Modeling Dark Energy - from a Cosmological Constant to Quintessence to Modified Theories of Gravity

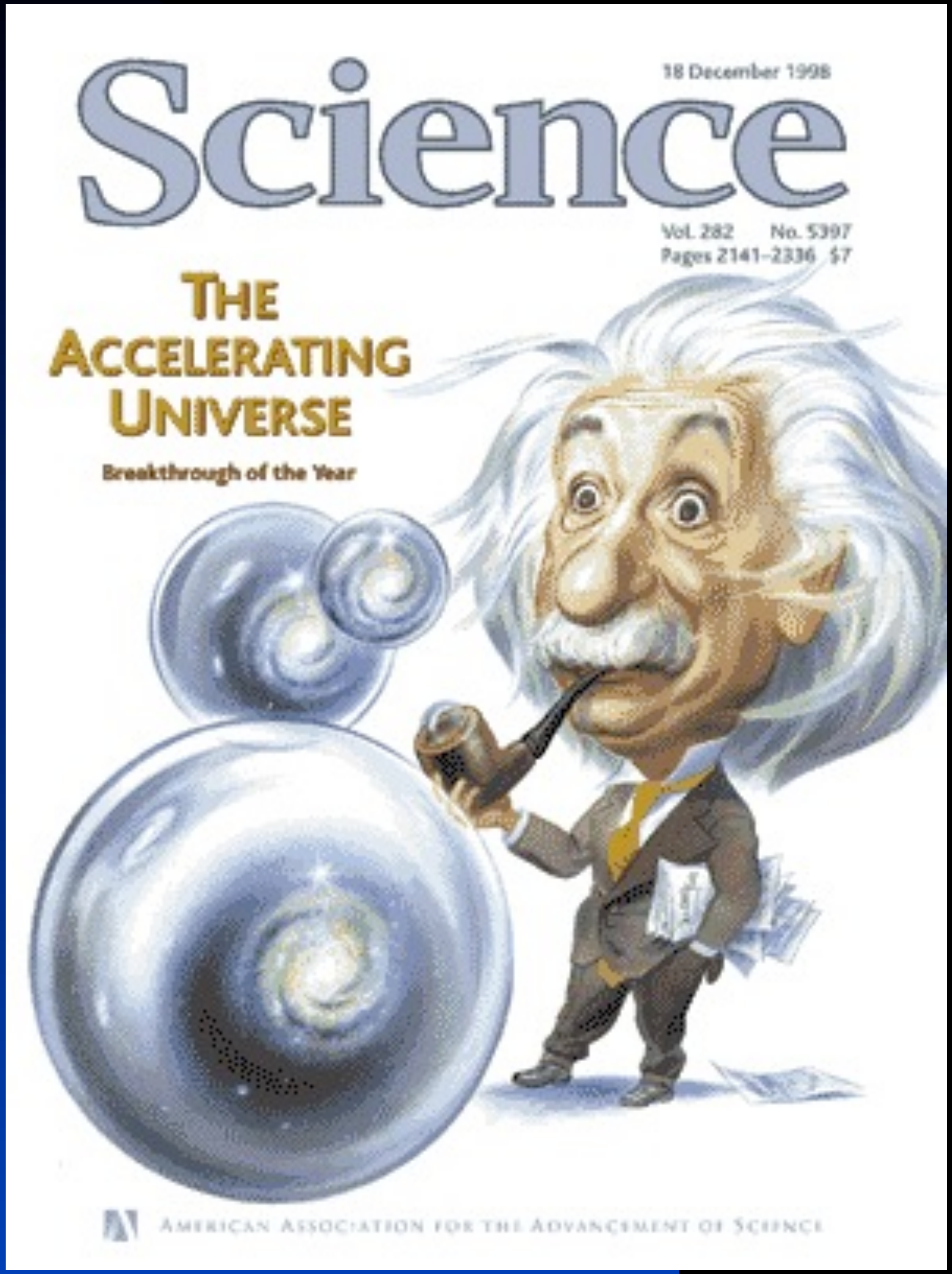
Ed Copeland -- Nottingham University

1. Models of Lambda -- and problems
2. Scalar field models -- and problems
3. Modified gravity -- and problems

Higgs Centre for Theoretical Physics, Edinburgh

The Ides of March -- Mar 15th 2013 -- Red Nose Day

Science Magazine -- Breakthrough of the year -- Dec 1998



``Einstein watches in surprise as a universe expands exponentially, its galaxies rushing apart ever faster. Evidence for an accelerating universe, the Breakthrough of the Year for 1998, resurrects Einstein's discarded idea of an energy called lambda, or λ, which counteracts gravity and pushes space apart.”

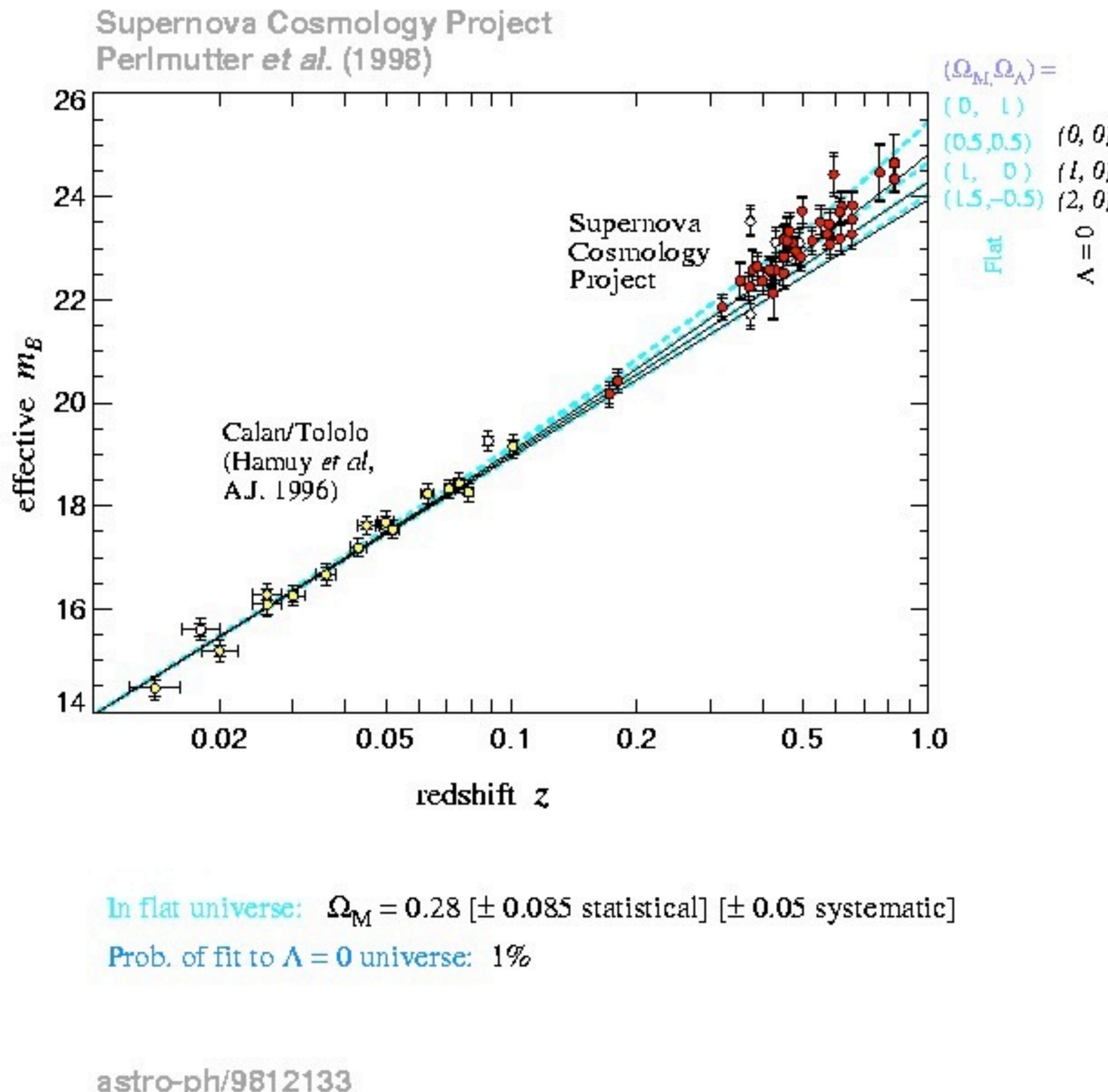
Led to the Nobel Prize for Physics in 2011

Saul Perlmutter - 0.5

Brian P Schmidt - 0.25

Adam Riess - 0.25

"for the discovery of the accelerating expansion of the Universe through observations of distant supernovae"



... and yet we still really have little idea what is causing this acceleration.

Is it a cosmological constant, an evolving scalar field, evidence of modifications of General Relativity on large scales or something yet to be dreamt up ?

Some basic equations

Friedmann:

$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{8\pi}{3} G\rho - \frac{k}{a^2} + \frac{\Lambda}{3}$$

a(t) depends on matter.

$$\text{Energy density } \rho(t): \quad \text{Pressure } p(t)$$

$$\text{Related through}: p = w\rho$$

w=1/3 – Rad dom: w=0 – Mat dom: w=-1– Vac dom

Eqns (Λ=0):

Friedmann + Fluid conservation

$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{8\pi}{3} G\rho - \frac{k}{a^2}$$

$$\dot{\rho} + 3(\rho + p)\frac{\dot{a}}{a} = 0$$

Combine

$$\frac{\ddot{a}}{a} = -\frac{8\pi}{3}G(\rho + 3p) - - - Accn$$

$$If \ \rho + 3p < 0 \Rightarrow \ddot{a} > 0$$

$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{8\pi}{3}G\rho - \frac{k}{a^2}$$

$$\dot{\rho} + 3(\rho + p)\frac{\dot{a}}{a} = 0$$

$$\rho(t) = \rho_0 \left(\frac{a}{a_0}\right)^{-3(1+w)} \quad ; a(t) = a_0 \left(\frac{t}{t_0}\right)^{\frac{2}{3(1+w)}}$$

$$\mathrm{RD} \quad : \quad w = \frac{1}{3} : \rho(t) = \rho_0 \left(\frac{a}{a_0}\right)^{-4} \ ; \ a(t) = a_0 \left(\frac{t}{t_0}\right)^{\frac{1}{2}}$$

$$\mathrm{MD} \quad : \quad w = 0 : \rho(t) = \rho_0 \left(\frac{a}{a_0}\right)^{-3} \ ; \ a(t) = a_0 \left(\frac{t}{t_0}\right)^{\frac{2}{3}}$$

$$\mathrm{VD} \quad : \quad w = -1 : \rho(t) = \rho_0 \ ; \ a(t) \propto e^{Ht}$$

A neat equation

$$\rho_c(t) \equiv \frac{3H^2}{8\pi G} \quad ; \quad \Omega(t) \equiv \frac{\rho}{\rho_c}$$

Friedmann eqn

$$\Omega_m + \Omega_\Lambda + \Omega_k = 1$$

Ω_m - baryons, dark matter, neutrinos, electrons, radiation ...

