The Copenhagen Interpretation Born Again

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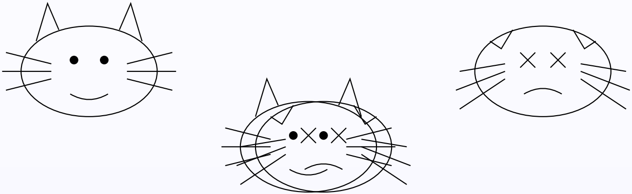

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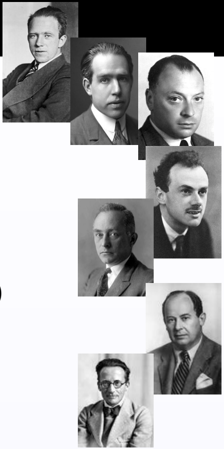
Copenhagen QM



- The pioneers of QM left us with a truly amazing universal theory of microscopic physics
  - *The rules are sufficient to make sense of the microscopic world, e.g. QED*

$$g_\mu\Big|_{\text{theory}} = 2.0023318361(10) \qquad g_\mu\Big|_{\text{exp.}} = 2.0023318416(13)$$

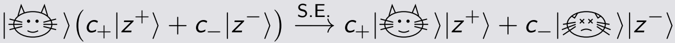
  - *Theoretical prediction involves calculating a 4-loop amplitude* **and applying Born's rule**

- So what's the problem?
  - *Uneasy relation between the microscopic and macroscopic*
  - *A bit ad-hoc so lots room for misunderstandings: non-locality; reduction/collapse; delayed choice; ...*

On the verge of an exciting era of quantum technologies isn't it time to finally understand the foundations of QM

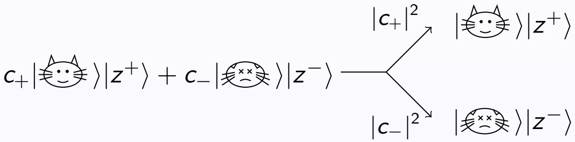
The Key Problem

Schrödinger's cat: linearity gives



- **Copenhagen interpretation:** two modes of time evolution:

1. Schrödinger equation $$\boxed{H|\Psi(t)\rangle = i\hbar\frac{\partial}{\partial t}|\Psi(t)\rangle}$$
2. Measurement: implement a simple stochastic process



1. Where and when does the **Heisenberg cut** occur?
2. Why pick out the macroscopically distinct states?

Is the Quantum Puzzle Solvable?

Do we need to?

Mermin: If I were forced to sum up in one sentence what the Copenhagen interpretation says to me, it would be "Shut up and calculate!"

Can it be done?

Weinberg: There is now in my opinion no entirely satisfactory interpretation of quantum mechanics. The Copenhagen interpretation assumes a mysterious division between the microscopic world governed by quantum mechanics and a macroscopic world of apparatus and observers that obey classical physics.

A Bit of a Muddle

Lots of different approaches:

- Have QM emerge from hidden (i.e. classical) variables
  - *non-local*
  - *contrived*
  - *no reasonable generalization to QFT*
- Only use the Schrödinger equation: many worlds interpretation
  - *loose Born's rule and difficult (i.e. very contrived) to get back*
  - *problem with decaying systems: continuum of worlds ??*

... for these ideas the emperor has no clothes

Have the Classical World Emerge From QM

- Have a clear set of rules with no room for misrepresentation
  - *Locality is fundamental*
  - *Classically sensible events occur but in a way that can be fundamentally random:* **Geiger counters really do click randomly with a probability we can calculate precisely from QM of nuclei using Born's rule**
  - *No silliness: no Alices, Bobs, minds, brains, consciousness, agents, users ... no philosophy required*
- Take account of up-to-date experiments:

> Modern atomic physics experiments on single atoms show that Bohr and Einstein's quantum jumps of 1910's occur in the measuring device and are not an objective behaviour of the atom, i.e. to the extent that state vector reduction is real (and we will argue that it is) it takes place in the measuring device

