely new technology makes possible a breakthrough: [Bern,Carrasco,Dixon,Johansson,Roiban, PRL103,081301(2009)]

... but must forget just about all textbook wisdom!

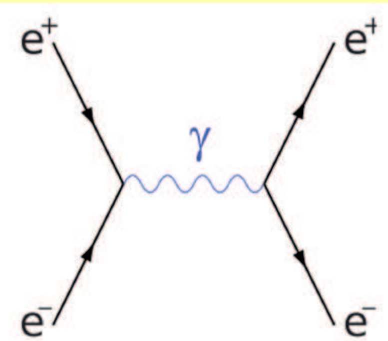
- **Obtain on-shell field theory amplitudes as limits of on-shell string amplitudes.**
- **Use methods from S-matrix theory: unitarity, analyticity, cutting rules, dispersion relations, etc.**
- **Gravity amplitudes from 'squaring' Yang Mills amplitudes by means of KLT rules** [Kawai,Lewellen,Tye(1986)]

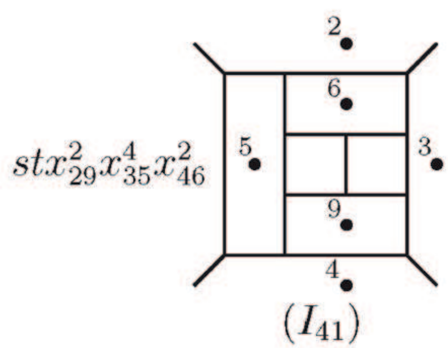
$$M_4^{tree}(1,2,3,4) = -is_{12}A_4^{tree}(1,2,3,4)A_4^{tree}(1,2,4,3)$$

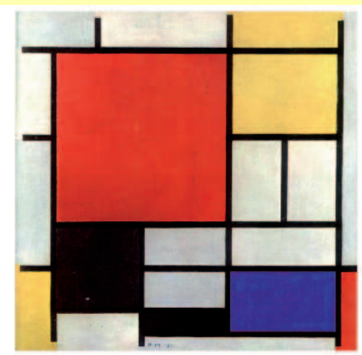
Four-loop finiteness of $N = 8$ supergravity

[Bern,Carrasco,Dixon,Johansson,Roiban, PRL103,081301(2009)]

- Exploit **Gravity = (Yang-Mills)^2**.
- **Use unitarity based arguments to reduce all amplitudes to integrals over products of tree amplitudes.**
- **All particles are on-shell $\rightarrow$ only 3-point vertices.**
- **Instead of $\mathcal{O}(10^{xxx})$ Feynman diagrams need only calculate $\mathcal{O}(50)$ 'Mondrian-like' diagrams!**







Beyond $L = 4$ loops

There is now mounting evidence from different sources that $N = 8$ supergravity is UV finite at $L < 7$ loops:

- **spectacular computational advances *(see above...)***
- **MHV, NMHV,... amplitude considerations *(idem)***
- **exploiting nonlinear symmetries (SUSY, $E_{7(7)}$,...)**

... and could thus be UV finite to *all* orders!

BUT $L = 7$ requires 10^{xxxxx} Feynman diagrams! AND:

- **Even if $N = 8$ supergravity is finite, (non-linear) E_7 and SUSY may not be enough to prove it.**
- **Even if it is finite we still do not understand what 'happens' to space and time at the Planck scale.**

A strange coincidence?