Ω_Λ - dark energy ; Ω_k - spatial curvature

$$\rho_c(t_0) \equiv 1.88h^2 * 10^{-29}\,\mathrm{gcm}^{-3}$$

Critical density

Current bounds on H(z) -- Komatsu et al 2010 - (WMAP7+BAO+SN)

$$\mathbf{H^2(z) = H_0^2 \left(\Omega_r (1+z)^4 + \Omega_m (1+z)^3 + \Omega_k (1+z)^2 + \Omega_{de} \exp\left(3 \int_0^z \frac{1+w(z')}{1+z'} dz' \right) \right)}$$

(Expansion rate) -- H_0=70.4 ± 1.3 km/s/Mpc

(radiation) -- $\Omega_r = (8.5 \pm 0.3) \times 10^{-5}$

(baryons) -- $\Omega_b = 0.0456 \pm 0.0016$

(dark matter) -- $\Omega_m = 0.227 \pm 0.014$

(curvature) -- $\Omega_k < 0.008$ (95%CL)

(dark energy) -- $\Omega_{de} = 0.728 \pm 0.015$

(de eqn of state) -- $1+w = 0.001 \pm 0.057$ -- looks like a cosm const.

If allow variation of form : $w(z) = w_0 + w' z/(1+z)$ then
$w_0 = -0.93 \pm 0.12$ and $w' = -0.38 \pm 0.65$ (68% CL)

Note -- important because distance measurements often rely on assumptions made about background cosmology.

Many approaches to Dark Energy:

- A true cosmological constant -- but why this value?
- Time dependent solutions arising out of evolving scalar fields -- Quintessence/K-essence.
- Modifications of Einstein gravity leading to acceleration today.
- Anthropic arguments.
- Perhaps GR but Universe is inhomogeneous.
- Hiding the cosmological constant -- its there all the time but just doesn't gravitate

Early evidence for a cosmological constant type term.

1987: Weinberg argued that anthropically ρ_{vac} could not be too large and positive otherwise galaxies and stars would not form. It should be not be very different from the mean of the values suitable for life which is positive, and he obtained $\Omega_{vac} \sim 0.6$

1990: Observations of LSS begin to kick in showing the standard $\Omega_{CDM} = 1$ struggling to fit clustering data on large scales, first through IRAS survey then through APM (Efstathiou et al)

1990: Efstathiou, Sutherland and Maddox - Nature (238) -- explicitly suggest a cosmology dominated today by a cosmological constant with $\Omega_{vac} < 0.8$!

1998: Type Ia SN show striking evidence of cosm const and the field takes off.

Why not simply stick with a cosmological constant and be done with it?

The CC gravitates in General Relativity:

$$\mathcal{L} = \sqrt{-g}\left(\frac{R}{16\pi G} - \rho_{\rm vac}\right)$$

$$G_{\mu\nu} = -8\pi G \rho_{\rm vac} g_{\mu\nu}$$

Now: $$\rho_{\rm vac}^{\rm obs} \ll \rho_{\rm vac}^{\rm theory}$$

Just as well because anything much bigger than we have and the universe would have looked a lot different to what it does look like. In fact structures would not have formed in it.

Estimate what the vacuum energy should be :

$$\rho_{\text{vac}}^{\text{theory}} \sim \rho_{\text{vac}}^{\text{bare}}$$

$$+$$

zero point energies of each particle

$$+$$

contributions from phase transitions in the early universe

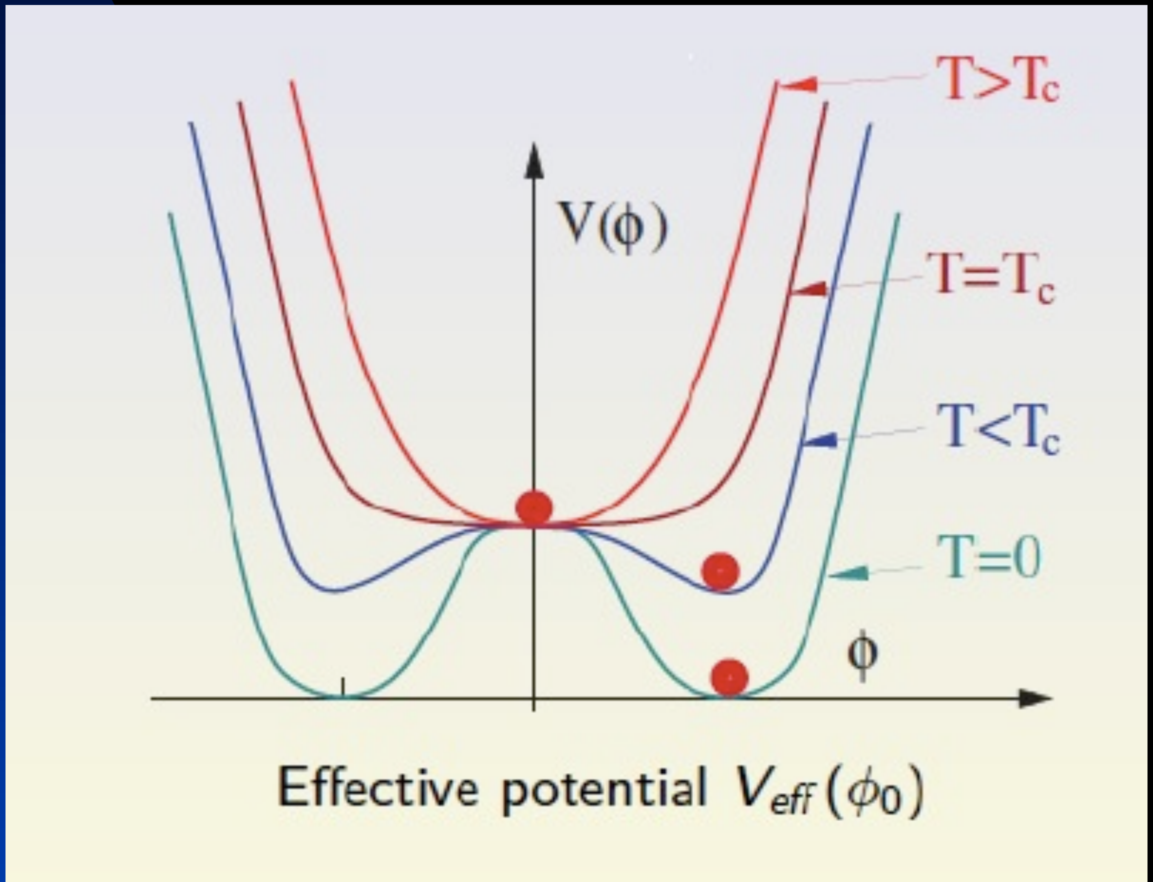
zero point energies of each particle

For many fields (i.e. leptons, quarks, gauge fields etc...):

$$< \rho > = \frac{1}{2} \sum_{\text{fields}} g_i \int_0^{\Lambda_i} \sqrt{k^2 + m^2} \frac{d^3k}{(2\pi)^3} \simeq \sum_{\text{fields}} \frac{g_i \Lambda_i^4}{16\pi^2}$$

where g_i are the dof of the field (+ for bosons, - for fermions).

contributions from phase transitions in the early universe



$$\Delta V_{\rm ewk} \sim (200\ \mathrm{GeV})^4$$

$$\Delta V_{\rm QCD} \sim (0.3\ \mathrm{GeV})^4$$

Quantum Gravity cut-off	$(10^{18}\ \mathrm{GeV})^4$	fine tuning to 120 decimal places
SUSY cut-off	$(\mathrm{TeV})^4$	fine tuning to 60 decimal places
EWK phase transition	$(200\mathrm{GeV})^4$	fine tuning to 56 decimal places
QCD phase transition	$(0.3\mathrm{GeV})^4$	fine tuning to 44 decimal places
Muon	$(100\mathrm{MeV})^4$	
electron	$(1\ \mathrm{MeV})^4$	fine tuning to 36 decimal places
	$(\mathrm{meV})^4$	Observed value of the effective cosmological constant today !

String - theory -- where are the realistic models?

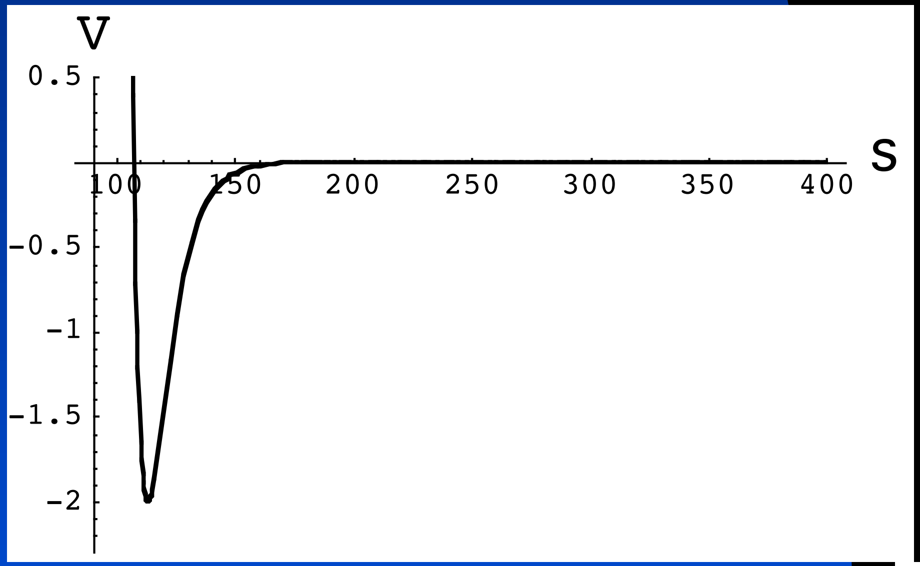
`No go' theorem: forbids cosmic acceleration in cosmological solutions arising from compactification of pure SUGR models where internal space is time-independent, non-singular compact manifold without boundary --[Gibbons]

Avoid no-go theorem by relaxing conditions of the theorem.