- The idea:

Have both the Schrödinger equation describing microscopic dynamics and a Born inspired continous stochastic process describing the macroscopic dynamics consistently co-existing

- And then:

The Copenhagen Interpretation will emerge as a set of effective rules when we want to put the focus on the microscopic system and not the measuring device

How to Build a Classical World

Two sources of inspiration:

- **QFT:** Wilson's revolution: theories are only effective (e.g. QED; standard model; quantum GR; chiral PT;...)
  - *Explicit cut off, i.e. coarse graining or resolution, scale*
  - *Set of coarse grained observables* $\mathcal{A} = \{\mathcal{O}_n\}$
  - *IR behaviour insensitive to cut off*
- **Quantum SM:** equilibration and thermalization are properties of expectation values of coarse grained observables $\langle\Psi(t)|\mathcal{O}_n|\Psi(t)\rangle$ and **not** $|\Psi(t)\rangle$ itself

Build the classical world in terms of a set of coarse grained observables $\mathcal{A} = \{\mathcal{O}_n\}$

- In most realistic situations, we can assume that $\mathcal{A}$ is a commuting set (even if $\mathcal{A}$ contains coarse grained position and momentum operators)

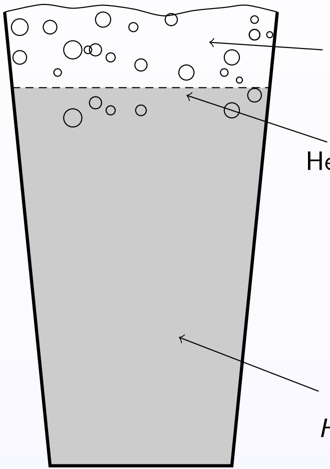
Imagine that the world is ...

macroscopic

partly deterministic
partly random

microscopic

deterministic



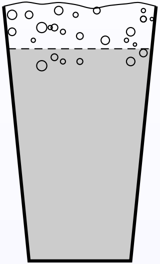
$$\left\{ \begin{array}{l} F = ma \\ \text{Prob}_i = \left| \langle \Psi_i | \Psi \rangle \right|^2 \\ \text{thermal ensembles} \end{array} \right.$$

Heisenberg Cut

$$H|\Psi\rangle = i\hbar \frac{\partial}{\partial t} |\Psi\rangle$$

Have the two modes of time evolution—Schrödinger and Born—occurring continuously:

1. An underlying microscopic deterministic dynamics, **the Schrödinger Equation**
2. A stochastic process, **a continuous time Markov Chain**, acting on the effective theory of the macroscopic



$$\rightarrow |\Psi_{i_1}\rangle \xrightarrow{T_{i_2 i_1}} |\Psi_{i_2}\rangle \rightarrow |\Psi_{i_3}\rangle \rightarrow$$

quantum microstate

$$H|\Psi\rangle = i\hbar\frac{\partial}{\partial t}|\Psi\rangle$$

underlying state

- Special set of states: the **Quantum Microstates** (cf. Boltzmann's classical SM)

$$\boxed{|\Psi\rangle = \sum_i c_i|\Psi_i\rangle}$$

- The **macrostate** will be the time average of $\langle\Psi_{i(t)}|\mathcal{O}_n|\Psi_{i(t)}\rangle$ for special set of **coarse grained observables** $\{\mathcal{O}_n\}$

The New Rules

- **Jaynes coarse graining ('57):** if all we know are the expectation values $\langle\Psi|\mathcal{O}_n|\Psi\rangle$ then the most unbiased description of the state is the density operator ρ

$$\langle\Psi|\mathcal{O}_n|\Psi\rangle = \mathrm{Tr}\left[\rho\,\mathcal{O}_n\right] = \sum_i |c_i|^2 \langle\Psi_i|\mathcal{O}_n|\Psi_i\rangle$$

with maximal entropy $S = -\,\mathrm{Tr}\left[\rho \log \rho\right] = -\sum_i |c_i|^2 \log |c_i|^2$