$SO(8) \to SU(3)\times U(1)$ **breaking and 'family color locking'**

$$
\begin{array}{rlll}
(u\,,\,c\,,\,t)_L & : & \mathbf{3}_c \times \bar{\mathbf{3}}_f \to \mathbf{8} \oplus \mathbf{1}\,, & Q = \frac{2}{3} - q \\
(\bar{u}\,,\,\bar{c}\,,\,\bar{t})_L & : & \bar{\mathbf{3}}_c \times \mathbf{3}_f \to \mathbf{8} \oplus \mathbf{1}\,, & Q = -\frac{2}{3} + q \\
(d\,,\,s\,,\,b)_L & : & \mathbf{3}_c \times \mathbf{3}_f \to \mathbf{6} \oplus \bar{\mathbf{3}}\,, & Q = -\frac{1}{3} + q \\
(\bar{d}\,,\,\bar{s}\,,\,\bar{b})_L & : & \bar{\mathbf{3}}_c \times \bar{\mathbf{3}}_f \to \bar{\mathbf{6}} \oplus \mathbf{3}\,, & Q = \frac{1}{3} - q \\
(e^-, \mu^-, \tau^-)_L & : & \mathbf{1}_c \times \mathbf{3}_f \to \mathbf{3}\,, & Q = -1 + q \\
(e^+, \mu^+, \tau^+)_L & : & \mathbf{1}_c \times \bar{\mathbf{3}}_f \to \bar{\mathbf{3}}\,, & Q = 1 - q \\
(\nu_e\,,\,\nu_\mu\,,\,\nu_\tau)_L & : & \mathbf{1}_c \times \bar{\mathbf{3}}_f \to \bar{\mathbf{3}}\,, & Q = -q \\
(\bar{\nu}_e\,,\,\bar{\nu}_\mu\,,\,\bar{\nu}_\tau)_L & : & \mathbf{1}_c \times \mathbf{3}_f \to \mathbf{3}\,, & Q = q
\end{array}
$$

Supergravity and Standard Model assignments agree if spurion charge is chosen as $q = \frac{1}{6}$ [Gell-Mann (1983)]

Realized at $SU(3)\times U(1)$ **stationary point!** [Warner,HN: NPB259(1985)412]

Mismatch of $\pm\frac{1}{6}$ **can be fixed by deforming U(1)** [Meissner,HN:1412.1715]

*[... just in case LHC keeps *not* finding new fundamental spin-*$\frac{1}{2}$ *fermions...]*

Exceptional Symmetries and Supergravity

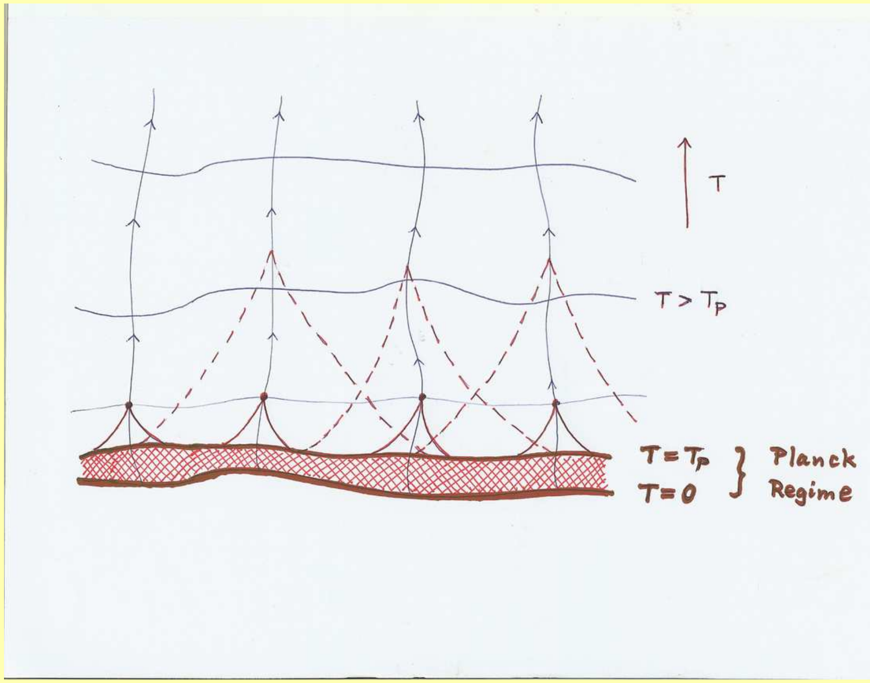
Continuing on the road towards more symmetry!