1. Allow internal space to be time-dependent scalar fields (radion)
2. Brane world set up require uplifting terms to achieve de Sitter vacua hence accn

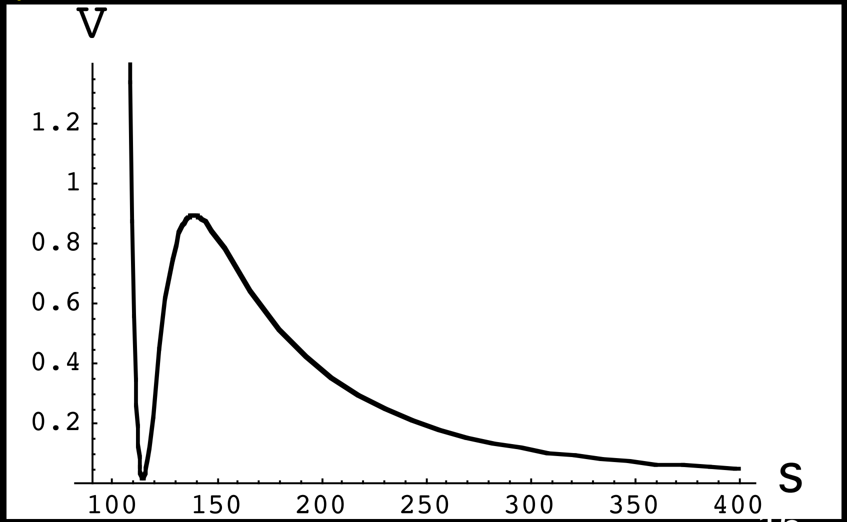
Example of stabilised scenario: Metastable de Sitter string vacua in TypeIIB string theory, based on stable highly warped IIB compactifications with NS and RR three-form fluxes. [Kachru, Kallosh, Linde and Trivedi 2003]

Metastable minima arises from adding positive energy of anti-D3 brane in warped Calabi-Yau space.



AdS minimum

$$V_{\mathrm{KKLT}} = V_{\mathrm{AdS}} + \frac{D}{\sigma^2}$$



Metastable dS minimum

The String Landscape approach

Type IIB String theory compactified from 10 dimensions to 4.

Internal dimensions stabilised by fluxes. Assumes natural AdS vacuum uplifted to de Sitter vacuum through additional fluxes !

Many many vacua ~ 10^{500} !

Typical separation ~ $10^{-500}\ \Lambda_{pl}$

Assume randomly distributed, tunnelling allowed between vacua --> separate universes .

Anthropic : Galaxies require vacua < $10^{-118}\ \Lambda_{pl}$ [Weinberg] Most likely to find values not equal to zero!

[Witten 2008]

Landscape gives a realisation of the multiverse picture.

There isn't one true vacuum but many so that makes it almost impossible to find our vacuum in such a Universe which is really a multiverse.

So how can we hope to understand or predict why we have our particular particle content and couplings when there are so many choices in different parts of the universe, none of them special ?

This sounds like bad news, we will rely on anthropic arguments to explain it through introducing the correct measures and establishing peaks in probability distributions.

Or perhaps, it isn't a cosmological constant, but a new field such as Quintessence which will eventually drive us to a unique vacuum with zero vacuum energy -- that too has problems, such as fifth force constraints, as we will see.

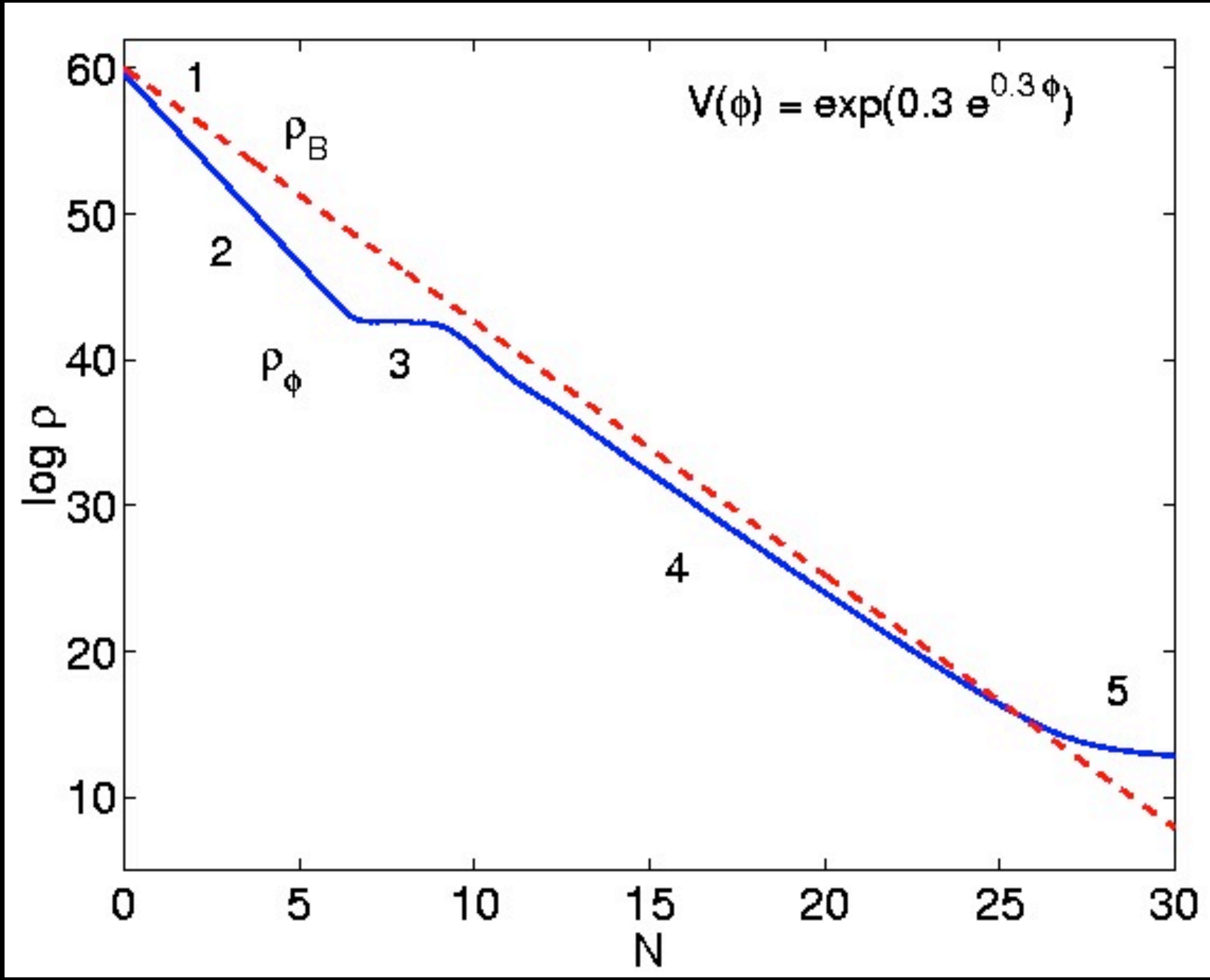
Slowly rolling scalar fields -- Quintessence

As of 14 Mar 2013, can really use this language !

Peebles and Ratra; Wetterich; Ferreira and Joyce

Zlatev, Wang and Steinhardt

1. PE → KE
2. KE dom scalar field energy den.
3. Const field.
4. Attractor solution: almost const ratio KE/ PE.
5. PE dom.