- Add two conditions:
  1. **Microscopic Born Rule:** probabilities $|c_i|^2 = |\langle\Psi_i|\Psi\rangle|^2$
  2. **Non-degeneracy:** $\langle\Psi_i|\mathcal{O}_n|\Psi_j\rangle \neq \mu_n \delta_{ij}$ for $i, j \in \{k, l\}$

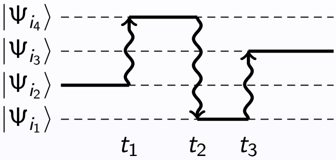
- $|\Psi_i\rangle$ are the **quantum microstates** associated to $\mathcal{A} = \{\mathcal{O}_n\}$ and $|\Psi\rangle$
  - *For abelian $\mathcal{A}$ they are the simultaneous eigenstates*

At a given time the macrostate is one of the quantum microstates $|\Psi_i\rangle$ (suitably averaged over time...)

Time Dependence: Quantum Jumps

- The macrostate from the perspective of $\mathcal{A}$ are the expectation values $\langle \Psi_{i(t)} | \mathcal{O}_n | \Psi_{i(t)} \rangle$ for a sequence $|\Psi_{i(t)}\rangle$ related by jumps:

$$i(t) = \begin{cases} i_2 & t < t_1 \\ i_4 & t_1 < t < t_2 \\ i_1 & t_2 < t < t_3 \\ & \ldots \end{cases}$$



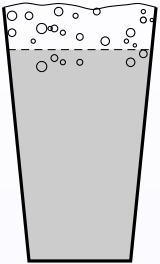
- Consistency with Schrödinger equation implies a continuous time Markov chain with transition rates for $|\Psi_j\rangle \to |\Psi_i\rangle$, $i \neq j$,

$$T_{ij} = \max\Big(-\frac{2}{\hbar} \operatorname{Im}\Big[\frac{c_i}{c_j} \langle \Psi_j | H | \Psi_i \rangle \Big], 0 \Big)$$

Bell's be-ables

- So the quantum microstate dynamics is fundamentally random
- Process driven by local interactions $\langle \Psi_j | H | \Psi_i \rangle$

To Summarize: Two Levels That Dovetail



$$\rightarrow |\Psi_{i_1}\rangle \xrightarrow{T_{i_2 i_1}} |\Psi_{i_2}\rangle \rightarrow |\Psi_{i_3}\rangle \rightarrow$$

quantum microstate

macrostate

$$H|\Psi\rangle = i\hbar \frac{\partial}{\partial t}|\Psi\rangle$$

underlying state

- Schrödinger equation and stochastic process now work in perfect harmony:

1. $H|\Psi\rangle = i\hbar \frac{\partial}{\partial t}|\Psi\rangle$ implies $i\hbar \frac{d}{dt} c_i = \sum_j \langle \Psi_i | H | \Psi_j \rangle c_j$

2. $\frac{d}{dt}|c_i|^2 = \sum_j T_{ij} |c_j|^2$

Ergodicity and the Classical Limit

- The stochastic process must account for the 3 different facets of the classical limit

Deterministic: $F = ma$; **Random:** $Z = \sum_n e^{-E_n/T}$; **Born:** $p_i = |\langle \Psi_i | \Psi \rangle|^2$

Ergodicity of the stochastic process and its breaking is the key

- In equilibrium $|c_i|^2 \approx$ constant and if all states can be reached from any other state then process is **ergodic** and **long time average** $\approx$ **ensemble average:** for large enough T

$$\frac{1}{T}\int_0^T dt\, \langle \Psi_{i(t)} | \mathcal{O}_n | \Psi_{i(t)} \rangle \approx \sum_i |c_i|^2 \langle \Psi_i | \mathcal{O}_n | \Psi_i \rangle = \langle \Psi | \mathcal{O}_n | \Psi \rangle$$

