The Hierarchy Problem
&
Compositeness

Riccardo Rattazzi
The Standard Model as an Effective Theory

with fundamental scale \( \Lambda_{UV}^2 \gg 1 \text{ TeV} \)
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\[
\mathcal{L}_{SM} = \mathcal{L}_{kin} + g A_\mu \bar{F} \gamma_\mu F + Y_{ij} \bar{F}_i H F_j + \lambda (H^\dagger H)^2
\]
The Standard Model as an Effective Theory
with fundamental scale $\Lambda_{UV}^2 \gg 1\, \text{TeV}$

$$\mathcal{L}_{SM} = \mathcal{L}_{\text{kin}} + g A_\mu \bar{F} \gamma_\mu F + Y_{ij} \bar{F}_i H F_j + \lambda (H^\dagger H)^2$$

$$+ \frac{b_{ij}}{\Lambda_{UV}} L_i L_j H H$$

$$+ \frac{c_{ijkl}}{\Lambda_{UV}^2} \bar{F}_i F_j \bar{F}_k F_\ell + \frac{c_{ij}}{\Lambda_{UV}} \bar{F}_i \sigma_{\mu\nu} F_j G^{\mu\nu} + \ldots$$

$$+ \ldots$$

$d=4$

$d>4$
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$\Lambda_{UV} \rightarrow \infty$ (pointlike limit) nicely accounts for ‘what we see’
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\[+ \ldots\]

\( \Lambda_{UV} \to \infty \) (pointlike limit) nicely accounts for ‘what we see’
\[ \Lambda_{UV} \quad \text{scale invariance} \quad \Lambda_{IR} \]

hierarchy’s naturalness \( \leftrightarrow \) fixed point stability

\( m_H^2 \equiv \text{strongly relevant} \) perturbation
$\Lambda_{UV}$ ———— hierarchy's naturalness $\leftrightarrow$ fixed point stability

$\sim$ scale invariance

$\Lambda_{IR}$ ————

unstable

stable

marginal

$m_H^2 \equiv$ strongly relevant perturbation

$|T - T_c| \ll T$
$\Lambda_{UV}$

$\sim$ scale invariance

$\Lambda_{IR}$

hierarchy’s naturalness $\iff$ fixed point stability

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picture courtesy of V. Rychkov who stole it anyway
Hierarchy see-saw

Standard Model up to some $\Lambda_{UV}^2 \gg 1 \text{ TeV}$
Hierarchy see-saw

Standard Model up to some $\Lambda_{UV}^2 \gg 1\,\text{TeV}$
Hierachy see-saw

Standard Model up to some \( \Lambda_{UV}^2 \gg 1 \text{ TeV} \)

\[ \Lambda_{UV}^2 \ H^\dagger \ H \]

\[ y_{ij} \ H \bar{F}_i F_j \]

\[ m_H^2 = \epsilon \Lambda_{UV}^2 \ll \Lambda_{UV}^2 \]

Tuning!
Natural SM: \[ \Lambda_{UV}^2 \lesssim 1 \text{ TeV} \]
Un-natural SM

The Higgs boson really is *The SM Higgs boson*
Flavor and approx B & L are theoretically appealing
... and experimentally boring

How would one understand the apparent tuning?

• Anthropic selection
• There is more than Effective Field Theory
• Intelligent design
Un-natural SM

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Supersymmetry

\[ m^2 H^\dagger H \]

\[ m^2 = 0 \quad \text{enhanced UV symmetry} \]

\[ m^2 \neq 0 \]

new states at

\[ g_W v_F \sim 100 \text{ GeV} \]

Compositeness

\[ \dim (H^\dagger H) \geq 4 \]

\[ v_F \neq 0 \]

new states at

\[ 4\pi v_F \sim 2 \text{ TeV} \]

In both cases, must use ingenuity to satisfy flavor and EW constraints
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Collider data display no signal of compositeness for quarks, leptons and *transversely polarized* vector bosons.

**Plausible scenario**

\[ E_{\text{TeV}} \]

\[ \Lambda_{\text{UV}} \]

\[ q, \ell, \gamma, W_T, Z_T, g \]

\[ W^\pm_L, Z^0_L, \ldots \]
Need $\Lambda_{UV} \gg \text{TeV}$ to filter out unwanted effects and produce a realistic Flavor story.