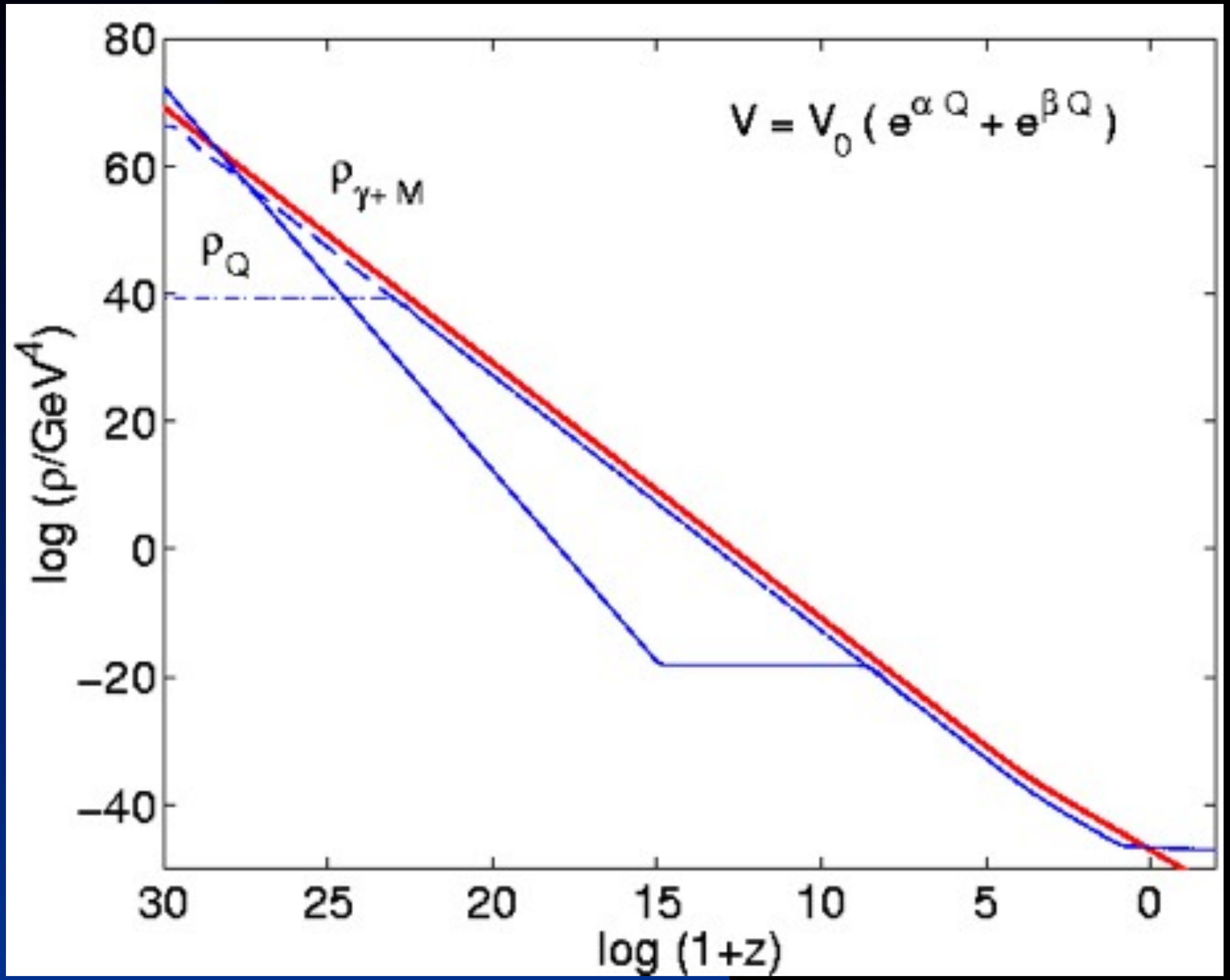


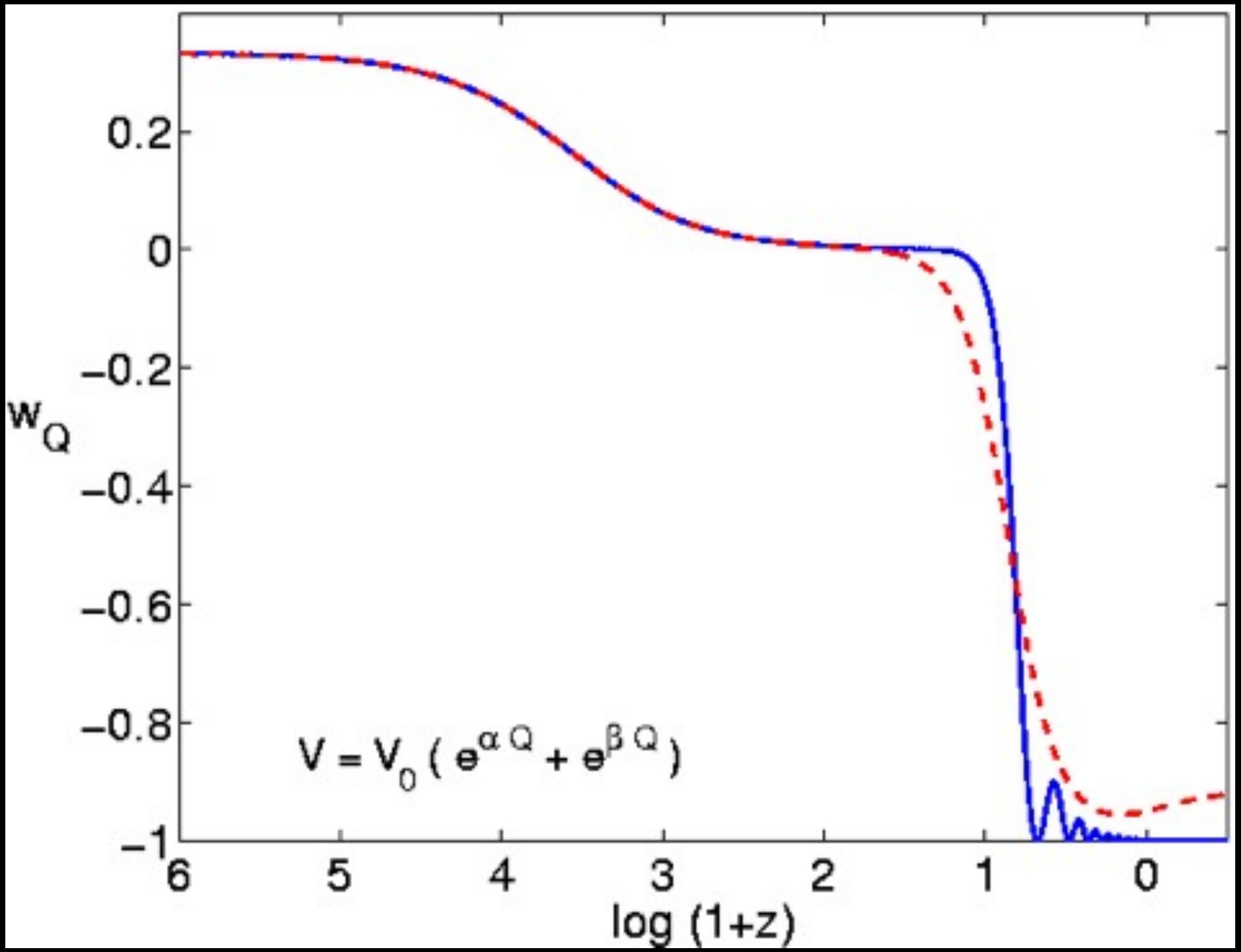
Nunes

Attractors make initial conditions less important

$$V(\phi) = V_1 + V_2 = V_{01}e^{-\kappa\lambda_1\phi} + V_{02}e^{-\kappa\lambda_2\phi}$$

EC and Nunes





$\alpha = 20; \beta = 0.5$

Scaling for wide range of i.c.

Fine tuning: $V_0 \approx \rho_\phi \approx 10^{-47}\ \text{GeV}^4 \approx (10^{-3}\ \text{eV})^4$

Mass: $m \approx \sqrt{\frac{V_0}{M_{pl}^2}} \approx 10^{-33}\ \text{eV}$

Fifth force !

1. Chameleon fields [Khoury and Weltman (2003) …]

Key idea: in order to avoid fifth force type constraints on Quintessence models, have a situation where the mass of the field depends on the local matter density, so it is massive in high density regions and light (m~H) in low density regions (cosmological scales).

2. Phantom fields [Caldwell (2002) …]

The data does not rule out w<-1. Can not accommodate in standard quintessence models but can by allowing negative kinetic energy for scalar field (amongst other approaches).

3. K-essence [Armendariz-Picon et al …]

Scalar fields with non-canonical kinetic terms. Advantage over Quintessence through solving the coincidence model?

Long period of perfect tracking, followed by domination of dark energy triggered by transition to matter domination -- an epoch during which structures can form. Similar fine tuning to Quintessence.

4. Interacting Dark Energy [Kodama & Sasaki (1985), Wetterich (1995), Amendola (2000) + others…]

Idea: why not directly couple dark energy and dark matter?

$$\begin{aligned} \text{Ein eqn} &: \quad G_{\mu\nu} = 8\pi G T_{\mu\nu} \\ \text{General covariance} &: \quad \nabla_\mu G^\mu_\nu = 0 \to \nabla_\mu T^\mu_\nu = 0 \end{aligned}$$

$$T_{\mu\nu} = \sum_i T^{(i)}_{\mu\nu} \to \nabla_\mu T^{\mu\,(i)}_\nu = -\nabla_\mu T^{\mu\,(j)}_\nu \;\; \text{is ok}$$

Couple dark energy and dark matter fluid in form:

$$\begin{aligned} \nabla_\mu T^{\mu\,(\phi)}_\nu &= \sqrt{\frac{2}{3}}\kappa\beta(\phi)T^{\alpha\,(m)}_\alpha \nabla_\nu\phi \\ \nabla_\mu T^{\mu\,(m)}_\nu &= -\sqrt{\frac{2}{3}}\kappa\beta(\phi)T^{\alpha\,(m)}_\alpha \nabla_\nu\phi \end{aligned}$$

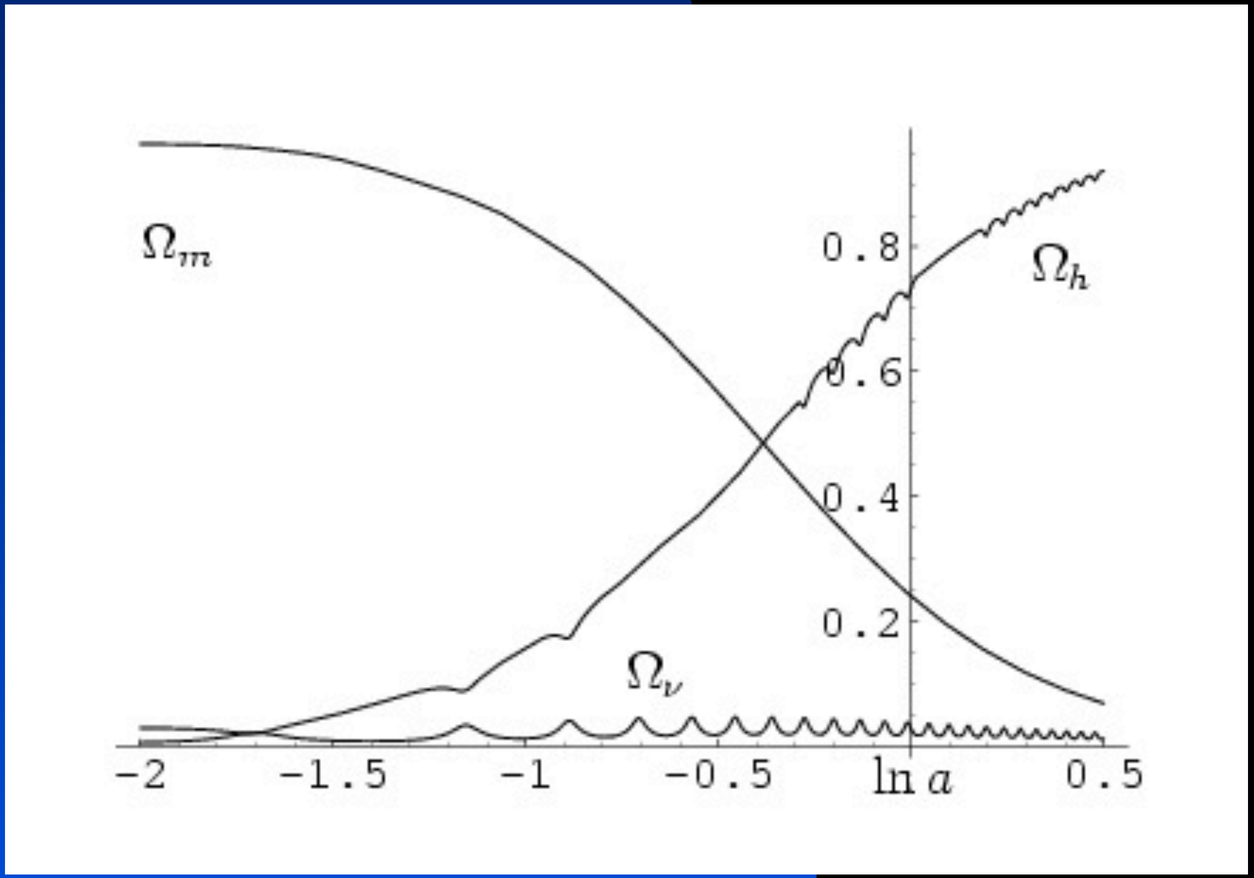
Ex: Including neutrinos -- 2 distinct DM families -- resolve coincidence problem [Amendola et al (2007)]