- *Much simpler than ergodicity in classical SM: (i) fundamentally random (ii) discrete set of states*

- fate of ergodicity depends on the matrix elements $\langle \Psi_i | H | \Psi_j \rangle$
  - *If these are generic then get ergodic behaviour*
  - *If $|\Psi_i\rangle$ and $|\Psi_j\rangle$ are macroscopically distinct then $\langle \Psi_i | H | \Psi_j \rangle \approx 0$ (decoherence) and get non-ergodic behaviour:*

Three generic types of behaviour:

1. Newton's laws: **ergodicity broken**, trajectories clustered around the classical trajectory
2. Thermal Ensembles: **ergodic** process in equilibrium
3. Quantum measurement: **partially broken ergodicity** choose out of a set of ergodically disjoint subsets associated to the different outcomes

Newton's Laws: the Particle

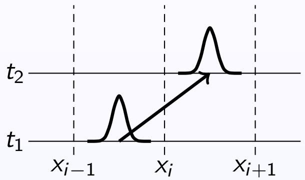
- Set of coarse grained position operators $\Pi_i = \int_{x_{i-1}}^{x_i} dx \, |x\rangle\langle x|$

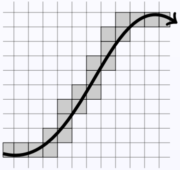
$$\Psi_i(x) = \begin{cases} c_i^{-1}\Psi(x) & x_{i-1} \leq x \leq x_i \\ 0 & \text{otherwise} \end{cases} \qquad |c_i|^2 = \int_{x_{i-1}}^{x_i} dx \, |\Psi(x)|^2$$

- Stochastic process allows hopping between neighbouring sites $\Psi_j \rightarrow \Psi_{j\pm1}$

$$\boxed{T_{j\pm1,j} = \max\left(\pm \frac{\hbar}{2iM|c_j|^2} \left[\Psi^* \frac{\partial \Psi}{\partial x} - \Psi \frac{\partial \Psi^*}{\partial x} \right]_{x = x_j, x_{j-1}}, 0 \right)}$$

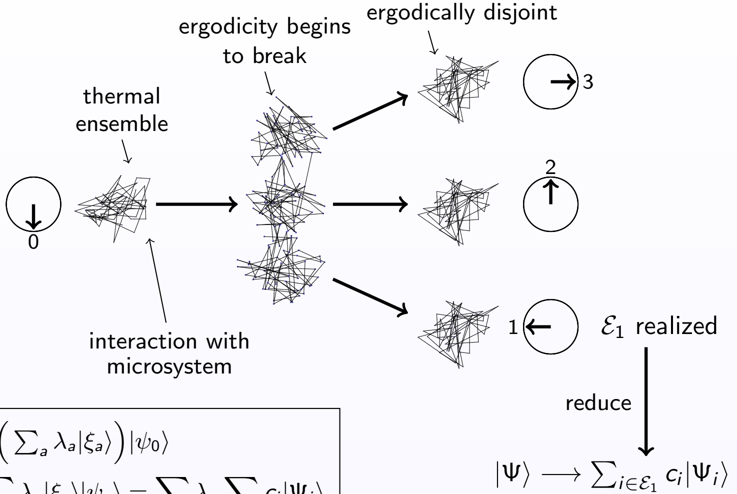
narrow
wave function





- Narrow wave functions break ergodicity and give rise to deterministic dynamics

Quantum Measurement ... No Problem



$$|\Psi(t)\rangle = \Big(\textstyle\sum_a \lambda_a|\xi_a\rangle\Big)|\psi_0\rangle$$
$$\longrightarrow \sum_a \lambda_a|\xi_a\rangle|\psi_a\rangle = \sum_a \lambda_a \sum_{i\in\mathcal{E}_a} c_i|\Psi_i\rangle$$

$\langle\Psi_i|H|\Psi_j\rangle \approx 0$ for $i \in \mathcal{E}_a$ and $j \in \mathcal{E}_b$ for $a \neq b$

Reduction of the State Vector

Reduction of the state vector to the ergodic component that is realized is an (ultimately unnecessary) piece of book keeping,
i.e. it is not a physical change of the system

Born's rule $\text{Prob}_a = |\lambda_a|^2$ follows when the initial state and final reduced states are ergodic and the measuring device is efficient

- application to statistical mechanics will be discussed later ...