Scale (conformal) invariant theories are thus an essential ingredient of model building.
80’s: asymptotically free gauge theory (Technicolor)

2000’s: Holographic CFTs (Randall-Sundrum)

‘Abstract CFT’ characterized by
- symmetry
- operator content

$\Lambda_{UV}$

• Higgs’ dynamics

$\text{TeV}$
The structure underlying the Higgs sector

\[ H = \begin{pmatrix} h_1 + ih_2 \\ h + ih_3 \end{pmatrix} \]

must be a pseudo-Golstone multiplet

\[ W^\pm, Z, h \]

Georgi, Kaplan '84
Banks '84

Arkani-Hamed, Cohen, Katz, Nelson '02
Agashe, Contino, Pomarol '04
Simplest options compatible with $\rho \simeq 1$

$SO(5)/SO(4)$ \quad \rightarrow \quad H$

$SO(6)/SO(5)$ \quad \rightarrow \quad H \oplus \eta$

$SO(6)/SO(4) \times U(1)$ \quad \rightarrow \quad H_1 \oplus H_2$

$SO(4,1)/SO(4)$ \quad \rightarrow \quad H$
Composite sector is *broadly* described by:

**Giudice, Grojean, Pomarol, RR, 2007**

✦ one mass scale $m_\rho$  (of order TeV)

✦ one coupling $g_\rho$

✦ decay constant $f \sim \frac{m_\rho}{g_\rho}$

\[ g_\rho \sim g_{KK} \quad g_\rho \sim \frac{4\pi}{\sqrt{N}} \]

\[ = g_\rho \bar{\Psi} \Psi \Phi \quad \times \quad = \frac{g_\rho^2}{m_\rho^2} \bar{\Psi} \Psi \bar{\Phi} \Psi \]

Thursday, January 10, 2013
Origin of Higgs potential

strongly coupled Higgs sector

SM matter & gauge fields

global symmetry weakly broken

\[ V(h) = \frac{m_{\rho}^4}{g_{\rho}^2} \times \frac{\lambda_t^2}{16\pi^2} \times F(h/f) \]

\[ \sim \frac{\lambda_t^2}{16\pi^2} m_{\rho}^2 H^\dagger H + \frac{g_{\rho}^2}{16\pi^2} \lambda_t^2 (H^\dagger H)^2 + \cdots \]
Origin of Higgs potential

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SM matter & gauge fields

"Yukawa"

gauge

global symmetry weakly broken

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Vacuum dynamics

unbroken $SU(2)_L \times U(1)$

$\begin{array}{c}
v = 0
\end{array}$

maximally broken

$\begin{array}{c}
v = f
\end{array}$

$V \propto F(h/f)$ generically $\frac{v^2}{f^2} = O(1)$

EW precision observables

$\mathcal{O} \sim \frac{v^2}{f^2} \times \mathcal{O}_{\text{TechniColor}}$
simply tune \( \frac{v^2}{f^2} \sim 0.05 \div 0.1 \)

be clever: Little Higgs

\[ V \propto F(h/f) \quad \text{generically} \quad \frac{v^2}{f^2} = O(1) \]

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Vacuum dynamics

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EW precision observables

\[ \mathcal{O} \sim \frac{v^2}{f^2} \times \mathcal{O}_{\text{TechniColor}} \ll \mathcal{O}_{\text{TechniColor}} \]
Two Ways to Flavor

Bilinear: ETC, conformalTC
Dimopoulos, Susskind
Holdom
....
Luty, Okui

Linear: partial compositeness
D.B. Kaplan
....
Huber
RS with bulk fermions
Yukawas are *irrelevant* $m_t$ hard to produce

$$\dim (\bar{q}qH) = 4 + (d_H - 1) > 4$$

question: how close can I push $d_H \rightarrow 1$
without getting back the hierarchy problem?

$$\dim (H^\dagger H)$$

Rattazzi, Rychkov, Tonni, Vichi ’08
Poland, Simmons-Duffin, Vichi ’11
Yukawas are irrelevant. $m_t$ hard to produce.

$$\dim(H) \equiv d_H > 1$$

$$\dim(\bar{q}qH) = 4 + (d_H - 1) > 4$$

Question: how close can I push $d_H \to 1$ without getting back the hierarchy problem?
Flavor from partial compositeness

\[ \mathcal{L}_{Yukawa} = \epsilon^i_q q^i_L \Psi^i_q + \epsilon^i_u u^i_L \Psi^i_u + \epsilon^i_d d^i_L \Psi^i_d \]

\[ Y_u^{ij} \sim \epsilon^i_q \epsilon^j_u g_* \]

\[ Y_d^{ij} \sim \epsilon^i_q \epsilon^j_d g_* \]