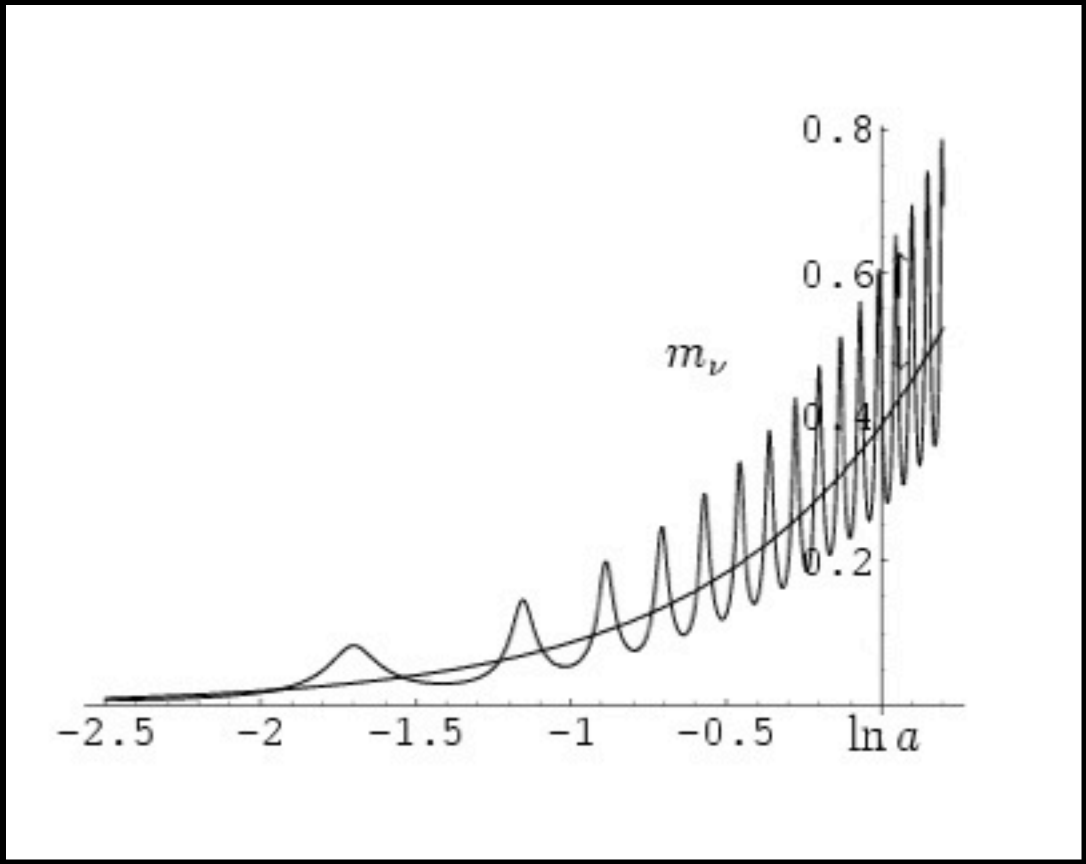
Depending on the coupling, find that the neutrino mass grows at late times and this triggers a transition to almost static dark energy.

Trigger scale set by time when neutrinos become non-rel

$$[\rho_h(t_0)]^{\frac{1}{4}} = 1.07 \left(\frac{\gamma m_\nu(t_0)}{eV}\right)^{\frac{1}{4}} 10^{-3} eV$$

$$w_0 \approx -1 + \frac{m_\nu(t_0)}{12\text{eV}}$$





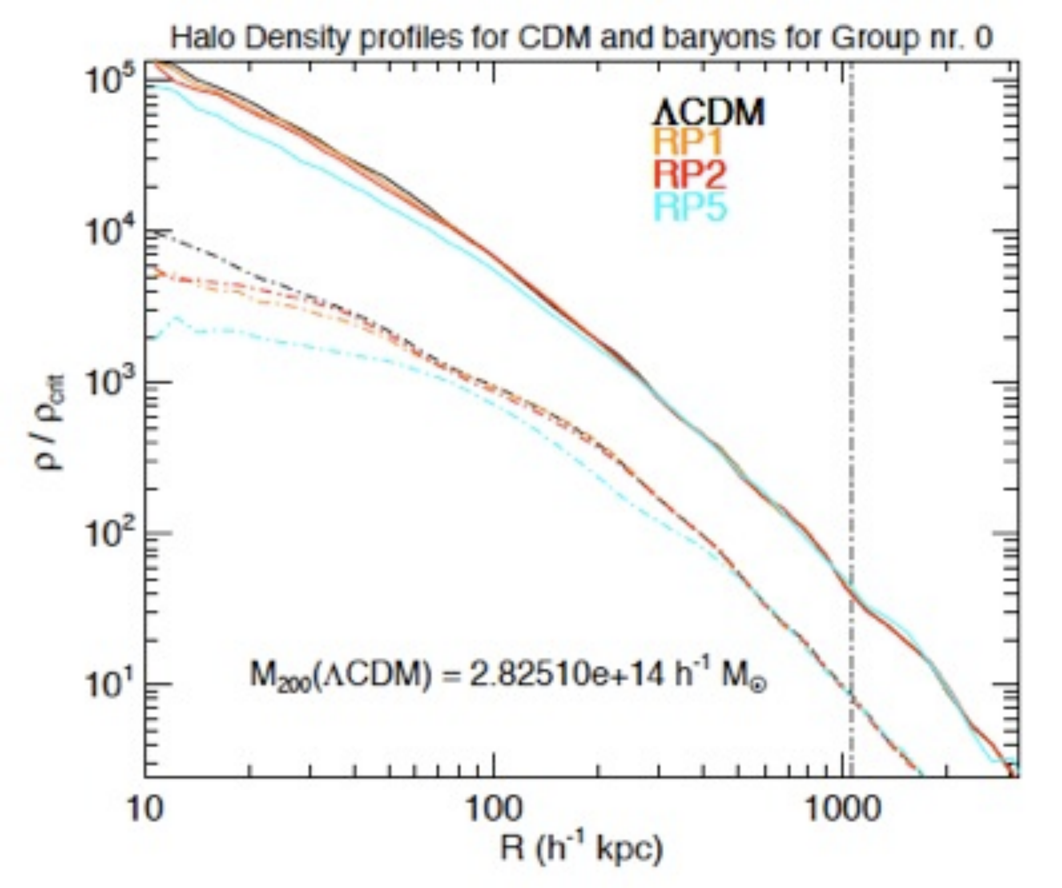
m_ν

Perturbations in Interacting Dark Energy Models [Baldi et al (2008), Tarrant et al (2010)]

Perturb everything linearly : Matter fluid example

$$\ddot{\delta}_c + \underbrace{\left(2H - 2\beta\frac{\dot{\phi}}{M}\right)}_{\text{extra friction}} \dot{\delta}_c - \frac{3}{2}H^2[(1 + \underbrace{2\beta^2}_{\text{modified grav interaction}})\underbrace{\Omega_c\delta_c}_{\text{vary DM particle mass}} + \Omega_b\delta_b] = 0$$

Include in simulations of structure formation : GADGET [Springel (2005)]



Density decreases as coupling β increases

Halo mass function modified.

Halos remain well fit by NFW profile.

Density decreases compared to ΛCDM as coupling β increases.

Scale dep bias develops from fifth force acting between CDM particles. enhanced as go from linear to smaller non-linear scales.

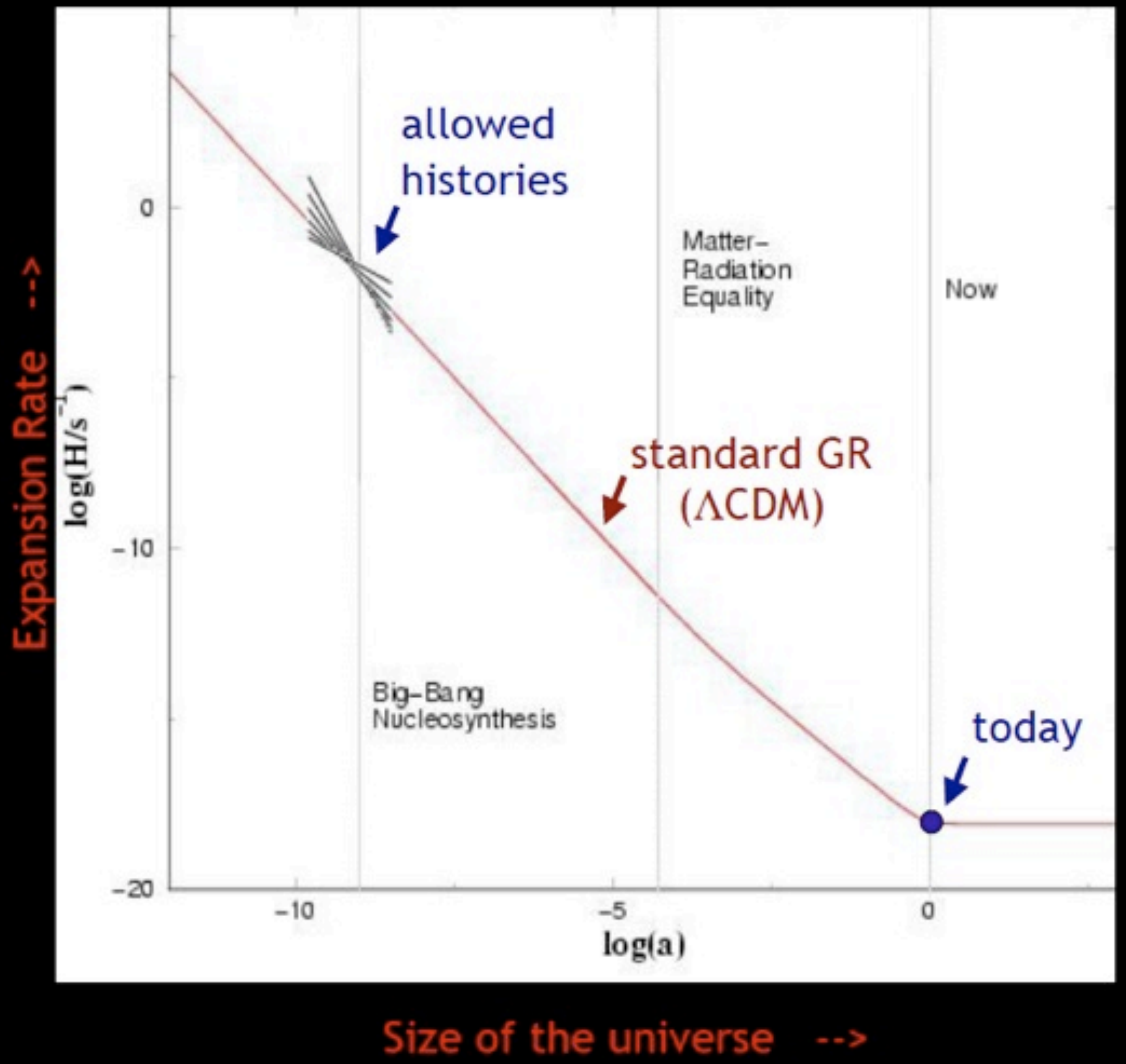
Still early days -- but this is where there should be a great deal of development.

Should we be modifying gravity instead of looking for dark energy ?

Has become a big industry but it turns out to be hard to do too much to General Relativity without falling foul of data.

BBN occurred when the universe was about one minute old, about one billionth its current size. It fits well with GR and provides a test for it in the early universe.

Any alternative had better deliver the same successes not deviate too much at early times, but turn on at late times .



[Carroll & Kaplinghat 2001]

Any theory deviating from GR must do so at late times yet remain consistent with Solar System tests. Potential examples include:

- f(R), f(G) gravity -- coupled to higher curv terms, changes the dynamical equations for the spacetime metric.

[Starobinski 1980, Carroll et al 2003, ...]

