Generating f_{NL} at $\ell < 60$

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Originally planned talk

BICEP2, THE CURVATURE PERTURBATION AND SUSY

based on 1403.7323 but not given

because

B POLARIZATION = TENSOR + DUST

and

Planck DUST is consistent with TENSOR = 0

Mortonson/Seljak 1405.5857, Flauger/Hill/Spergel 1405.7351

This talk: Generating $f_{\rm NL}$ at $\ell < 60$

Based on 1405.3562

SUMMARY

For CMB at $\ell < 60$, ie. scale $k^{-1} = x_{\rm ls}/\ell > 200\,$Mpc;

- OBSERVATION gives
  - Dipole asymmetry
  - Reduced power

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- Then get ADDED EFFECTS
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- Upper bound on quadrupole $\Rightarrow$ Lower bound on $f_{\rm NL}$
- $f_{\rm NL}$ may **already** be present in data

Dipole asymmetry

Let's assume, in each small patch of sky,

$$\Delta T(\hat{\mathbf{n}}) = (1 + A_\ell(\hat{\mathbf{p}} \cdot \hat{\mathbf{n}})\, \Delta T_{\rm iso}(\hat{\mathbf{n}}),$$

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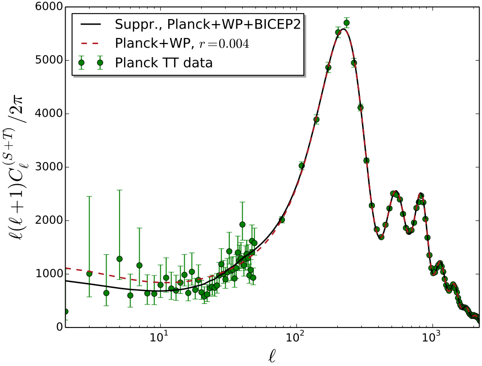
But same assumption for $\ell = 601$ to 2048 gives $A < 0.0045$ Fender & Hotchkiss 1307.6069

Lack of power



Upper line is ΛCDM model with constant $n_{\rm s} = 0.96$. Lower lower obtained by Contaldi, Pelos & Sorbo, 1403.4596, assuming $\mathcal{P}_\zeta(k) = \mathcal{P}_\zeta(k_0)\,(1 - K\theta(k_0 - k))$ with $K = 0.35$ and $k_0^{-1} = x_{\rm ls}/38$.

My assumption for $\mathcal{P}_z(k)$

To account for both effects take, within small region at location $\mathbf{x}$

$$\zeta_{\mathbf{k}}(\mathbf{x}) = \left(1 - C(k) + A(k)\hat{\mathbf{p}} \cdot \mathbf{x}/x_{\rm ls}\right) \zeta_{\mathbf{k}}$$

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$$\begin{aligned} A(k) &= A\theta(k_* - k), \quad A = 0.07 \\ C(k) &= C\theta(k_* - k), \quad C = 0.17 \end{aligned}$$

with say $k_* = x_{\rm ls}/60$. Corresponds to

$$\mathcal{P}_\zeta(k,\mathbf{x}) \simeq \left(1 - 2C(k) + 2A(k)\hat{\mathbf{p}}\cdot\mathbf{x}/x_{\rm ls}\right)\mathcal{P}_\zeta(k)$$

Contributions to ζ

Assume ζ generated by curvaton-type field with super-horizon contribution $\delta\chi_{\rm L}$.

$$
\begin{aligned}
\zeta(\mathbf{x}) &= N(\chi(\mathbf{x})) - N(\chi_0) \\
&= N'(\chi_0))\left(\delta\chi_{\rm S}(\mathbf{x}) + \delta\chi_{\rm L}(\mathbf{x})\right) + \frac{1}{2}N''(\chi_0))\left(\delta\chi_{\rm S}(\mathbf{x}) + \delta\chi_{\rm L}(\mathbf{x})\right)^2 \\
&\equiv \left(\zeta_{\rm S}(\mathbf{x}) + \zeta_{\rm L}(\mathbf{x})\right) + \frac{3}{5}f_{\rm NL}(k)\left(\zeta_{\rm S}(\mathbf{x}) + \zeta_{\rm L}(\mathbf{x})\right)^2
\end{aligned}
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where $\zeta_{\rm S} \equiv N'(\chi_0)\delta\chi_{\rm S}(\mathbf{x})$ and $\zeta_{\rm L} \equiv N'(\chi_0)\delta\chi_{\rm L}(\mathbf{x})$, and $f_{\rm NL}(k)$ is the reduced bispectrum of ζ in the equilateral configuration

Byrnes/Nurmi/Tasinato/Wands 0911.2780.

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$$
\frac{6}{5}f_{\rm NL}(k)\zeta_{\rm L}(\mathbf{x}) = -C(k) + A(k)\hat{\mathbf{p}}\cdot\mathbf{x}/x_{\rm ls} + \cdots
$$

Generating $f_{\rm NL}(k)$

$$\left|\frac{df_{\rm NL}(k)}{d\ln k}\right| = \frac{5}{6}\sqrt{\frac{r}{8}}\frac{M_{\rm P}|V'''|}{3H^2}$$

where r is the tensor fraction Byrnes *et. al.* 1007.4277.

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Need $f_{\rm NL}(k)$ approximating step function, requires approximate step function for V''.

Grischuk-Zeldovich constraint

Let's assume $\mathcal{P}_{\zeta_{\rm L}}(k) = P^2\delta(\ln k - \ln k_{\rm L})$ and assume we live at a typical location.

$$|C(k)| \simeq \frac{6}{5} P |f_{\rm NL}(k)|$$

$$\left| \frac{A(k)}{3.7 f_{\rm NL}(k)} \right| \simeq P \frac{k_{\rm L}}{aH}$$

$$C_2^{\rm GZ} \simeq 0.21 \left(\frac{k_{\rm L}}{aH} \right)^4 P^2 \qquad \text{(GZ effect)}$$

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Need $\sqrt{C_2^\mathrm{GZ}} \lesssim \sqrt{\frac{1}{5}\sum |a_{2m}|^2} = 6.5\times 10^{-6}$. Using the step function for $A(k)$ and $C(k)$ this requires $|f_\mathrm{NL}(k)| \gtrsim 170$ at $k^{-1} > x_\mathrm{ls}/60$.

GZ constraint for open universe

There is an open universe model that can generate ζ_L with $k_{\rm L}^{-1}$ bigger than the curvature scale Linde/Mezhlumian astro-ph/9506017; Yamamoto/Sasaki/Tanaka astro-ph/9605103; Sasaki/Tanaka astro-ph/9605104. This has been used to implement the present model Liddle/Cortes 1306.5698.

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The GZ constraint is now DHL 1405.3562 $|f_{\mathrm{NL}}(k)| \gtrsim 48$.

Erickeck-Kamionkowski-Carroll constraint

A different contribution to the quadrople comes Erickeck/Kamionkowski/Carroll 0806.0377 from

$$\zeta_{\rm EKC} = \frac{3}{5} f_{\rm NL}(k) \zeta_{\rm L}^2 = \frac{5}{12} \frac{\left(-C(k) + A(k) \hat{\mathbf{p}} \cdot \mathbf{x}/x_{\rm ls}\right)^2}{f_{\rm NL}(k)}$$

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Weaker than GZ constraint for flat universe but a bit stronger for open universe.

Conclusion

Plausible prediction $|f_{\rm NL}| \gtrsim 100$ at $\ell \lesssim 60$.

Let's see if this is visible in existing CMB data