\( \Psi \) = composite with dimension \( \sim \frac{5}{2} \)

\( \epsilon^i_q, \epsilon^i_u, \epsilon^i_d \) = dimensionless

- Hypothesis seems a bit wishful to me, but I see no other option
- Problems of minimal technicolor greatly alleviated, but not eliminated
Flavor transitions controlled by selection rules

$\Delta F=2$

\[ \epsilon_q^i \epsilon_d^j \epsilon_q^k \epsilon_d^\ell \times \frac{g_\rho^2}{m_\rho^2} (\bar{q}^i \gamma^\mu d^j)(\bar{q}^l \gamma_\mu d^l) \]

$\Delta F=1$

\[ \epsilon_q^i \epsilon_u^j g_\rho \times \frac{v}{m_\rho^2} \times \frac{g_\rho^2}{16\pi^2} \bar{q}^i \sigma_{\mu\nu} u^j G_{\mu\nu} \]
### Bounds & an intriguing hint

**Keren-Zur, Lodone, Nardecchia, Pappadopulo, RR, Vecchi '12**

<table>
<thead>
<tr>
<th>$\epsilon_k$</th>
<th>$m_\rho \gtrsim 10$ TeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\epsilon'/\epsilon, \ b \to s\gamma$</td>
<td>$m_\rho \gtrsim \frac{g_\rho}{4\pi} \times (10 - 15)$ TeV</td>
</tr>
<tr>
<td>$d_n$</td>
<td>$m_\rho \gtrsim \frac{g_\rho}{4\pi} \times (20 - 40)$ TeV</td>
</tr>
</tbody>
</table>

**CP violation in D decays**

$$\Delta a_{CP} = a_{KK} - a_{\pi\pi} = -(0.67 \pm 0.16)\%$$

$$m_\rho \simeq \frac{g_\rho}{4\pi} \times 10$$ TeV

- **connection with weak scale not perfect**
- **Not crazy at all to see deviation in D’s first !**
- **d$_n$ should be next**
\[ \mu \to e\gamma \quad \frac{\sqrt{m_\mu m_e}}{m_\rho^2} \bar{\mu} \sigma_{\alpha\beta} e F^{\alpha\beta} \]

MEG: \( \text{Br}(\mu \to e\gamma) < 2.4 \times 10^{-12} \quad m_\rho \gtrsim 150 \text{ TeV} \)

Partial compositeness clearly cannot be the full story
Must assume strong sector possesses some flavor symmetry

Range of possibilities

\[ U(1)_e \times U(1)_\mu \times (1)_\tau \]

\[ \cdots \cdots \]

\[ SU(3) \times SU(3) \times \cdots \]

Redi, Weiler ’11
Barbieri et al. ’12
Higgs potential, fine-tuning and expectations at the LHC
Higgs’s mass versus top-partners’

\[ V(h) = O(\lambda^2_L) + O(\lambda^2_R) \]

\[ \lambda_L \lambda_R \sim \lambda_t g_T \]

**best option**

\[ t_R \] is fully composite SO(5) singlet

\[ \lambda_L \sim \lambda_t \]

\[ \lambda_R \sim g_T \]

\[ V(h) = \frac{m_T^4}{g_T^2} \times \frac{\lambda^2_t}{16\pi^2} \times F(h/f) \]
Higgs’s mass versus top-partners’

\[ V(h) = O(\lambda_L^2) + O(\lambda_R^2) + \ldots \]

\[ \lambda_L \lambda_R \sim \lambda_t g_T \]

best option for \( t_R \) is fully composite \( \text{SO}(5) \) singlet

\[ \lambda_L \sim \lambda_t \]
\[ \lambda_R \sim g_T \]

\[ V(h) = \frac{m_T^4}{g_T^2} \times \frac{\lambda_t^2}{16\pi^2} \times F(h/f) \]