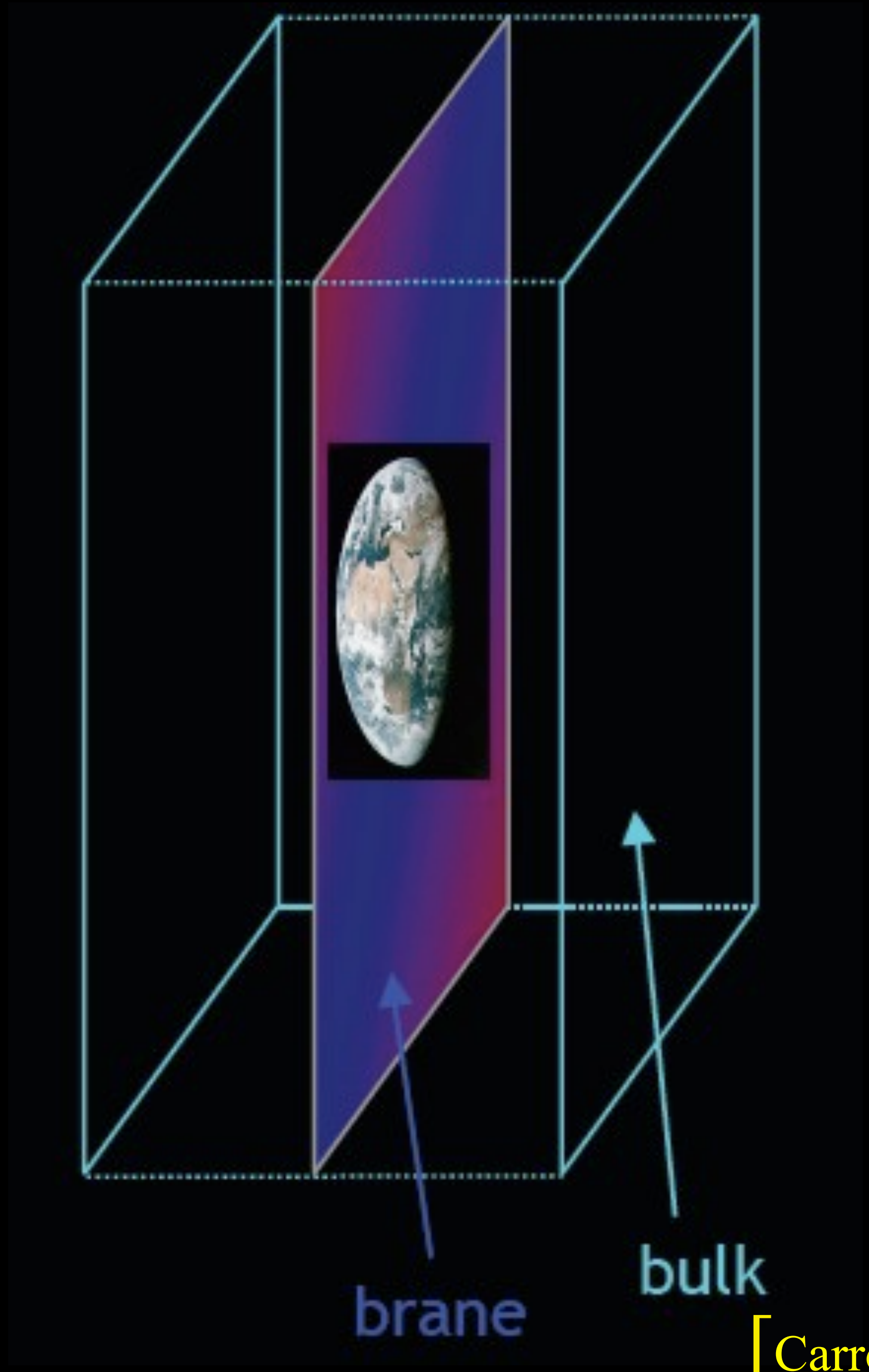
- Modified source gravity -- gravity depends on nonlinear function of the energy.

- Gravity based on the existence of extra dimensions -- DGP gravity

We live on a brane in an infinite extra dimension. Gravity is stronger in the bulk, and therefore wants to stick close to the brane -- looks locally four-dimensional.

Tightly constrained -- both from theory [ghosts] and observations

- Scalar-tensor theories including higher order scalar-tensor lagrangians -- recent examples being Gallileon models



[Carroll]

f (R) models [Lots and lots of people...]

$$S = \int \mathrm{d}^4x \sqrt{-g} \left[\frac{R + f(R)}{2\kappa^2} + \mathcal{L}_\mathrm{m} \right]$$

No Λ

Usually f (R) struggles to satisfy both solar system bounds on deviations from GR and late time acceleration. It brings in extra light degree of freedom --> fifth force constraints.

Ans: Make scalar dof massive in high density solar vicinity and hidden from solar system tests by chameleon mechanism.

Requires form for f (R) where mass of scalar is large and positive at high curvature.

Has to look like a standard cosmological constant

Designer f (R) or f(G) models [Hu and Sawicki (2007), ...]

Construct a model to satisfy observational requirements:

1. Mimic LCDM at high z as suggested by CMB

2. Accelerate univ at low z

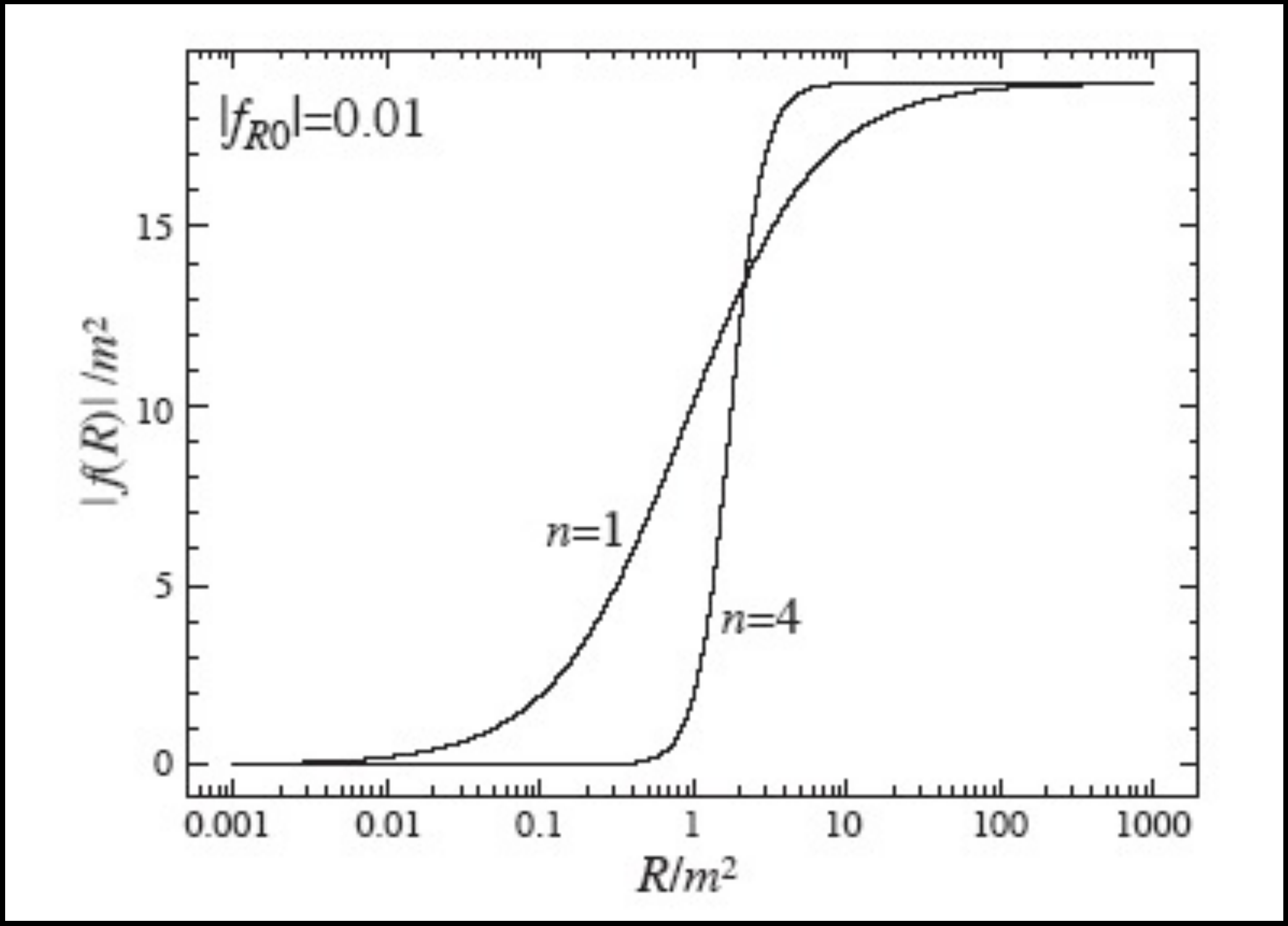
3. Include enough dof to allow for variety of low z phenomena

4. Include phenom of LCDM as limiting case.

$$\lim_{R\to\infty} f(R) = \text{const.}\,,$$
$$\lim_{R\to 0} f(R) = 0\,,$$

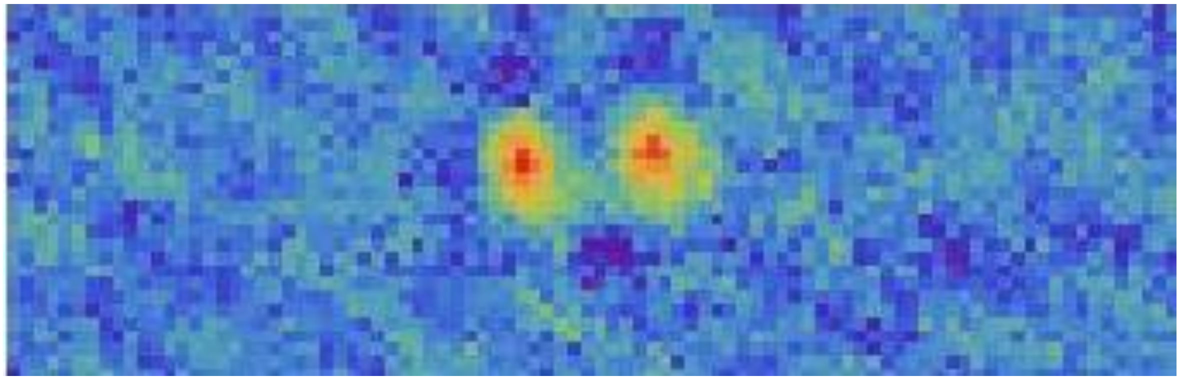
$$f(R) = -m^2 \frac{c_1 (R/m^2)^n}{c_2 (R/m^2)^n + 1}\,,$$