- **Recall: $N = 8$ supergravity in $D = 4$ has more symmetry than meets the eye: $E_{7(7)}$!** [Cremmer,Julia,1979]
- **An unsuspected link: all of the *five exceptional Lie groups* G_2, F_4, E_6, E_7, E_8 with their non-compact real forms appear naturally in extended supergravities!**
- **For instance all maximal supergravities in $D \neq 4$: $E_{n(n)}$ for maximal supergravity in $D = 11 - n$.**

Below $D = 3$ symmetries become *infinite-dimensional*:

- **$E_{9(9)} \equiv E_8^{(1)}$ for maximal supergravity in $D = 2$.**
- **... suggests $E_{10(10)}$ for $D = 1$: no space, only time?!?**

Another hint: BKL and Spacelike Singularities

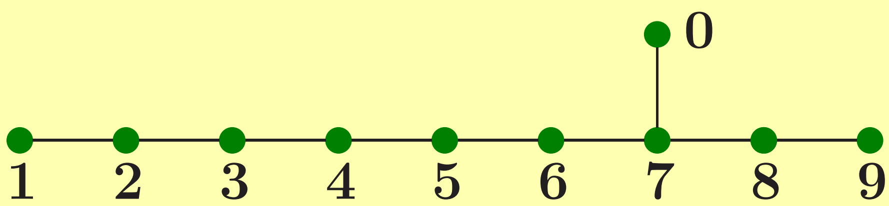


For $T \to 0$ spatial points decouple and the system is effectively described by a continuous *superposition of one-dimensional systems* $\to$ effective dimensional reduction to $D = 1$! [Belinski,Khalatnikov,Lifshitz (1972)]

What is E_{10}?

(No one knows, really....)

E_{10} is the 'group' associated with the Kac-Moody Lie algebra $\mathfrak{g} \equiv \mathfrak{e}_{10}$ defined via the Dynkin diagram [e.g. Kac]