Mrazek et al, ‘11
Pomarol, Riva ‘12
\[ V = \frac{3\lambda_t^2 m_T^2}{16\pi^2} \left( ah^2 + bh^4/f^2 + \ldots \right) \]
$$V = \frac{3\lambda_t^2 m_T^2}{16\pi^2} \left( a h^2 + b h^4 / f^2 + \ldots \right)$$
\[ V = \frac{3\lambda_t^2 m_T^2}{16\pi^2} \left( a h^2 + b h^4 / f^2 + \ldots \right) \]

Total tuning $\sim$ area

\[ \left( \frac{430 \text{ GeV}}{m_T} \right)^2 \times \frac{4}{g_T^2} \]
The main test of naturalness is the search for fermionic top partners

but how to proceed? given we do not have in our hand
a truly compelling and calculable model

how to help our experimental colleagues to express the
results of their searches in the light of more interesting
scenarios than, say, a fourth family
Simplified Model Ideology

*in the end model builders mistrust full fledged models*

De Simone, Matsedonkyi, RR, Wulzer ’12
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De Simone, Matsedomkyi, RR, Wulzer ’12

Combining

• choice for $\psi$ quantum numbers
• hypothesis on UV origin of $\lambda_t$
• symmetry, selection rules, power counting

→ a handful of models

→ a handful of parameters

Focus on on just $\Psi$
Production

Decay

Ex.: $\Psi_{5/3} \in \left(\frac{1}{2}, \frac{1}{2}\right)$

Contino, Servant 2008
\[ \Psi_{5/3} \text{ et } \Psi_{-1/3} \in \left( \frac{1}{2}, \frac{1}{2} \right) \] constrained by 4th family search \( b' \rightarrow Wt \)
same sign dileptons (trileptons) + b + 3 (2) jets

\[ \lambda = 3 \]
\[ \lambda = 0.3 \]

\[ M_{5/3} \]

\[ C_1 \]

\[ \xi \equiv \frac{\nu^2}{f^2} = 0.2 \]

At 14 TeV with ab\(^{-1}\), one expects a reach of up to 2 TeV mass

\[ \text{Mrazek, Wulzer '10} \]
Effective Lagrangian for a composite light Higgs boson
(+ pseudo-Goldstone hypothesis)

Giudice, Grojean, Pomarol, RR, 2007

Two leading operators in effective lagrangian
\[ \# \frac{1}{2 f^2} \partial_\mu |H|^2 \partial^\mu |H|^2 \]

\[ \# \frac{y_i}{f^2} (\bar{f}_i f_i H) |H|^2 \]

\[ a \times \frac{2 m^2_V}{v} \approx 1 - \frac{1}{2} \frac{v^2}{f^2} < 1 \]

robust consequence of coset structure

\[ c_i \times \frac{m_i}{v} \approx 1 + O\left(\frac{v^2}{f^2}\right) < 1 \]

generic but not a theorem

\[ b \times \frac{m^2_V}{v^2} \approx 1 - 2 \frac{v^2}{f^2} \]

\[ \propto \frac{m_i}{f^2} \]

New!
\[
\Delta \epsilon_1 = -\frac{3g^2 \tan \theta_w^2}{32\pi^2} (1 - a^2) \ln(m_\rho/m_h)
\]

\[
\Delta \epsilon_3 = \frac{g^2}{96\pi^2} (1 - a^2) \ln(m_\rho/m_h)
\]

Barbieri, Bellazzini, Rychkov, Varagnolo 07

\[
v^2 \sim 0.1 \div 0.2
\]

is tolerable

\[
\Delta \epsilon_1 \gtrsim 10^{-3}
\]

\[
M_\Psi < 1 \text{ TeV}
\]
\[ \Delta \epsilon_1 = -\frac{3g^2 \tan \theta_W^2}{32\pi^2} (1 - a^2) \ln(m_\rho/m_h) \]

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Barbieri, Bellazzini, Rychkov, Varagnolo 07

\[ \frac{v^2}{f^2} = 1 - a^2 \]

\[ \frac{v^2}{f^2} \sim 0.1 \div 0.2 \]

is tolerable

\[ \Delta \epsilon_1 \gtrsim 10^{-3} \]

\[ M_\Psi < 1 \text{ TeV} \]
vector resonances

\[ W \lambda \rho \lambda W \]

\[ \Delta \epsilon_3 \sim \frac{m_W^2}{m_\rho^2} \]

\[ \Delta \epsilon_3 < 10^{-3} \quad m_\rho > 3 \text{ TeV} \]