Copenhagen Rules Sharpened

- The rules of the Copenhagen interpretation are reproduced but in a way that are sharpened:

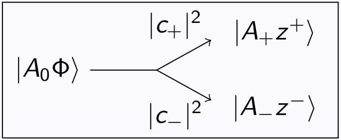
1. Reduction of the state vector is a book keeping procedure performed by a measuring device and is local to that device (not a physical process) **Quantum jumps are properties of the measuring device not the atom!**
2. There is no physical global collapse of the state vector and no global notion of a reduced state vector
3. The dynamics of the stochastic process are local: there is no non-locality in QM and the new rules don't add any (see later)
4. One cannot reduce the state vector backwards in time to say what the state was before a measurement (see later)

Simple Measuring Devices

- If we want to concentrate on the microscopic system and not the measuring device then we can just "coarse grain away" the thermal ensembles: the "really coarse grained" quantum microstate is then the macrostate
- e.g. simple qubit detector: 3 quantum microstates $|A_i\rangle$, $i = 0, \pm$

$$|A_0\rangle\big(c_+|z^+\rangle + c_-|z^-\rangle\big) \longrightarrow c_+|A_+z^+\rangle + c_-|A_-z^-\rangle$$

quantum microstate
$=$ macrostate
($=$ reduced state)



So the normal Copenhagen rules are recovered as the effective description of a very coarse grained measuring device

A Moan ... Non-locality Nonsense

- There is a lot of talk of **quantum non-locality**, spooky action at a distance

The violation of Bell's theorem in the real world implies that we have to give up on local realism...right?

So that means that Bell's theorem implies that QM is non-local....right?

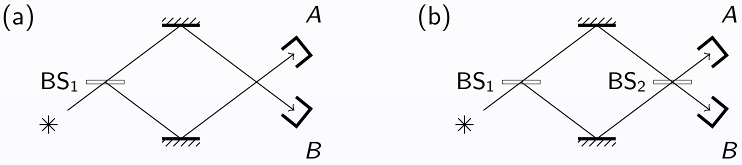
State vector collapse (reduction) in the Copenhagen interpretation is non local...right?

wrong, wrong, wrong ...

- QM is explicitly local (this is clear in QFT) and is not covered by Bell's theorem
- Bell's theorem has nothing to say about **local quantum realism** apart from the fact that it cannot be mimicked
- State reduction is a local book keeping procedure and not a physical procedure let alone a non-local one

Further Moan … Delayed Choice Nonsense

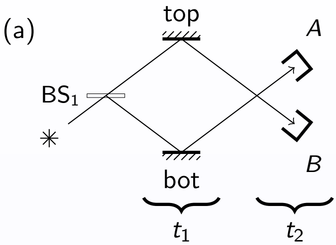
QM is weird: we can affect quantum states in the past by choosing what to measure now...right?



 **Warning fallacy alert** 

For (a) we get **which way** information so detection at, say, A means that photon took the bottom route and hence if instead we insert BS_2 this **erases** the **which way** information even after photon has passed BS_1 (delayed choice), i.e. what we do in the future can affect the state in the past

- But for (a) photon detected at A we **cannot** say photon took the bottom route:



$$|\Psi(t_1)\rangle = |A_0 B_0\rangle \frac{1}{\sqrt{2}}\big(|\text{top}\rangle + |\text{bot}\rangle\big)$$

$$\xrightarrow{\text{S.E.}} |\Psi(t_2)\rangle = \frac{1}{\sqrt{2}}|A_0 B_+ \text{top}\rangle + \frac{1}{\sqrt{2}}|A_+ B_0 \text{bot}\rangle \longrightarrow \begin{cases} |A_0 B_+ \text{top}\rangle \\ |A_+ B_0 \text{bot}\rangle \end{cases}$$