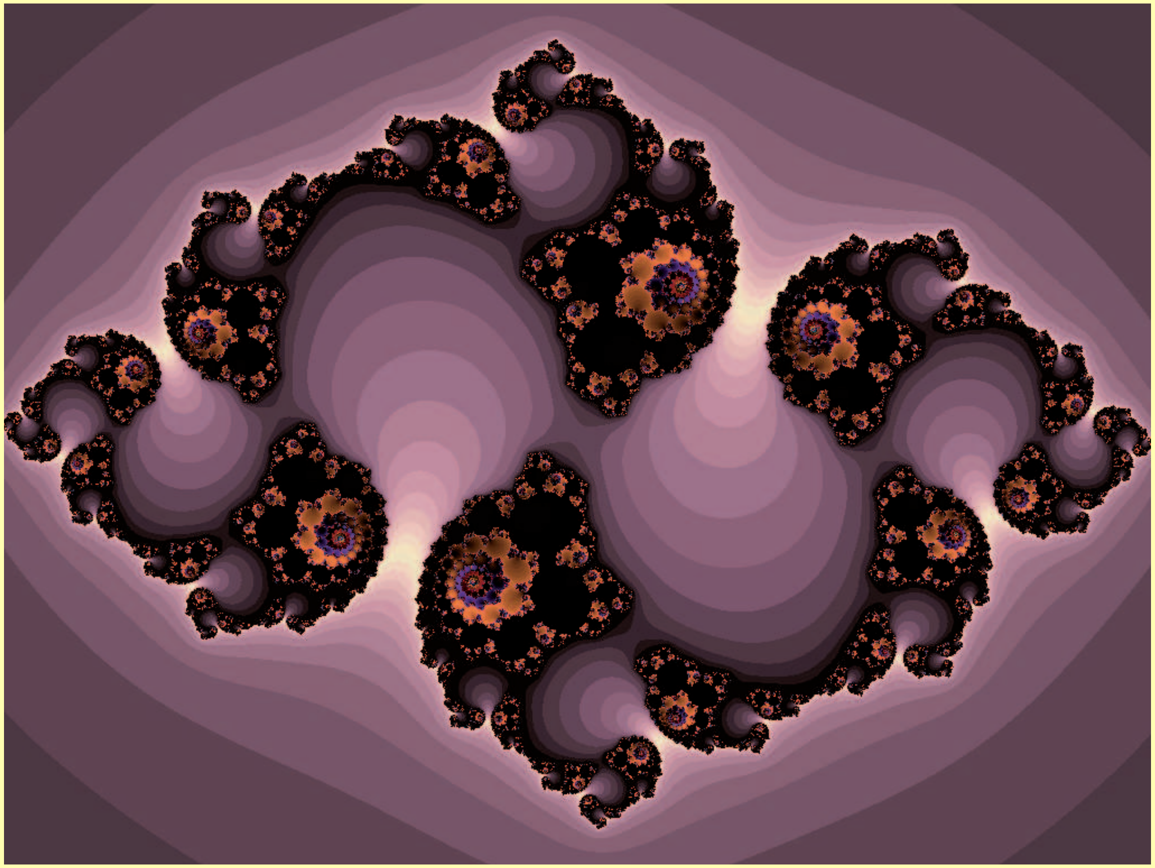


Defined by generators $\{e_i, f_i, h_i\}$ and relations via Cartan matrix A_{ij} ('Chevalley-Serre presentation')

$$\begin{aligned} [h_i, h_j] &= 0, \qquad\qquad [e_i, f_j] = \delta_{ij} h_i, \\ [h_i, e_j] &= A_{ij} e_j, \qquad\quad [h_i, f_j] = -A_{ij} f_j, \\ (\mathrm{ad}\, e_i)^{1-A_{ij}} e_j &= 0 \quad (\mathrm{ad}\, f_i)^{1-A_{ij}} f_j = 0. \end{aligned}$$

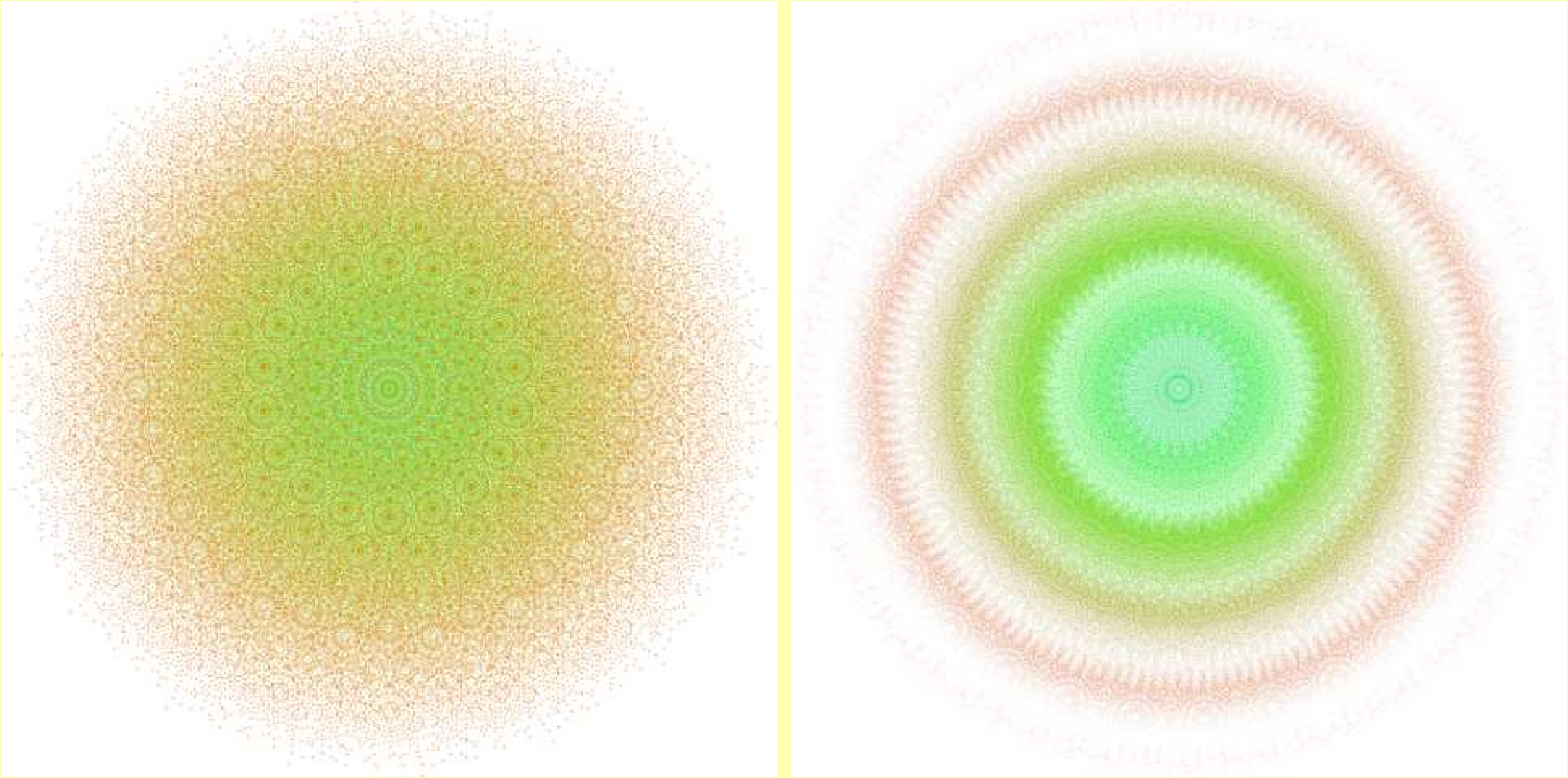
$\mathfrak{e}_{10}$ is the free Lie algebra generated by $\{e_i, f_i, h_i\}$ modulo these relations → infinite dimensional as A_{ij} is *indefinite* → Lie algebra of *exponential growth* !