time not ripe to search for them at LHC
• overall reduction of fermion couplings is mildly favored over SM
• composite Higgs scenario not unfit
• 2HDM definitely unfit: either \( c_t > 1, c_b < 1 \) or \( c_t < 1, c_b > 1 \)
Alternative lagrangian, associated with new light & weakly coupled states

\[ \mathcal{L}_{\text{eff}} = c_\gamma \frac{\alpha_s}{4\pi} \frac{h}{v} G_{\mu\nu} G^{\mu\nu} + c_\gamma \frac{\alpha}{4\pi} \frac{h}{v} F_{\mu\nu} F^{\mu\nu} \]
Future perspective on Higgs couplings
LHC $\sim 3 \text{ ab}^{-1}$: Single Higgs production

$\Delta a = 3\%$

$\Delta c_f = 5\%$

$\frac{v^2}{f^2} \sim 0.05$

LHC $\sim 1 \text{ ab}^{-1}$: Double Higgs production

$\frac{v^2}{f^2} \sim 0.1$

Grober, Muhlleitner 2010
Contino, Ghezzi, Moretti, Panico, Piccinini, Wulzer 2012

Linear Collider $\sim 1 \text{ ab}^{-1}$: Single Higgs production

$\frac{v^2}{f^2} \sim 0.01$
CLIC and beyond the Infinite
Composite $h$ fails to fully unitarize VV scattering

\[
\mathcal{A}(VV \rightarrow VV) = \frac{s}{v^2} (1 - a^2) \quad \text{Goldstone} \quad \Rightarrow \frac{s}{f^2}
\]

\[
\mathcal{A}(VV \rightarrow hh) = \frac{s}{v^2} (a^2 - b) \quad \Rightarrow \frac{s}{f^2}
\]

LHC ab\(^{-1}\): sensitive to $\frac{v^2}{f^2} \gtrsim 0.3$

CLIC at 3TeV and 1 ab\(^{-1}\): $\frac{v^2}{f^2} \gtrsim 0.01$

Contino, Grojean, Pappadopulo, RR, Thamm in preparation
for \( \frac{v^2}{f^2} \gtrsim 0.1 \) can test coset structure by considering

\[
\begin{array}{c}
\text{symmetric coset} \\
\text{homogeneous space}
\end{array}
\]

versus

\[
\pi \to -\pi \quad \text{unbroken around any point}
\]

Processes with odd number of legs are suppressed

Notice: in effective lagrangian non trivial correlation of dim 6 and 8 operator coefficients
Precision versus energy frontier

indirect and semi-direct

sensitive to

\[ \frac{1}{f^2} \sim \frac{g^2}{m^2_\rho} \]

HL-LHC  ILC

5%  1%

factor 2 in \( f \)

direct

HL-LHC  ILC

13 TeV  33 TeV

factor 2 in \( m_\rho \)
The most amazing thing about the Standard Model Higgs boson

Being Elementary and Lonely

I

• Baryon & Lepton number conservation
• Natural Flavor Conservation (CKM)
• custodial symmetry

{ emergent accidents of effective theory

II

the apparent tuning of its mass parameter
Higgs mass  \hspace{1cm} \text{Yukawas} \hspace{1cm} \text{unwanted effects}
Flavor demands a rich spectrum of composite fermion operators, with non trivial quantum numbers under color, electroweak and flavor symmetry.

In UV \( \dim(\Psi) \sim 5/2 \), mimicking \( \dim(H_q) \sim 5/2 \) in SM.

Bosonic resonances: EW precision data want them heavy and strongly coupled. \( m_\rho > 3 \text{ TeV} \) with \( g_\rho \) as big as possible.

Fermionic top partners: Naturalness wants them below 1 TeV, while \( m_h = 125 \) wants them more weakly coupled.

If the scenario of Composite Higgs is realized in Nature it rather clear the underlying theory must be significantly more complex than a generic rescaled version of QCD!!
Scherzo

Gauge Invariance and Mass
Higgsless Standard Model in AdS$_4$

- AdS has 2+1 boundary which is reached in finite coordinate time by lightlike geodesics

\[ ds^2 = \frac{L^2}{z^2} ( -dt^2 + dx^2 + dy^2 + dz^2 ) \]

- eaten Goldstone is electron-antielectron two particle state
- theory weakly coupled
- no Higgs boson

\[ m_W^2 \propto \frac{\alpha}{4\pi} \frac{1}{L^2} \]

Rattazzi, Redi 09