State of photon at time t_1 is $|\text{top}\rangle + |\text{bot}\rangle$ not $|\text{top}\rangle$ or $|\text{bot}\rangle$

Cannot reduce the state vector backwards in time: state reduction is a local book keeping procedure done at A or B just after photon hits

EPR: There's No Non-locality

- Take a pair of qubits in the state $|\Phi\rangle = \frac{1}{\sqrt{2}}\big(|z^+z^-\rangle - |z^-z^+\rangle\big)$
- Take two simple qubit detectors with 3 quantum microstates

$$|A_0\rangle\big(c_+|z^+\rangle + c_-|z^-\rangle\big) \longrightarrow c_+|A_+z^+\rangle + c_-|A_-z^-\rangle$$

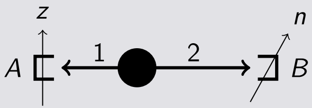
$$|B_0\rangle\big(c_+|n^+\rangle + c_-|n^-\rangle\big) \longrightarrow c_+|B_+n^+\rangle + c_-|B_-n^-\rangle$$

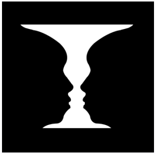
- Consider the EPR-Bohm experiment

$$|\Psi(t_1)\rangle = |A_0B_0\rangle\tfrac{1}{\sqrt{2}}\big(|z^+z^-\rangle - |z^-z^+\rangle\big)$$

$$|\Psi(t_2)\rangle = \tfrac{1}{\sqrt{2}}\big(|A_+B_0z^+z^-\rangle - |A_-B_0z^-z^+\rangle\big)$$

$$|\Psi(t_3)\rangle = \tfrac{1}{\sqrt{2}}\big(\sin\theta|A_+B_+z^+n^+\rangle + \cos\theta|A_+B_-z^+n^-\rangle - \cos\theta|A_-B_+z^-n^+\rangle + \sin\theta|A_-B_-z^-n^-\rangle\big)$$