Infinite Complexity from simple recursion



A Mandelbrot set generated from $z_{n+1} = f_c(z_n)$.

Vistas into E_{10}...

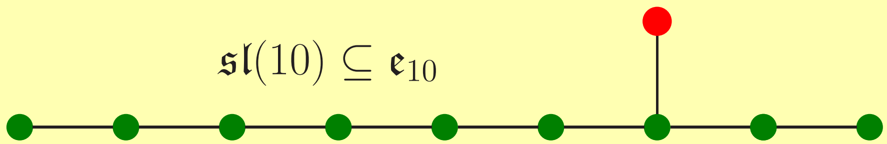


[from: Teake Nutma (formerly University of Groningen, now at AEI)]

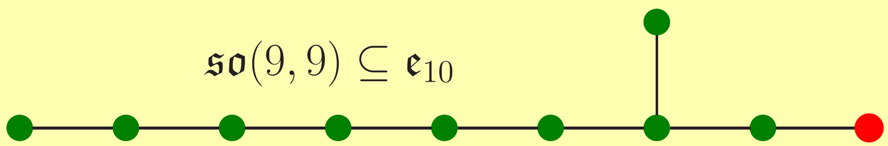
What is so special about E_{10}?

- E_{10} occupies a **uniquely distinguished place among all infinite-dimensional Lie algebras** (much like E_8 among the finite-dimensional Lie algebras)
- Near the initial singularity (for $0 < T < T_{Planck}$) the dynamics of maximal supergravity 'asymptotes' to a **'cosmological billiards'** system based on E_{10}.
- E_{10} may provide ***Lie-algebraic mechanism* for the 'de-emergence' of space and time** near the big bang.
- E_{10} 'knows all' about maximal supersymmetry.