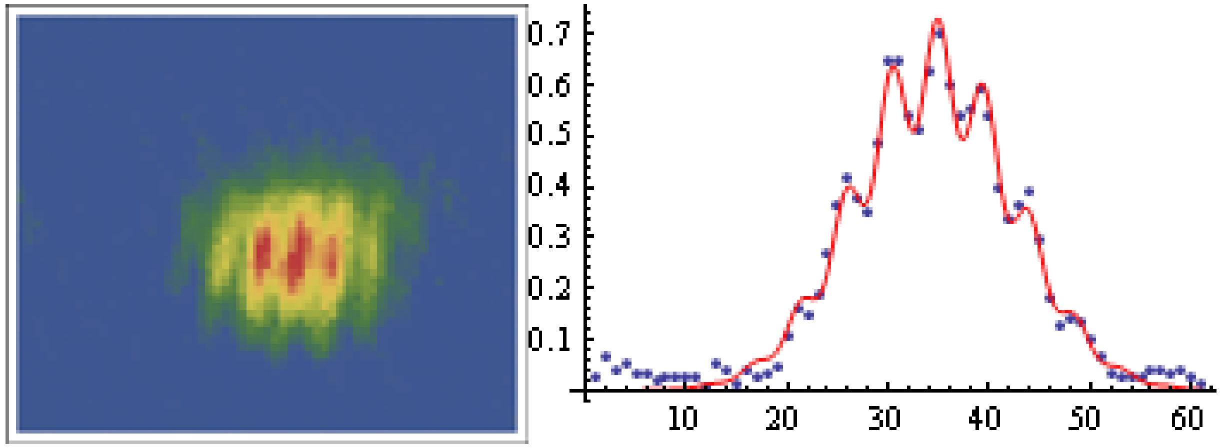
$$f_{RR} \equiv \frac{d^2 f(R)}{dR^2} > 0$$



Dark Energy Direct Detection Experiment [Burrage, EC, Hinds]

Coherent waves in Bose-Einstein condensates can be used for interferometry





Credit: Centre for Cold Matter, Imperial

Interference of waves in condensates at different heights has already detected gravitational effects

(Dimopoulos, Geraci 2003. Baumgärtner et al. 2010)

Interference of waves in condensates held in different environments can be used to directly detect screening mechanisms

What should we do to help determine the nature of DE ?

1. We need to define properly theoretically predicted observables, or determine optimum ways to parameterise consistency tests (i.e. how should we parameterise w(z)?)

2. Need to start including dynamical dark energy, interacting dark matter-dark energy and modified gravity models in large scale simulations.

3. Include the gastrophysics + star formation especially when considering baryonic effects in the non-linear regimes.

4. On the theoretical side, develop models that go beyond illustrative toy models. Are there examples of actual Landscape predictions? De Sitter vaccua in string theory is non trivial.

5. Recently massive gravity and galileon models have been developed which have been shown to be free of ghosts. What are their self-acceleration properties?

6. Will we be able to reconstruct the underlying Quintessence potential from observation?

7. Never mind evidence of evolution, will we ever be able to determine whether $w \neq -1$?

8. Look for alternatives, perhaps we can shield the cosmological constant from affecting the dynamics through self tuning-- The Fab Four

9. Given the complexity (baroque nature ?) of some of the models compared to that of say Λ, we should be using Bayesian model selection criterion to help determine the relevance of any one model.

Many more things to be done on a phenomenological and theoretical side.

Things are getting very exciting with the Dark Energy Survey beginning to take data and proposed longer term Euclid mission.

Self tuning - with the Fab Four

"The Beatles [have made] negligible contributions to cosmological theory"
PRL, June 2011

In GR the vacuum energy gravitates, and the theoretical estimate suggests that it gravitates too much.

Basic idea is to use self tuning to prevent the vacuum energy gravitating at all.

The cosmological constant is there all the time but is being dealt with by the evolving scalar field.

with Charmousis, Padilla and Saffin

PRL 108 (2012) 051101; PRD 85 (2012) 104040

with Padilla and Saffin

JCAP 1212 (2012) 026

Horndeski's theory: [G.W. Horndeski, Int. Jour. Theor. Phys. 10 (1974) 363-384

Most general scalar-tensor theory with second order field equations:

$$\begin{aligned}\mathcal{L}_H = & \ \delta^{ijk}_{\mu\nu\sigma}\Big[\kappa_1 \nabla^\mu \nabla_i \phi R_{jk}{}^{\nu\sigma} - \frac{4}{3}\kappa_{1,\rho}\nabla^\mu\nabla_i\phi\nabla^\nu\nabla_j\phi\nabla^\sigma\nabla_k\phi \\ & +\kappa_3\nabla_i\phi\nabla^\mu\phi R_{jk}{}^{\nu\sigma} - 4\kappa_{3,\rho}\nabla_i\phi\nabla^\mu\phi\nabla^\nu\nabla_j\phi\nabla^\sigma\nabla_k\phi\Big] \\ & +\delta^{ij}_{\mu\nu}\left[(F+2W)R_{ij}{}^{\mu\nu} - 4F_{,\rho}\nabla^\mu\nabla_i\phi\nabla^\nu\nabla_j\phi + 2\kappa_8\nabla_i\phi\nabla^\mu\phi\nabla^\nu\nabla_j\phi\right] \\ & -3[2(F+2W)_{,\phi}+\rho\kappa_8]\nabla_\mu\nabla^\mu\phi + \kappa_9(\phi,\rho),\end{aligned}$$

$$\rho = \nabla_\mu\phi\nabla^\mu\phi$$

$\kappa_1,\ \kappa_3,\ \kappa_8,\ \kappa_9 - -$Four indep func of$\phi$ andρ

W can be set to zero and F can be derived from κ's.

Equivalent to Deffayet et al, PRD80 (2009) 064015

(see also Kobayashi et al 1105.5723 [hep-th])

The action which leads to self tuning solutions can be rewritten in a more natural way in which we see how the scalar fields couple directly to various curvature invariants:

$$
\begin{aligned}
\mathcal{L}_{john} &= \sqrt{-g}V_{john}(\phi)G^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi \\
\mathcal{L}_{paul} &= \sqrt{-g}V_{paul}(\phi)P^{\mu\nu\alpha\beta}\nabla_\mu\phi\nabla_\alpha\phi\nabla_\nu\nabla_\beta\phi \\
\mathcal{L}_{george} &= \sqrt{-g}V_{george}(\phi)R \\
\mathcal{L}_{ringo} &= \sqrt{-g}V_{ringo}(\phi)\hat{G}
\end{aligned}
$$

where $\hat{G} = R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} - 4R_{\mu\nu}R^{\mu\nu} + R^2$

and $P^{\mu\nu\alpha\beta} = -\frac{1}{4}\varepsilon^{\mu\nu\lambda\sigma}\, R_{\lambda\sigma\gamma\delta}\, \varepsilon^{\alpha\beta\gamma\delta}$

In other words it can be seen to reside in terms of the four arbitrary potential functions of ϕ coupled to the curvature terms.

Covers most scalar field related modified gravity models studied to date.

Assume no derivative couplings to matter to avoid violation of Equivalence Principle.

Can assume matter only couples to metric.

Begin the Cosmology

$$g_{\mu\nu}dx^\mu dx^\nu = -dt^2 + a^2(t)\left[\frac{dr^2}{1-\kappa r^2} + r^2\, d\Omega_{(2)}\right]$$

$$L_H^{\rm eff}(a,\dot a,\phi,\dot\phi) = a^3 \sum_{n=0}^{3}\left(X_n - Y_n\frac{\kappa}{a^2}\right) H^n$$

$$\begin{aligned}
X_0 &= -\tilde{Q}_{7,\phi}\dot\phi + \kappa_9\\
Y_0 &= \tilde{Q}_{1,\phi}\dot\phi + 12\kappa_3\dot\phi^2 - 12F\\
X_1 &= -12F_{,\phi}\dot\phi + 3(Q_7\dot\phi - \tilde{Q}_7) + 6\kappa_8\dot\phi^3\\
Y_1 &= -Q_1\dot\phi + \tilde{Q}_1\\
X_2 &= 12\kappa_{1,\phi}\dot\phi^2 - 12\kappa_3\dot\phi^2 + 24\kappa_{3,\rho}\dot\phi^4\\
&\quad -12F - 24F_{,\rho}\dot\phi^2\\
X_3 &= 8\kappa_{1,\rho}\dot\phi^3
\end{aligned}$$

$$\begin{aligned}
Q_1 &= \frac{\partial\tilde{Q}_1}{\partial\dot\phi} = -12\kappa_1\\
Q_7 &= \frac{\partial\tilde{Q}_7}{\partial\dot\phi} = 6F_{,\phi} - 3\dot\phi^2\kappa_8
\end{aligned}$$

Friedmann equation: $$\mathcal{H}(a,\dot a,\phi,\dot\phi)=\frac{1}{a^3}\left[\dot a\frac{\partial L_H^{\rm eff}}{\partial\dot a}+\dot\phi\frac{\partial L_H^{\rm eff}}{\partial\dot\phi}-L_H^{\rm eff}\right]=-\rho_m$$

At most cubic in Hubble parameter H

$$\mu_3H^3+\mu_2H^2+\mu_1H+\mu_0=\rho_m$$

Scalar eom: $$\mathcal{E}(a,\dot a,\ddot a,\phi,\dot\phi,\ddot\phi) = \frac{\partial L_H^{\rm eff}}{\partial\phi}-\frac{d}{dt}\left[\frac{\partial L_H^{\rm eff}}{\partial\dot\phi}\right] = 0$$

Linear in both $\ddot\phi$ and $\ddot a$.

Self tuning in Horndeski.

1. Vacuum solution is always Minkowski whatever the vacuum energy

2. Solution remains Minkowski even after a phase transition where the vacuum energy changes instantaneously.

In other words the vacuum energy does not gravitate at all because of the influence of the evolving scalar field and curvature.

$$< \rho_m >_{\rm vac} = \rho_\Lambda, \quad H^2 = -\frac{\kappa}{a^2}, \quad \phi = \phi_\Lambda(t)$$

piecewise constant but discontinuous at transition

``On shell in a" always satisfied

continuous everywhere and not constant

Scalar field eqn of motion should be trivial ``on-shell-in-a''

The scalar is completely determined by the vacuum Friedmann equation.