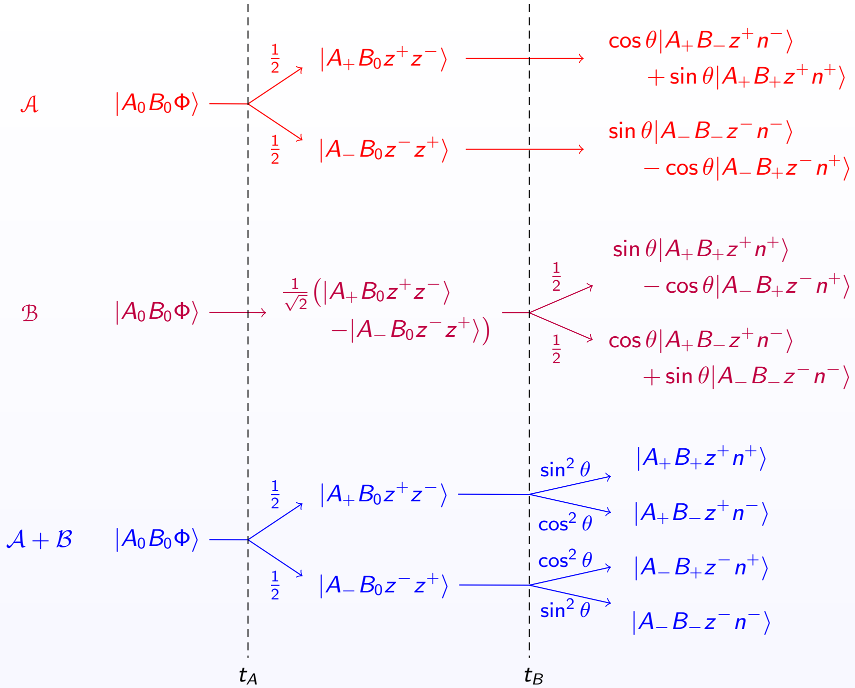




- There are 3 different perspectives/stochastic processes:
  - $\mathcal{A}$ *observables of measuring device* A
  - $\mathcal{B}$ *observables of measuring device* B
  - $\mathcal{A} \cup \mathcal{B}$ *observables of measuring devices* $A + B$
- Only the $\mathcal{A} \cup \mathcal{B}$ view can "see" the correlations
- **Locality:** stochastic processes $\mathcal{A}$ driven by local interactions between q_1 and measuring device A

> Quantum microstate of A can only change at $t = t_A$ and B at $t = t_B$: in particular, interaction of A and q_1 cannot induce a transition of the stochastic process of $\mathcal{B}$ (ditto for B and q_2)

- Mapping out the 3 processes ...



𝒜
$|A_0 B_0 \Phi\rangle$
$\frac{1}{2}$
$|A_+ B_0 z^+ z^-\rangle$
$\cos\theta|A_+ B_- z^+ n^-\rangle + \sin\theta|A_+ B_+ z^+ n^+\rangle$
$\frac{1}{2}$
$|A_- B_0 z^- z^+\rangle$
$\sin\theta|A_- B_- z^- n^-\rangle - \cos\theta|A_- B_+ z^- n^+\rangle$
ℬ
$|A_0 B_0 \Phi\rangle$
$\frac{1}{\sqrt{2}}\left(|A_+ B_0 z^+ z^-\rangle - |A_- B_0 z^- z^+\rangle\right)$
$\frac{1}{2}$
$\sin\theta|A_+ B_+ z^+ n^+\rangle - \cos\theta|A_- B_+ z^- n^+\rangle$
$\frac{1}{2}$
$\cos\theta|A_+ B_- z^+ n^-\rangle + \sin\theta|A_- B_- z^- n^-\rangle$
$\mathcal{A} + \mathcal{B}$
$|A_0 B_0 \Phi\rangle$
$\frac{1}{2}$
$|A_+ B_0 z^+ z^-\rangle$
$\sin^2\theta$
$|A_+ B_+ z^+ n^+\rangle$
$\cos^2\theta$
$|A_+ B_- z^+ n^-\rangle$
$\frac{1}{2}$
$|A_- B_0 z^- z^+\rangle$
$\cos^2\theta$
$|A_- B_+ z^- n^+\rangle$
$\sin^2\theta$
$|A_- B_- z^- n^-\rangle$
t_A
t_B

EPR: The Conclusions

- Formalism is completely local: all that happens is that an underlying correlation (that cannot be mimicked by a classical theory) is revealed
- Reduction takes place at the measuring devices and not before and is a local book keeping procedure
  - *Avoids the delayed choice type fallacy discussed earlier*
  - *Measurement at A is not changing the state of q_2 instantaneously: reduction is not a global procedure*
- The view from $\mathcal{A} \cup \mathcal{B}$ is a refinement of that of $\mathcal{A}$ or $\mathcal{B}$ alone
  - *If we have more observables then the quantum microstates are more detailed*
  - *The $\mathcal{A} \cup \mathcal{B}$ view describes the usual QM correlations that violate Bell's theorem*

Classical Limit and Statistical Mechanics

- Classical SM is an old theory with many conceptual problems:
  1. The conceptual status of probability in a deterministic theory
  2. Definition of entropy and the second law
  3. Description of non-equilibrium and the approach to equilibrium
  4. Relation between ensembles and long, or infinite, time averages **(ergodicity)**

Boltzmann (dynamical) **v.** Gibbs (inferential)

- Many of these issues can be swept away by a quantum approach to SM
  - *Schrödinger (1927)*
  - *von Neumann's Quantum Ergodic Theorem (1929)*