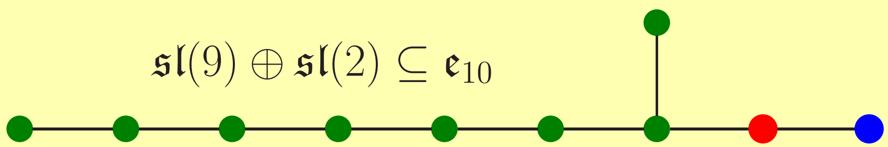
E_{10} Versatility



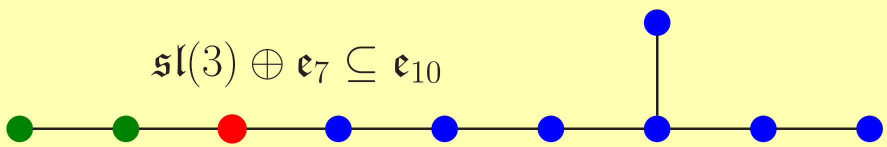
$D = 11$ **SUGRA**



mIIA $D = 10$ **SUGRA**

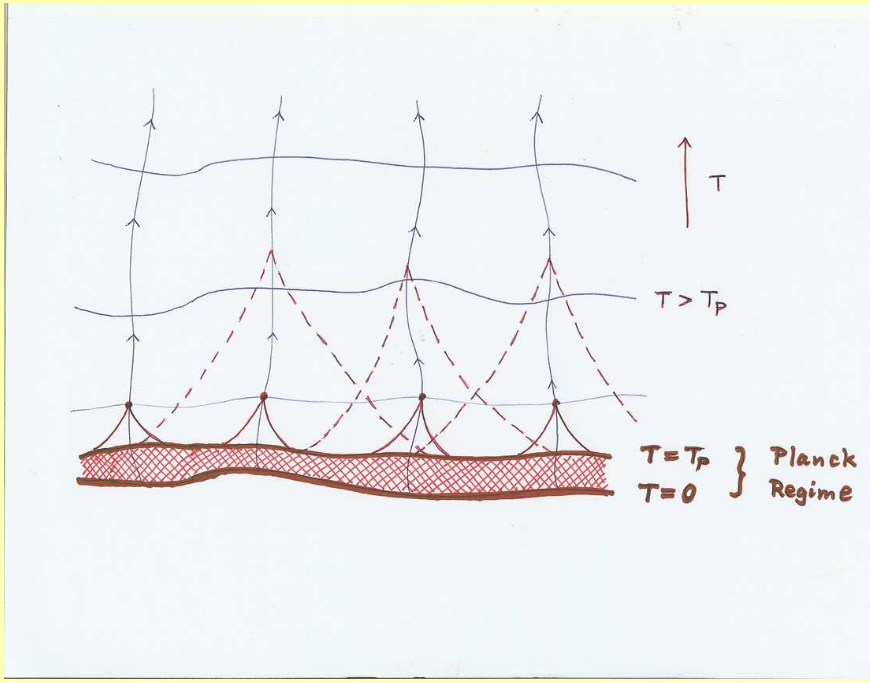


IIB $D = 10$ **SUGRA**



$\mathcal{N} = 8$**,** $D = 4$ **SUGRA**

E_{10}: The Basic Picture



Conjecture: **for $0 < T < T_P$ space-time 'de-emerges', and space-time based (quantum) field theory is replaced by (quantized) $E_{10}/K(E_{10})$ σ-model** [Cf. DN, 0705.2643]

Outlook

- **Symmetry by no means exhausted as a *guiding principle of physics* but many open questions remain.**
- **Both $N = 8$ supergravity and E_{10} are *uniquely distinguished* by their symmetry properties.**
- **E_{10} 'knows all' about maximal supersymmetry and unifies many known $(S, T, U, ...)$ string dualities.**

  [so supersymmetry may not be as fundamental as we thought...]
- **Exponentially increasing complexity of E_{10} algebra $\rightarrow$ an element of *non-computability* for $T \rightarrow 0$?**
- **Either way, we have to come to grips with *infinity* as we try to understand the origin of the universe.**

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THANK YOU