In this self tuning vacuum:

1. the matter tells the scalar how to move - this requires that the ``on-shell-in-a'' gravity equation be dependent on $\dot{\phi}$

2. the scalar tells the spacetime not to curve, but crucially only in the vacuum - the scalar equation should not be independent of $\ddot{a}$

Some equations for Fab Four Cosmology:

$$
\begin{aligned}
\mathcal{L}_{john} &= \sqrt{-g}V_{john}(\phi)G^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi \\
\mathcal{L}_{paul} &= \sqrt{-g}V_{paul}(\phi)P^{\mu\nu\alpha\beta}\nabla_\mu\phi\nabla_\alpha\phi\nabla_\nu\nabla_\beta\phi \\
\mathcal{L}_{george} &= \sqrt{-g}V_{george}(\phi)R \\
\mathcal{L}_{ringo} &= \sqrt{-g}V_{ringo}(\phi)\hat{G}
\end{aligned}
$$

$$\hat{G} = R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} - 4R_{\mu\nu}R^{\mu\nu} + R^2$$

$$P^{\mu\nu\alpha\beta} = -\frac{1}{4}\varepsilon^{\mu\nu\lambda\sigma}\, R_{\lambda\sigma\gamma\delta}\, \varepsilon^{\alpha\beta\gamma\delta}$$

$$\mathcal{H}_{\text{john}} + \mathcal{H}_{\text{paul}} + \mathcal{H}_{\text{george}} + \mathcal{H}_{\text{ringo}} = -[\rho_\lambda + \rho_{\text{matter}}]$$

$$\mathcal{E}_{john} + \mathcal{E}_{paul} + \mathcal{E}_{george} + \mathcal{E}_{ringo} = 0$$

$$g_{\mu\nu}dx^\mu dx^\nu = -dt^2 + a^2(t)\left[\frac{dr^2}{1-\kappa r^2} + r^2\, d\Omega_{(2)}\right]$$

$$\mathcal{H}_{john} = 3V_{john}(\phi)\dot{\phi}^2\left(3H^2 + \frac{\kappa}{a^2}\right))$$

$$\mathcal{H}_{paul} = -3V_{paul}(\phi)\dot{\phi}^3 H\left(5H^2 + 3\frac{\kappa}{a^2}\right)$$

$$\mathcal{H}_{george} = -6V_{george}(\phi)\left[\left(H^2 + \frac{\kappa}{a^2}\right) + H\dot{\phi}\frac{V'_{george}}{V_{george}}\right]$$

$$\mathcal{H}_{ringo} = -24V'_{ringo}(\phi)\dot{\phi}H\left(H^2 + \frac{\kappa}{a^2}\right)$$

$$\mathcal{E}_{john} = 6\frac{d}{dt}\left[a^3V_{john}(\phi)\dot{\phi}\Delta_2\right] - 3a^3V'_{john}(\phi)\dot{\phi}^2\Delta_2$$

$$\mathcal{E}_{paul} = -9\frac{d}{dt}\left[a^3V_{paul}(\phi)\dot{\phi}^2 H\Delta_2\right] + 3a^3V'_{paul}(\phi)\dot{\phi}^3 H\Delta_2$$

$$\mathcal{E}_{george} = -6\frac{d}{dt}\left[a^3V'_{george}(\phi)\Delta_1\right] + 6a^3V''_{george}(\phi)\dot{\phi}\Delta_1 + 6a^3V'_{george}(\phi)\Delta_1^2$$

$$\mathcal{E}_{ringo} = -24V'_{ringo}(\phi)\frac{d}{dt}\left[a^3\left(\frac{\kappa}{a^2}\Delta_1 + \frac{1}{3}\Delta_3\right)\right]$$

Note each term vanishes identically when $\Delta = 0$

where:

$$\Delta_n = H^n - \left(\frac{\sqrt{-\kappa}}{a}\right)^n$$

fab four cosmology

TABLE I: Examples of interesting cosmological behaviour for various fixed points with $\sigma = 0$.

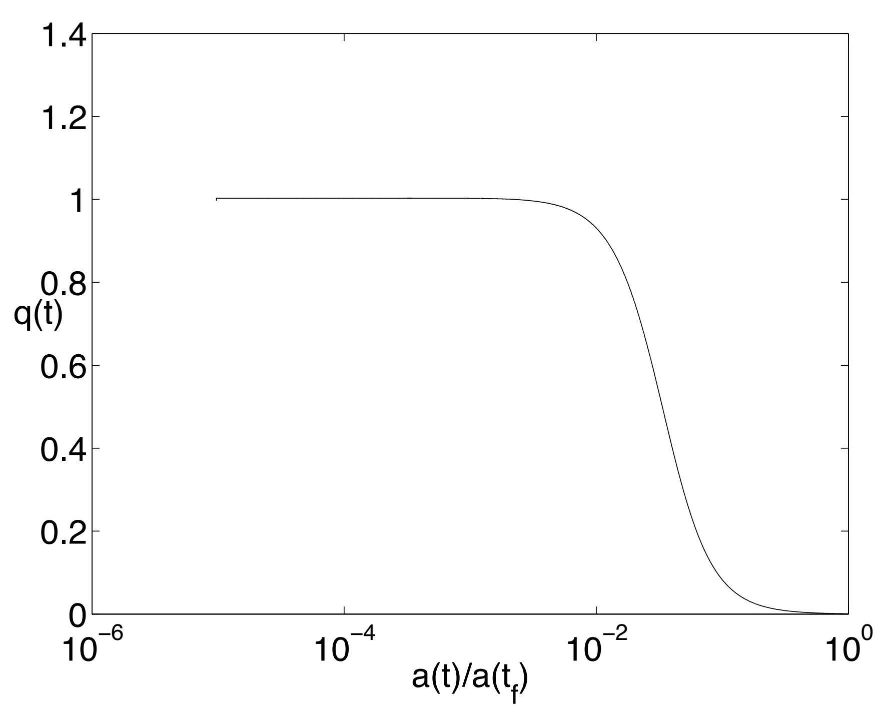
Case	cosmological behaviour	$V_j(\phi)$	$V_p(\phi)$	$V_g(\phi)$	$V_r(\phi)$
Stiff fluid	$H^2 \propto 1/a^6$	$c_1\phi^{\frac{4}{\alpha}-2}$	$c_2\phi^{\frac{6}{\alpha}-3}$	0	0
Radiation	$H^2 \propto 1/a^4$	$c_1\phi^{\frac{4}{\alpha}-2}$	0	$c_2\phi^{\frac{2}{\alpha}}$	$-\frac{\alpha^2}{8}c_1\phi^{\frac{4}{\alpha}}$
Curvature	$H^2 \propto 1/a^2$	0	0	0	$c_1\phi^{\frac{4}{\alpha}}$
Arbitrary	$H^2 \propto a^{2h}, \quad h \neq 0$	$c_1(1+h)\phi^{\frac{4}{\alpha}-2}$	0	0	$-\frac{\alpha^2}{16}h(3+h)c_1\phi^{\frac{4}{\alpha}}$

$$q = -\frac{a\ddot{a}}{\dot{a}^2}$$

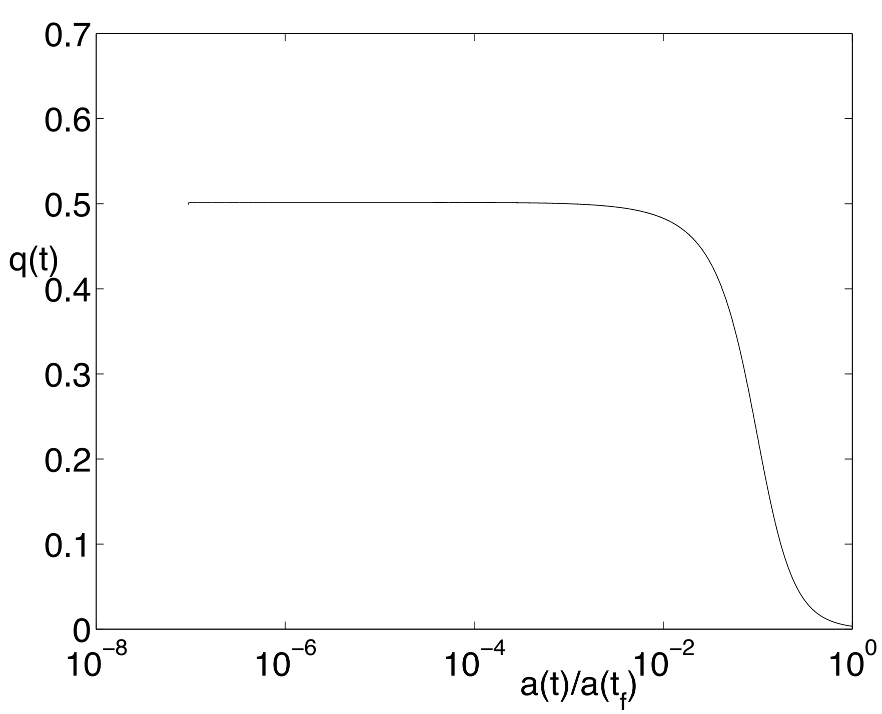
$$a \sim t^p \sim t^{-1/h}$$

$$q = -\frac{p(p-1)}{p^2} = -(1+h)$$

"radiation"



"matter"



Stability? see Kobayashi et al: 1105.5723; De Felice et al: 1108.4242

Tensor pertns: $$\frac{1}{8}\int dt d^3x a^3 \left[\mathcal{G}_T \dot{h}_{ij}^2 - \frac{\mathcal{F}_T}{a^2}(\nabla h_{ij})^2\right]$$

Scalar pertns: $$\int dt d^3x a^3 \left[\mathcal{G}_S \dot{\xi}^2 - \frac{\mathcal{F}_S}{a^2}(\nabla \xi)^2\right]$$

Find stable $F_T>0$, $G_T>0$, $F_S>0$, $G_S>0$ for say:

Case	cosmological behaviour	$V_j(\phi)$	$V_p(\phi)$	$V_g(\phi)$	$V_r(\phi)$
Matter I	$H^2 \propto 1/a^3$	$c_1\phi^{\hat{n}+4}$	$c_2\phi^{\hat{n}+6}$	0	$\frac{2\hat{n}-3}{16(2\hat{n}+7)(\hat{n}+6)}c_1\phi^{\hat{n}+6}$
Matter II	$H^2 \propto 1/a^3$	$c_1\phi^{\hat{n}+4}$	0	$c_2\phi^{\hat{n}+3}$	$-\frac{(\hat{n}+3)(2\hat{n}+5)}{8(2\hat{n}+7)(\hat{n}+6)}c_1\phi^{\hat{n}+6}$

Also true for radiation and inflation ...

But can we put them together somehow ?

Possible to have a self tuning `classical' solution in which the system adjusts itself to the Minkowski vacuum irrespective of the magnitude of the cosmological constant and whether it changes. It relies on breaking the assumption of Poincare invariance demanded by Weinberg in his original no-go theorem. In particular we have to have the scalar field evolving in time.

Remains to be seen whether we can satisfy solar system tests and obtain realistic cosmological solutions.

The role of quantum corrections remains to be evaluated (although initial calculaations suggest they can be controlled). They could spoil the party, although we note the crucial role played in the geometrical structure of the model.

There is always the question of stability of the solutions

Gregory Hormdeski left physics in 1981 having obtained a faculty position at Waterloo, Canada. He was on leave in Amsterdam and went to a Van Gogh exhibition.

His love of art was too strong and the inspiration he took from Van Gogh overpowering. He now works from his studio in Santa Fe.

Summary

•Data currently consistent with a pure cosmological constant -- but why that value?

•Why is the universe inflating today?

•Is w = - 1, the cosmological constant ? If not, then what value has it?

•Is w(z) -- dynamical. How should this be parameterized when considering surveys like DES and Euclid?

•New Gravitational Physics -- perhaps modifying Einstein equations on large scales? Key differences arising in perturbations.

•Perhaps we will only be able to determine it from anthropic arguments and not from fundamental theory.

•or -- we can avoid the need for a lambda term all together?