Don't obviously have classical SM arising in the classical limit

Two Big Issues: Equilibration and Thermalization

Coarse grained observables $\mathcal{A} = \{\mathcal{O}_n\}$

- **Equilibration:** after initial transient region Tr $\left[\rho(t)\mathcal{O}_n\right]$ stays close to equilibrium values Tr $\left[\rho_0\mathcal{O}_n\right]$ for most times (with fluctuations)

$$\rho(t) = \sum_{mn} \rho_{mn} e^{i(E_n - E_m)t/\hbar} |m\rangle\langle n| \qquad \rho_0 = \sum_n \rho_{nn} |n\rangle\langle n|$$

Note: the state $\rho(t)$ does not equilibrate itself!
$\Longrightarrow$ macrostate determined by Tr $\left[\rho(t)\mathcal{O}_n\right]$

- **Thermalization:** e.g. isolated system, to what extent is

$$\mathrm{Tr}\left[\rho(t)\mathcal{O}_n\right] \overset{?}{=} \mathrm{Tr}\left[\rho_{\mathrm{mc}}\mathcal{O}_n\right] \qquad \rho_{\mathrm{mc}} = \frac{1}{N} \sum_{E_n \in [E, E+\Delta E]} |n\rangle\langle n|$$

- von Neumann's **Quantum Ergodic Theorem** proves equilibration and thermalization: for any pure state $|\Psi(t)\rangle \in \mathcal{H}_{[E,E+\Delta E]}$

$$\langle\Psi(t)|\mathcal{O}_n|\Psi(t)\rangle \approx \mathrm{Tr}\left[\rho_{\mathrm{mc}}\mathcal{O}_n\right]$$

But in this quantum approach to SM there is no notion of probabilities or microstates or CSM arising in the classical limit

- In our approach there are ab initio probabilities and quantum precursors of the microstates, i.e. we re-instate a Boltzmann dynamical picture

- In an interacting non-integrable system, bases $\{|n\rangle\}$ and $\{|\Psi_i\rangle\}$ are randomly related. Implies $\langle\Psi_j|H|\Psi_i\rangle$ are randomly distributed and in equilibrium the stochastic process is ergodic

$$\frac{1}{T}\int_0^T dt\,\langle\Psi_{i(t)}|\mathcal{O}_n|\Psi_{i(t)}\rangle \approx \sum_i |c_i|^2\langle\Psi_i|\mathcal{O}_n|\Psi_i\rangle = \langle\Psi|\mathcal{O}_n|\Psi\rangle \approx \mathrm{Tr}[\mathcal{O}_n\rho_{\mathrm{mc}}]$$

ergodicity ($\approx$) — microscopic Born rule ($=$) — quantum ergodic theorem ($\approx$)

To Sum Up

- We have defined clear rules for which the usual Copenhagen interpretation emerges

> Definite classically sensible things happen in a random way if a microscopic system is involved, e.g. Geiger counter

- The usual rules (formulated correctly) are perfect effective rules when one doesn't want to focus on the measuring device but on the underlying quantum system (shut up and calculate!)
- Have underlying deterministic Schrödinger evolution and effective stochastic evolution of coarse grained effective theory living in harmony
- The resolution scale, **the Heisenberg cut**, can be changed, cf. Wilson's cut off and the renormalization group in QFT. Just like in QFT, IR behaviour, the ergodic subsets are robust

- New approach, via ergodicity, can account for all the facets of the classical limit
  - *Newton's Law; thermal ensembles; quantum measurement*

- The rules of the Copenhagen interpretation are sharpened: each local measuring device has its own reduced state which is chosen by ergodicity breaking
  - *Reduction is not physical and there is no global collapse*
  - *Cannot reduce backwards in time*

- Quantum mechanics with the new rules is perfectly local $\langle \Psi_i | H | \Psi_j \rangle$

- A new approach to quantum SM that deserves further study
  - *Puts the Boltzmann back in quantum SM*